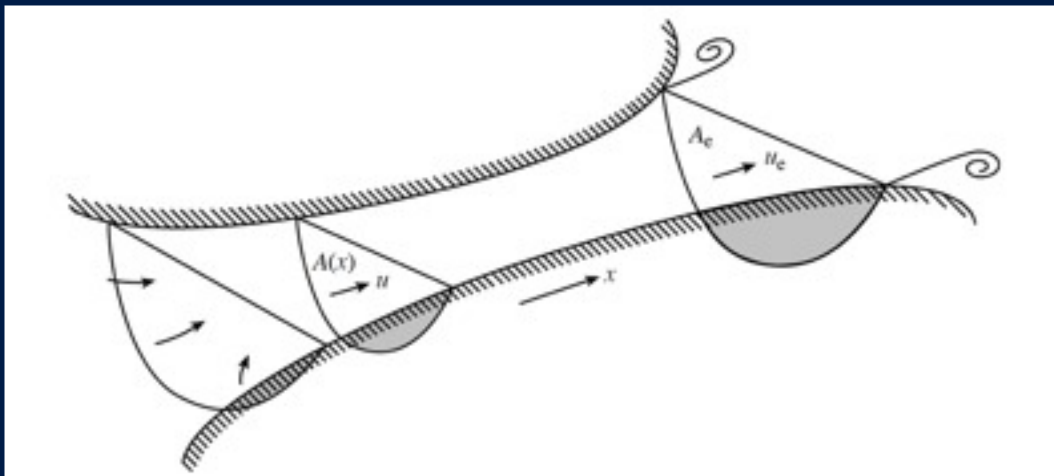
TIDAL ENERGY RESEARCH GROUP
DEPARTMENT OF ENGINEERING SCIENCE

On the Garrett & Cummins limit

Tom Adcock



Friday 11th week, Hilary Term, 2012

The most basic tidal channel

- Channel connecting two oceans
- Sinusoidal head difference
- Channel length << tidal wavelength — volume flow rate is constant along length
- Drag is proportional to Q^2
    - Natural drag in channel
    - Turbine drag
- Classic paper analysing this by Garrett & Cummins (2005) in Proc. R. Soc. A

Is the channel length really much shorter than the wavelength?

- The naturally occurring tide will have a much longer wavelength than most channels
- In this analysis we are going to use a time-varying resistance which will generate higher harmonics and hence shorter wavelengths
- Write

$$u\,|u| = U^2\left(\frac{8}{3\pi}\sin(\omega t) + \frac{8}{15\pi}\sin(3\omega t)\right) + \dots \qquad k(t) = k_0 + k_2\sin(2\omega t + \phi_2) + k_4\sin(4\omega t + \phi_4) + \dots$$

- These will be multiplied together — hence higher harmonics
- I will explore this further numerically when I next have a free weekend

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Equations (after non-dimensionalisation)

Inertia

Driving head

Natural friction

Turbine drag

$$\frac{dQ}{dt} = \cos(t) - (\lambda_0 + \lambda_1)\, Q\, |Q|$$

$$P = \frac{1}{2\pi} \int_0^{2\pi} \lambda_1 Q^2 \,|Q|\, \mathrm{d}t$$

Power averaged over tidal cycle

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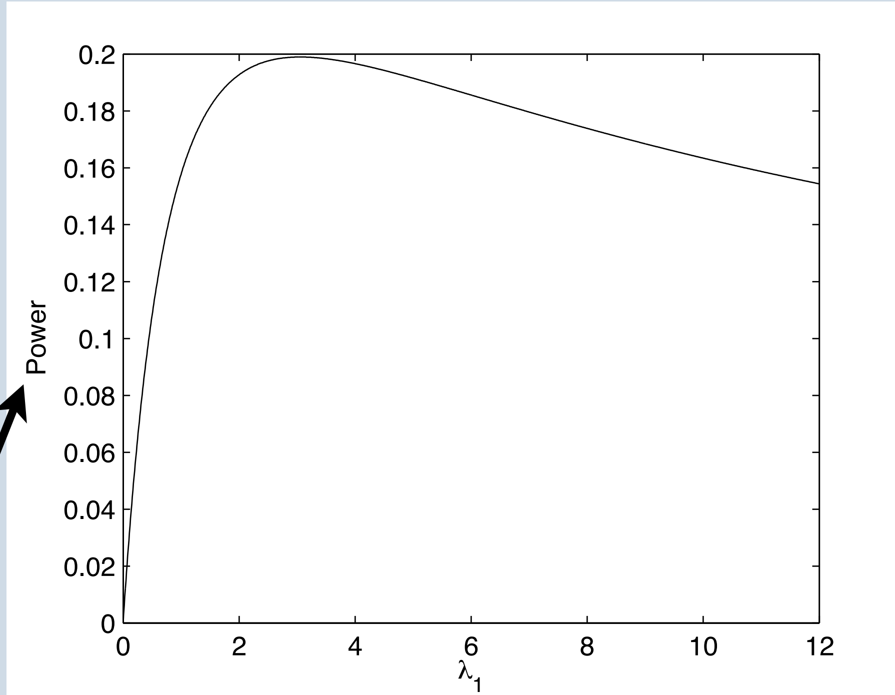
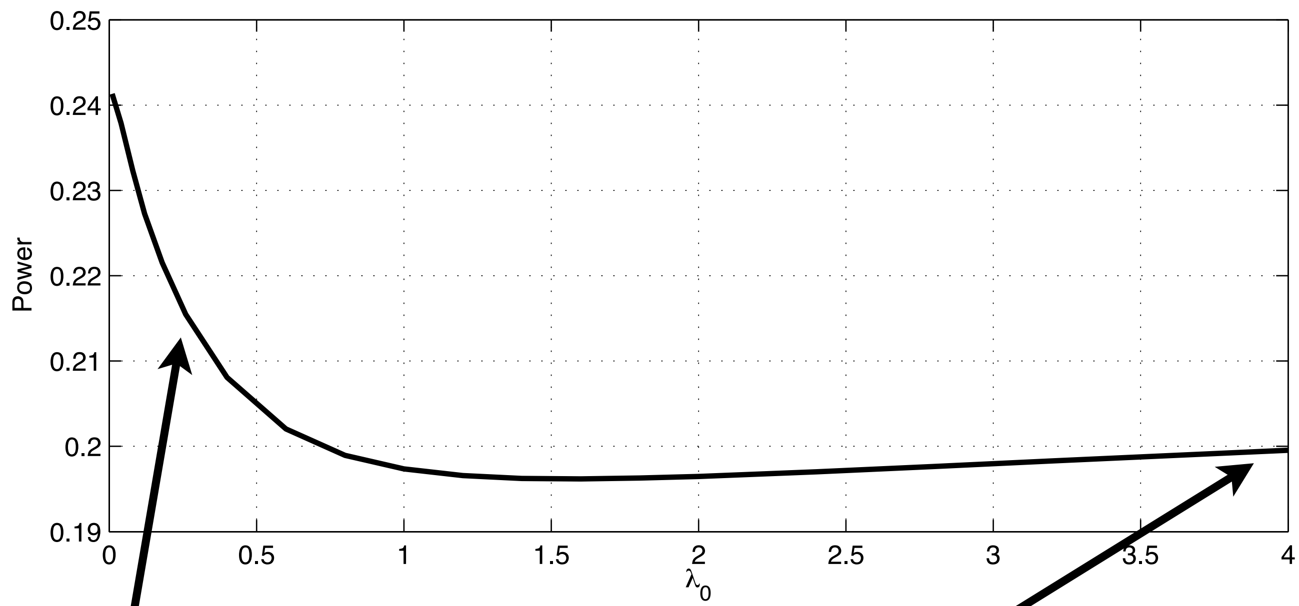
Finding the optimum turbine drag

Example: λ_0=0.8



Power normalised by $\rho g a Q_{nat}$

Garrett & Cummins limit



Inertia dominated

Friction dominated (Quasi-steady)

Why should λ_1 not vary in time?

- Define λ_1 at N points over the half tidal cycle
- Linearly interpolate between these
- Use optimisation routine to find form of λ_1 which maximises power output

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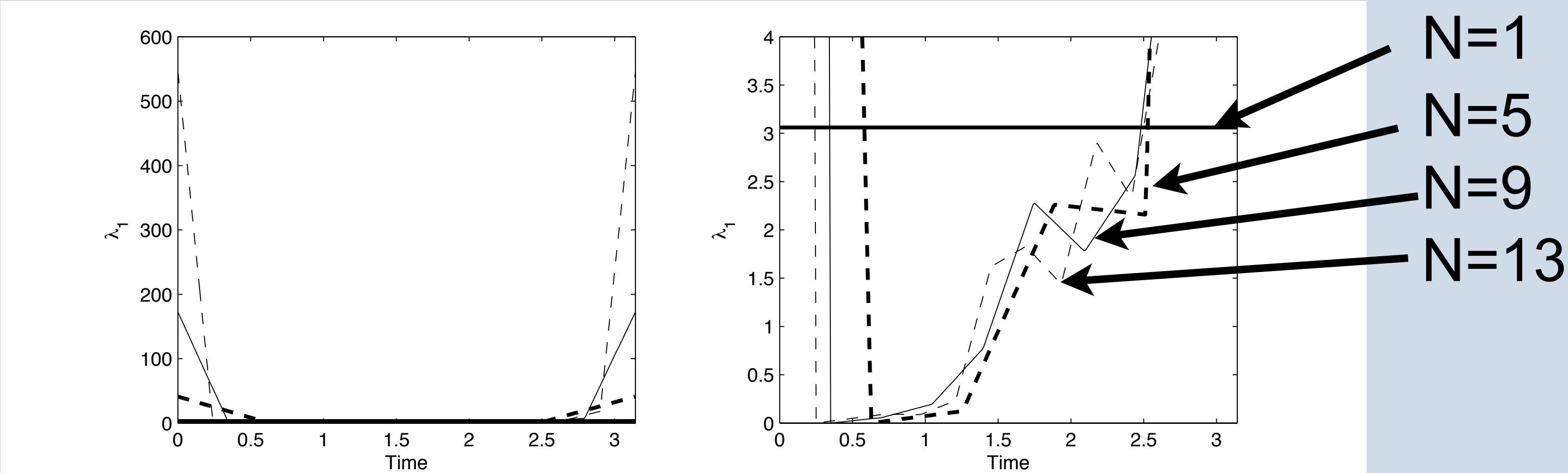
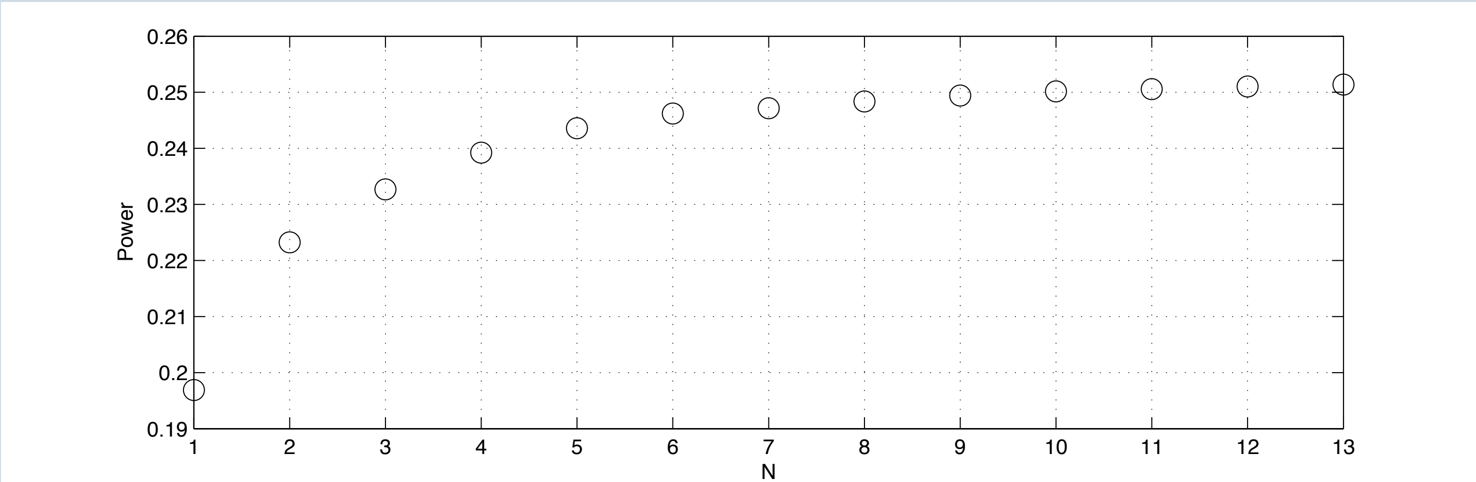
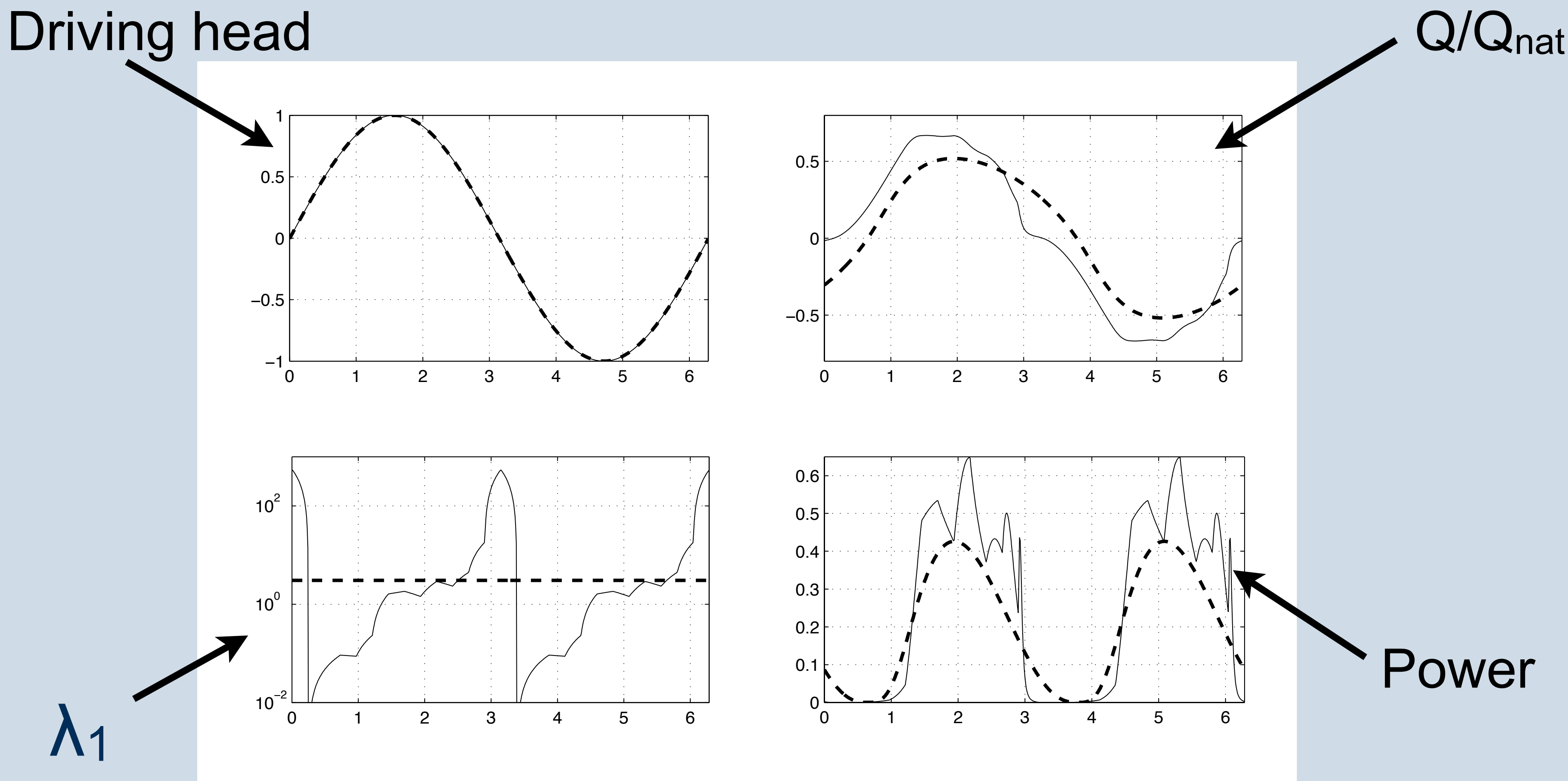
Example λ_1=0.8 — convergence

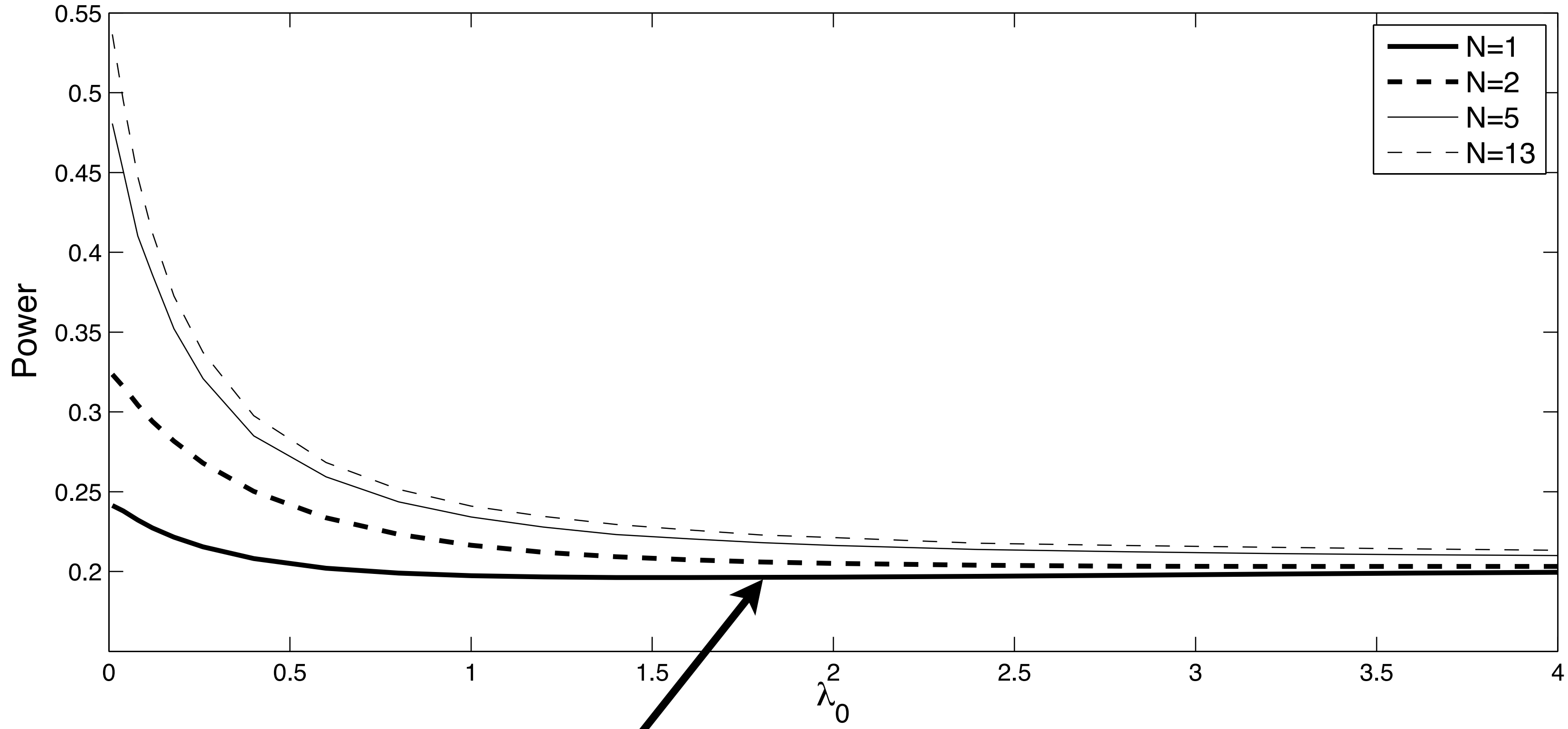




Example λ_1=0.8 — tidal dynamics



Power extraction



Garrett & Cummins

More realistic turbines

- Represent turbines using actuator disc theory
  - Use small Froude number theory as keeps analysis simple
  - Very close to results with finite Froude number model
- Turbine Ct and Cp a function of blockage, B, and wake induction factor, α_4
- Choose a blockage and find value of α_4 which maximises power available at the turbine
- Constant α_4 analysis by Vennel (2010) in JFM

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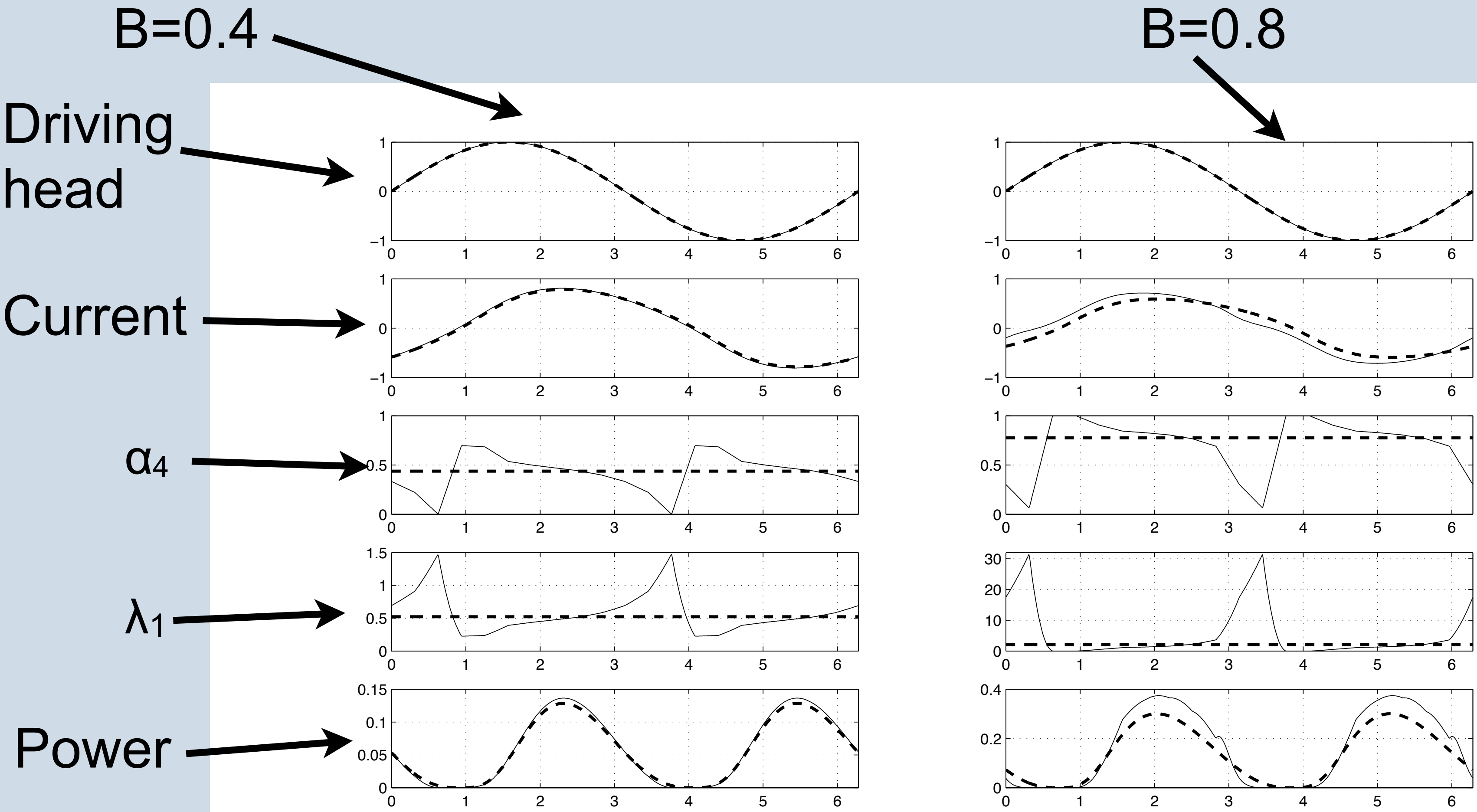
Time varying wake induction factor

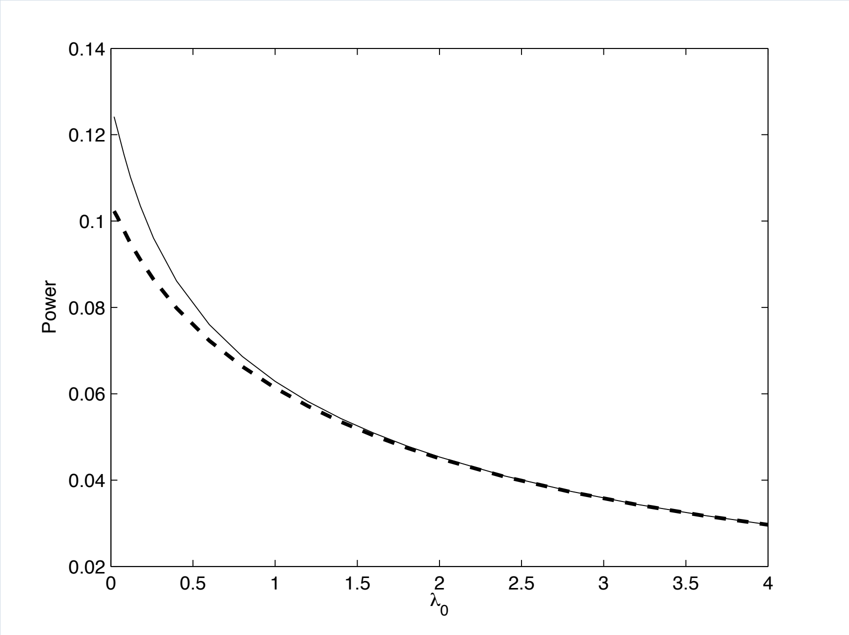
- Define λ_1 at N points over the half tidal cycle
- Linearly interpolate between these
- Use optimisation routine to find form of λ_1 which maximises power output

Tidal dynamics in channel at maximum power

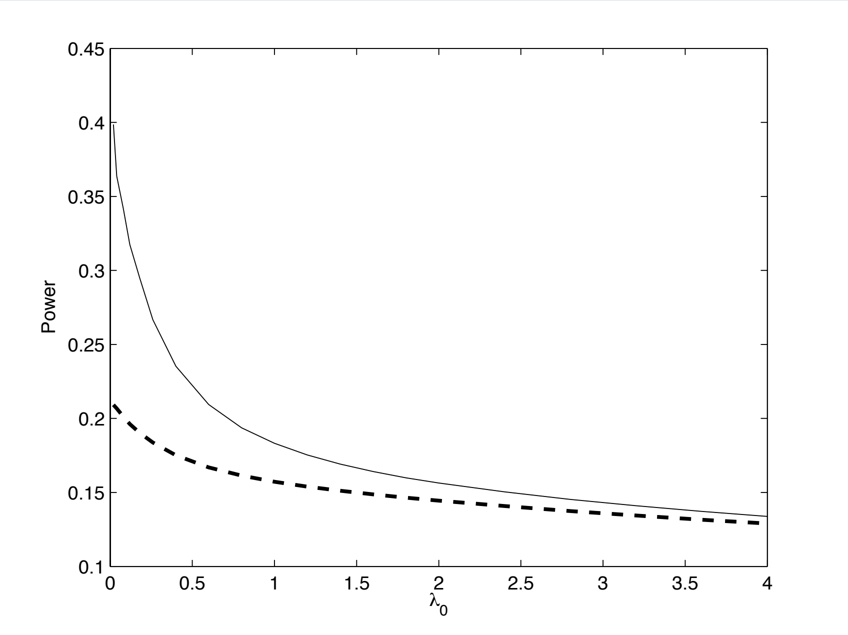


Power extraction — 1 row of turbines

B=0.4



B=0.8



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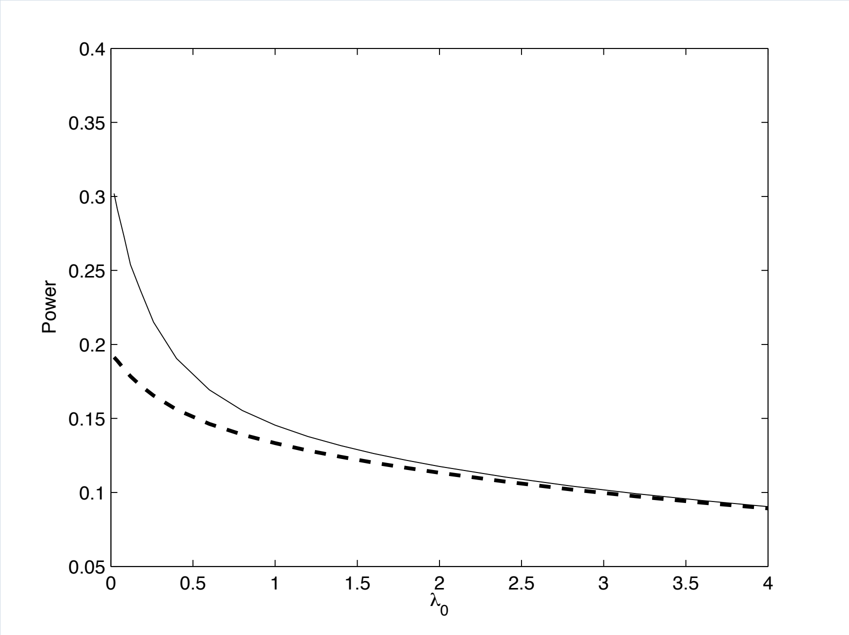
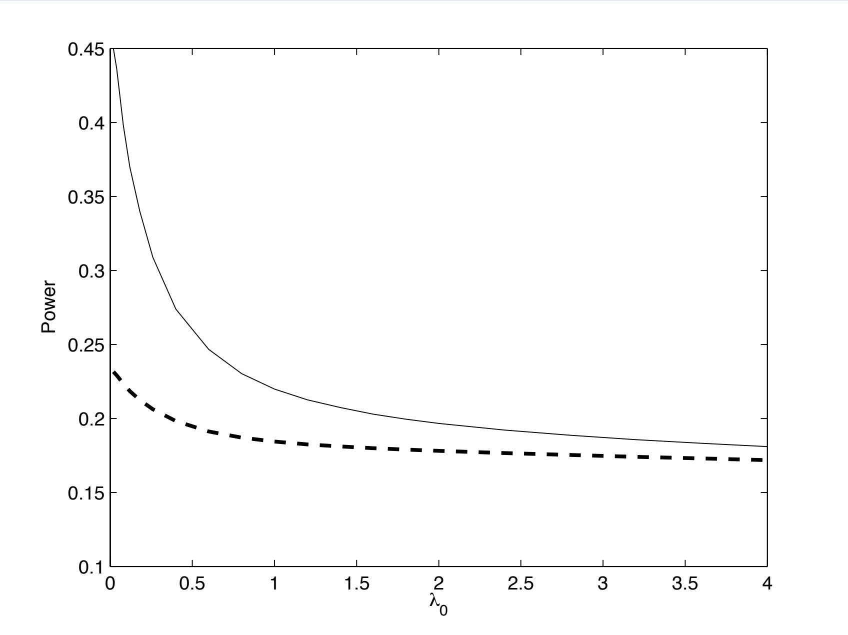
Power extraction — 5 rows of turbines

B=0.4



B=0.8



Conclusions

- For low friction/high inertia channels it is possible to extract more energy than the value given by Garrett & Cummins (2005).
- The analysis of Vennel (2010) also (slightly) underestimates the energy which may be extracted used actuator discs.
- No real turbine is going to be able to produce the thrust needed by this analysis.
- Small improvement in energy output may be possible