The power potential of a tidal turbine array with turbine power capping

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Summary: The fluctuating nature of the tidal resource means that it is not economical to design turbines to extract the maximum power during peak tide, so turbines will be limited to a more economical rated power. The impacts of such power capping have thus far not been well understood. This paper combines Blade Element Momentum theory with Linear Momentum Actuator Disc Theory derived for tidal turbines to determine the effect of power capping on the power of a tidal turbine array. It is found that the effect of increasing blockage, the ratio of the swept rotor area to the cross-sectional area of the surrounding flow passage, is to cause the turbine to spin more slowly and reach power capping at a lower upstream flow speed than unblocked turbines.

Introduction

It is recognised that significantly harnessing the global tidal resource will require the deployment of many turbines in an array configuration; one estimate for extracting 1.9GW from the Pentland Firth in the UK requires thousands of turbines [1]. Analytical and numerical methods have been developed to model the large arrays of turbines required to extract this power. One key result is that there is a scale separation between array-scale flow events and turbine-scale flow events [2, 3]. Analytical models provide a useful basis to understand the dynamics of the flow around an array of turbines, but they are restricted to idealised parameters to represent the turbines.

Some of these short-comings can be overcome with numerical techniques, such as depth-averaged simulations, to capture more complex flow features; multi-scale flow, shear, etc. Although numerical simulations offer an improved method for determining the power potential of a turbine array, correctly representing the turbines to capture the core and bypass flows requires careful implementation. One method of implementing this is to use a sub-grid scale actuator disc model [4, 5].

Although depth-averaged simulations provide a means to determine an upper bound on the extractable power of a turbine array, it is often (as in actuator disc type studies) assumed power extraction is proportional to the cube of the velocity. This may be reasonable at lower flow speeds, but becomes unrealistic at higher flow speeds as turbines will be limited to a maximum rated power, due to constraints such as generator capacity. Capping at higher flow speeds means that the turbine characteristic departs from the cubic relationship with flow velocity, and thus the flow dynamics around the turbines and the array will be altered. This paper introduces a power capping methodology into the depth-averaged simulation framework, allowing real turbine data to be used to parameterise the turbines and providing a more realistic upper bound to the extractable power potential of a turbine array.

Methods

Figure 1 shows a plan view of a long turbine array partially spanning a wide channel. There are two scales of flow; the array-scale flow, which scales on the size of the array and evolves slowly, and the turbine-scale flow, which scales on the size of the turbine and evolves much more quickly. The flow velocity through the array, u_a, is the upstream velocity boundary condition to the turbine-scale problem. The thrust applied by the turbine to the flow is a function of the velocity through the device, u_d, and the rated power, P_R. P_R is specified as an operating parameter of the turbine, as is the 'rated velocity', u_r, at which rated power is achieved.

The thrust applied by the turbine on the flow is determined using Blade Element Momentum (BEM) theory, which is coupled with the Linear Momentum Actuator Disc Theory (LMADT) for a turbine in a channel with a rigid lid proposed by Garrett and Cummins [6]. This represents a novel coupling between momentum theory for flow around a turbine and the BEM theory. Critically, it allows blockage, the ratio of the swept area of the turbine rotors to the cross-sectional area of the flow passage surrounding the turbine, which strongly affects power potential of a tidal turbine, as shown by Garrett and Cummins and others, to be accounted for.

Given rotor geometry and blade parameters, lift and drag coefficients, C_l and C_d respectively, the velocity induction factor in turbine wake, relative the far upstream flow speed, $a_4 = 1 - u_4/u_a$ is:

$$a_4 = \frac{\sigma(1-a_2)^2(C_l\cos\phi - C_d\sin\phi) - b_4(2+b_4)\sin^2\varphi}{2(1-a_2)\sin^2\phi}; \quad a_2 = 1 - \frac{u_d}{u_a}; b_4 = 1 - \frac{u_b}{u_a} \qquad (1)$$

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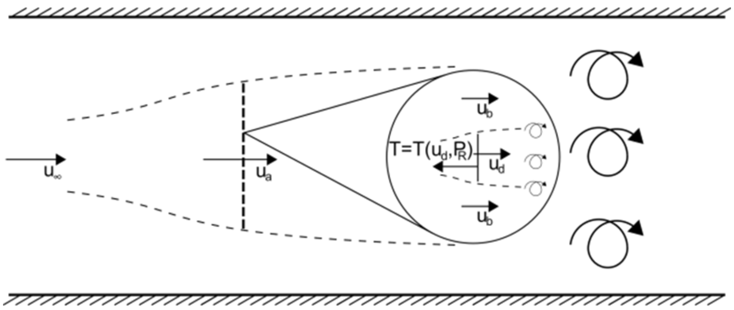

Figure 1: Plan view of a large turbine array, with inset showing flow around individual turbine

where a_2 is the turbine velocity and b_4 the bypass velocity normalised by the upstream velocity, φ is the angle between the plane of rotation and the relative velocity vector, and σ is the solidity of the rotor. If the radius of the blade element is r, the induced rotational velocity a' is calculated as:

$$a' = \frac{\sigma(C_l \sin\phi - C_d \cos\phi)}{4 sin\varphi cos\varphi - \sigma(C_l \sin\phi - C_d \cos\phi)} \tag{2}$$

An iterative solution method is used where an initial value for α_2, α_4, and a' are assumed, based on an unblocked rotor, and the velocity in the axial and tangential directions of the plane of rotation are calculated. The lift and drag forces are calculated based on the lift and drag coefficients for the aerofoil being used, which is then used to update the estimates for α_2, α_4, and a'. This is repeated iteratively until converged. The blocked turbine thrust and power are then calculated and power capping implemented when rated power is reached. The thrust/power relationship is then applied to turbine arrays in shallow water numerical models, which shall be presented in the workshop.

Conclusions

The theory described above has been applied to a turbine with the NREL S809 aerofoil section. It is found that the turbine blockage ratio plays an important role in the dynamics of turbine power capping. One effect of increasing the blockage of a turbine is that the velocity through the turbine is increased relative to that in an unblocked condition. Keeping the thrust of the turbine equal, a higher blockage turbine rotates more slowly than a lower blockage turbine. In addition to this, turbines in higher blockage reach power capping operation at a lower upstream velocity (u_a in figure 1). This may be beneficial to the economics of deploying turbines in arrays of many turbines where the inter-turbine spacing is kept low.

The role of power capping may be particularly important when tidal dynamics are considered in the context of a tidal channel or basin, as power capping will be in effect for only a portion of the tidal cycle, and will play an important role in determining what an economically and technically feasible power capping limit P_R should be set for the turbines. This is the subject of ongoing investigation.

Acknowledgements:

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