Tidal stream energy: designing for blockage

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Summary: The paper presents a new analytic model for the analysis of finite length turbine fences in channels driven by sinusoidally oscillating driving head. Thus the thrust presented by the turbines properly reduces the flow rate through the channel leading to a solution for overall power that is dependent upon blockage as well as channel characteristics. For a given channel, optimum, in terms of power per turbine, fence deployments can then be deduced. The model can be further extended to incorporate realistic turbine power and thrust curves.

Introduction

Significant advances have been made in recent years on the analytic modelling of tidal turbine arrays and farms. It is now well established that tidal turbines do not behave precisely as wind turbines and are subject to higher power extraction limits than the Betz limit due to the flow confinement effects (blockage) provided by the channel and neighbouring turbines. The work of Garrett & Cummins [1] set in place the upper energy extraction limit for a homogeneously arrayed turbine fence that completely spans the width of the channel under the assumption of an undeforming free surface. Later extensions by Whelan et al. [2] and Vennell [3] allowed respectively for the deformation of the free surface and the response of the channel flow rate to the resistance presented by the turbines. Later work by Nishino & Willden [4] extended the model of Garrett & Cummins to consider the practical case of a long but finite length turbine fence partially spanning the width of a channel; hereafter referred to as the NW12 model. This model, working on the basis of scale separation between turbine and array scale flow events, led to the development of an energy extraction limit for a closely packed turbine fence in an infinitely wide channel of 79.8% of the kinetic energy in the undisturbed approach stream, achieved at a local blockage (ratio of turbine to local flow passage area) of 0.4.

Whilst these models have made significant advances they are still a long way short of representing a realistic turbine installation, even in a simplified analytic sense. Whilst the NW12 model allows for finite length fences, and thus incomplete channel usage as will be required for shipping lanes, bathymetric variations etc., it is restricted to a constant flow rate through the channel, i.e. the channel is unresponsive to the resistance presented by the turbines. Further, all of the models discussed above are for actuator disk representations of turbines that are able to extract energy at will, in an optimal fashion, from the flow by presenting it with an ever increasing resistance. Real turbines will be subject to power capping, which changes the relationship between flow speed and turbine thrust. Moreover, whilst the analytic models tell us that higher power limits can be obtained if high thrust can be imposed they say nothing about whether these high thrusts can be achieved in practice.

This paper presents a new analytic model that embeds the multi-scale finite fence model of NW12 in the channel dynamics model of Garret & Cummins [5] (GC05). A further development of this model sees the local turbine actuator disk model replaced by local blockage corrected power and thrust characteristics for a realistic turbine. The resulting model enables a finite length fence of turbines partially spanning a wide head driven channel to be analysed. Finally, we will, in the presentation, address the issue of whether turbines can be designed to present increased thrust and take advantage of the local blocking effects.

Finite Fence Channel Dynamics Model

The one-dimensional channel dynamics model of GC05 can be cast in non-dimensional form:

$$\frac{dQ'}{dt'} - \cos t' = -\frac{1}{2} Q'|Q'| \left(\frac{\sqrt{ag}}{\omega L}\right)^2 \left(B_A C_{TA} + C_f \frac{L}{h}\right) \tag{1}$$

in which the flow is driven through the (here presumed) rectangular cross-section channel, length L, width w, height h, by a sinusoidally varying head difference between the ends of the channel, amplitude a frequency ω. $Q' = Q/Q_0$ is non-dimensional flow rate, in which Q_0 is the peak flow rate in the undisturbed channel, $t' = \omega t$ is non-dimensional time. The resistance to the flow has two contributions; from bed friction, included through the friction coefficient, C_f, and due to turbine array thrust, included through the array thrust coefficient $C_{TA} = T_A / \frac{1}{2}\rho U^2 w_A h$, in which T_A and $U = Q/wh$ are the array thrust and velocity, w_A the width of the array.

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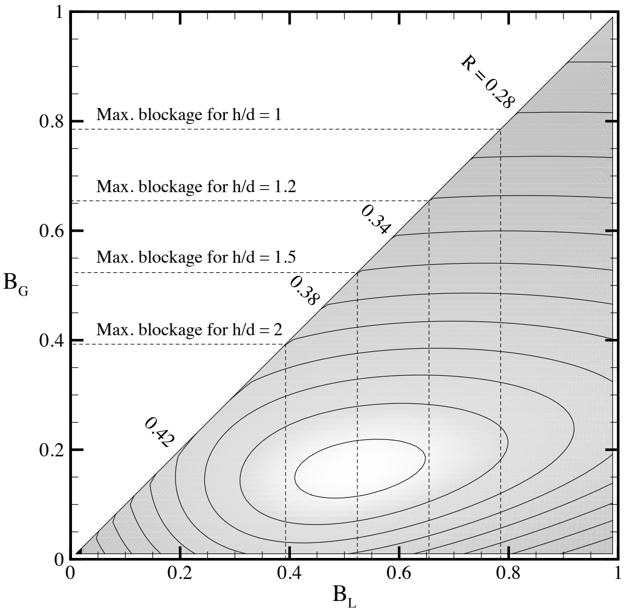

Fig. 1: Return parameter, $R = C_{PC} / B_G$, over local and global blockage space for channel parameters $\omega L / \sqrt{ag} = 0.635$, $L / h = 500$ & $C_f = 0$. Dashed lines indicate achievable blockages for given depth to turbine diameter ratios, h / d.

The array blockage ratio $B_A = w_A / w = B_G / B_L$ represents the proportion of the channel width occupied by the fence, whilst the local and global blockage ratios, B_L and B_G, the proportions of the local flow passage area and channel area occupied by turbines. The model assumes a smooth entry and exit to the channel and is closed through the specification of C_{TA} from the partial fence model of NW12.

Fig. 1 presents example solution to the finite fence channel dynamics model for given channel parameters in terms of the return parameter, $R = C_{PC} / B_G$; the ratio of power generated to turbine frontal area (note $C_{PC} = \bar{P} / \rho g a Q_0$ is the channel power coefficient and $\bar{P}$ the time average power generated). The local blockage infers the intra-turbine spacing so that for fixed B_L, increasing B_G corresponds to increasing the number of turbines and therefore length of the turbine fence. Increasing global blockage in this manner is seen to at first increase R, and thus power per turbine, due to the effect of restricting the array by-pass, but further increasing the fence length leads to a reduction in R through reduction in channel flow rate, although total power yield C_{PC} does continue to increase in this case (not shown here). There clearly exists an optimum point of operation if the desired outcome is to maximize power yield per turbine as opposed to total power yield.

Conclusions

To fully model the energy extraction by a tidal fence it is clearly necessary to consider the dynamics of the channel as the overall flow resistance plays an important role in limiting the potential power yield of the fence. Optimum points of operation (maximum yield per turbine) can be determined using this relatively simple finite fence channel dynamics model. Inclusion of realistic turbine thrust and power characteristics, as well as support structure drag and bed friction leads to more complex solution and will be presented at the workshop. Performance of real turbines to achieve high thrust singularly and in fence configuration will also be presented.

Acknowledgements:

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References:

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