The Potential of Sub-Arrays to Increase Tidal Farm Power

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Summary: Building on theoretical models previously developed for partial arrays of tidal turbines, a theoretical model has been developed to investigate the behaviour of a long fence array split into multiple shorter fences, or sub-arrays. Three flow scales are used for turbine, sub-array and channel flow, allowing the governing equations to be applied at each scale. Through varying blockage ratios, it is discovered that increasing the local blockage ratio, if over-blocked flow can be avoided, has the greatest potential to increase power yield. It is also found that splitting a sufficiently long fence array into sub-arrays can increase the power that can potentially be extracted.

Introduction

An analytical model for a single partial fence array of tidal turbines partially filling a wide channel has been previously developed by Nishino and Willden [1,2]. The previous work proposed a scale separation between the flow immediately surrounding each individual device and the flow around the complete array, with dynamic and kinematic matching between the two scales which allows an analytical solution to be found. The present work extends this concept of scale separation to a third scale, that around a 'sub-array'. This allows the investigation of a long row array that is split into multiple shorter arrays, which may be necessary or practical in reality where bathymetry is complex or where multiple devices are mounted on each support structure.

Methods

An analytical model of a tidal 'farm' including multiple sub-arrays was developed using Linear Momentum Actuator Disc Theory (LMADT). The basic model of a single turbine in a channel with constant mass flux was developed by Garrett and Cummins [3], who used LMADT to model the flow through and around a single turbine. This is a quasi-inviscid model, where wake expansion and pressure equalisation between the core flow and the bypass flow occur upstream of any mixing. This assumption allows the equations of conservation of mass, momentum and energy to be applied along the core flow and the bypass flow separately, and relationships between the thrust and power coefficients and the device induction factor can be found. The work of Nishino and Willden replaced the single device in this model with a partial fence array of devices, introducing the concept of separation of scales.

A multiple sub-array model has therefore been developed by extending this concept further, replacing each single device within the partial array model with a sub-array, such that there are three flow scales under consideration as shown in Fig. 1: the tidal farm scale, where a wide channel has a single 'device' within it, which is the entire tidal farm; the array scale, where multiple identical sub-array channels within the total farm contain single 'devices' within them, which are the sub-arrays; and the local scale, where each actual individual turbine sits in a local channel within its sub-array. The number of turbines and sub-arrays is considered to be high enough to justify the assumption of flow scale separation between each scale and the ones above/below it.

Kinematic matching between scales is ensured by equating the upstream flow speed at each scale to the 'device' location flow speed in the scale above it: i.e., the upstream flow speed in the turbine scale channel is equal to the sub-array's core flow speed at the array position. Dynamic matching between scales is simple, as the total thrust imposed on the flow by the total array must equal the sum of all the individual thrusts imposed by the turbines. These matching conditions allow the (numerical) solution of the system of governing equations at all scales if a local turbine induction factor is specified.

Results

The theoretical global power and thrust coefficients have been investigated over a parameter space representing a wide variety of different array configurations. Blockage ratios for the local, array and farm scale were defined as in Fig. 1 and varied independently, and for every blockage combination a range of local turbine induction factors were simulated to find the maximum power coefficient available. The case of fixed global blockage, B_G (which equals the product of the local, array and farm blockage ratios), was investigated in some

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detail, since it physically represents a channel with a fixed number of turbines of known area to be placed within it. For the case of $B_G = 0.131$, a maximum C_{PG} of 1.087 was found to be possible at $B_L = 0.65$, $B_A = 0.56$ and $B_F = 0.36$, as shown in Fig. 2a). This is 7.5% higher than the maximum C_{PG} of 1.011 achieved in the single partial array model of Nishino and Willden at the same global blockage, with $B_L = 0.49$ and $B_F = 0.27$. (B_A as defined in this model is equal to 1 in the single partial array model, as there is no separation between sub-arrays.) Basin efficiency (the ratio of extractable power to total power removed from the flow) was also investigated. As might be expected, basin efficiency, as shown in Fig. 2b), generally decreases in this model as extractable power is maximised, as this is accomplished through increased thrust. However, it should be noted that the lowest values of basin efficiency do not exactly correspond to the highest values of C_{PG}, so there is some opportunity to achieve a compromise solution between the two where required.

The order of blockage ratios in terms of their magnitude to achieve maximum C_{PG} as seen in this individual case, where $B_L > B_A > B_F$, was repeated at all global blockages considered within this analysis. Over the range of global blockages considered ($0 < B_G < 0.5$, which it is considered highly unlikely that any real tidal farm could ever exceed), it was also seen that the greatest increases in C_{PG} when moving from the single partial fence model to the multiple sub-array model were achievable at low global blockage. The specific case of $B_G = 0$ was considered, an infinite width channel that for tidal turbines is analogous to the wind turbine in free atmosphere, where C_{PGmax} takes the Lanchester-Betz limit of 0.593. For the multiple sub-array model, $C_{PGmax} = 0.865$ (an 8% increase on $C_{PGmax} = 0.798$ in the single partial fence case).

Conclusions

A new theoretical model has been proposed to investigate the efficiency of a long, cross-stream tidal farm, comprised of multiple sub-arrays partially filling a wide channel. This model is based on three scales of fluid flow; around the turbine, the sub-array and the entire tidal farm. The power coefficient for extractable power is found to be a function of the blockage ratios at all scales, and the flow speed induction factor through the turbine. It is found that the maximum power coefficient available can theoretically be increased above that achievable in the single turbine or single partial array models, though at some associated cost in basin efficiency.

Acknowledgements:

This work was done with the support of the SuperGen UK Centre for Marine Energy Research (UKCMER).

References:

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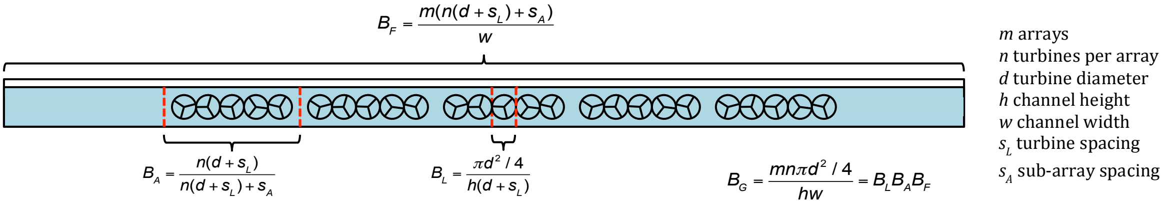
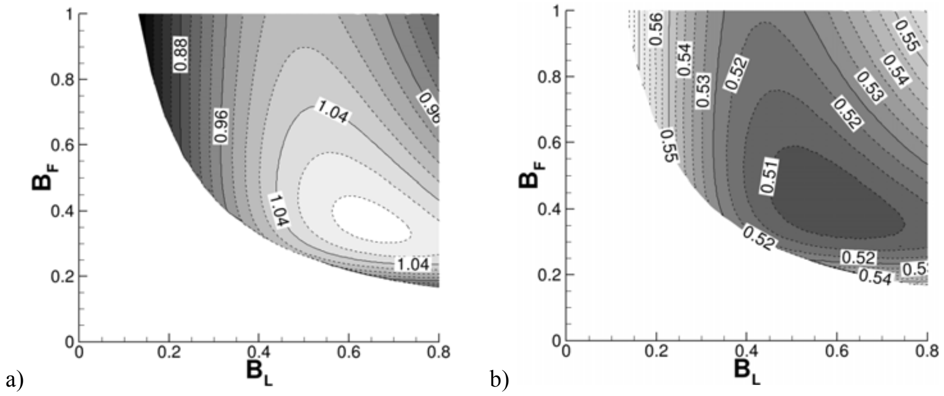


Figure 1 – An illustration of a tidal turbine farm containing multiple sub-arrays, with definitions of blockage ratios



a) b)

Figure 2 – Results for a) C_{PGmax} and b) η (basin efficiency) against B_L and B_F for the case of $B_G = 0.131$