Wind and tidal turbines in uniform flow

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Summary: Estimating the power that can be extracted by wind or tidal turbines in uniform flow is complicated because the resistance offered by turbines acts to divert flow around them. In this paper we briefly review different models which can be used to understand this flow diversion. We start with Linear Momentum Actuator Disc Theory (LMADT), summarising some extensions that have been made in recent years to model more complex arrangements of turbines. We then consider some future extensions that could be made to LMADT and numerical approaches that complement LMADT. As an example of the latter we use a point vortex model to investigate (i) a non-concentrically placed actuator disc in a channel; and (ii) a V-formation of 5 discs.

Existing work on modelling turbines with actuator disc theory

Linear Momentum Actuator Disc Theory (LMADT) was used almost 100 years ago to model a single wind turbine as an actuator disc in laterally unbounded flow. Assuming the disc offered a streamwise resistance to the flow, arguments of mass, energy and momentum were used selectively to quantify the diversion of flow around the disc and the power extracted by the disc. This analysis led to the result that at most 16/27 times the upstream kinetic flux (passing through an area equal to that of the disc) can be extracted by the disc, which is often referred to as the Betz limit. In practice this limit has proven useful as a benchmark in the wind industry, whilst the combination of LMADT with blade element theory has provided a valuable wind turbine design tool [1].

Motivated by this success, a number of extensions to LMADT have been made in recent years. Most of these extensions have been introduced in the context of tidal stream turbines, and include: extensions to model isolated turbines in bounded flows; extensions to model rows of turbines; and extensions to model arrays of turbines. Beginning with isolated turbines, LMADT was initially extended to consider the performance of a tidal turbine placed in the middle of a uniform channel [2, 3]. This allowed the important effect of blockage ratio, defined as disc area to area of bounded flow, to be quantified (with the analytical results proving to be in reasonable agreement with later numerical simulations of turbines concentrically placed in a variety of channel shapes [4]). Following this work, theoretical extensions to LMADT considered the effect of a deforming flow boundary, which is a more correct representation for a tidal channel with a free surface [3, 5]. Collectively, this work on uniform and deforming bounded flows has allowed for the introduction of turbine efficiency as a metric to described turbine performance (defined as the power extracted by the disc to the total power removed from the flow). In many situations turbine efficiency is an equally important metric as the amount of power extraction (see [8] for further discussion). Moving beyond one disc and towards rows of discs, [6] has more recently introduced a concept of scale separation to consider partial fences of turbines placed within laterally unbounded or bounded flows. This work has provided, for the first time, a means to account collectively for flow diversion around individual turbines and around rows of turbines, and the theoretical model has been shown to agree well with CFD simulations of rows of up to 40 actuator discs [7]. Finally, stepping from rows of turbines towards arrays of turbines, the ideas of [6] have recently been combined with an assumption about streamwise spacing between discs to consider turbine farms comprised of staggered and multiple rows of discs [8]. This work is the first to suggest analytically that (i) staggering turbines may not be the most efficient means of farm layout in uni- or bi-directional flows, compared with placing turbines closely together in one single row, and (ii) that the optimal spanwise spacing between turbines placed in rows is strongly dependant on the size of the farm (i.e. how many rows of turbines).

Further modelling turbines as actuator discs

Each of the extensions to LMADT noted above have allowed for an increase in the complexity of problem that can be handled analytically. However, despite the number of extensions that have been made thus far, it is apparent that future extensions to LMADT are still possible. For example, [8] has shown that discs placed in non-uniform flow can be modelled using LMADT, and a dedicated study on turbines in sheared flows could be undertaken based on this. Extensions to the scale separation idea of [6] also appear likely, both for different farm layouts in channels, and for problems involving additional scales of separation. It may also be possible to learn more about the effect of turbine support structure on power extraction by introducing combined blockage and drag in uniformly bounded flow. Finally extensions may be possible to treat the effects of different channel geometry and/or seabed drag on the power extraction of discs.

Aside from these theoretical possibilities, many important scenarios also exist in which turbines may be represented to first approximation as actuator discs (i.e. as discs introducing a uniform resistance) but LMADT cannot be used to quantify the flow field. Figure 1 and Figure 2 present two examples where this is the case: (i) a non-concentrically placed disc in a uniform channel and (ii) a V-formation of 5 actuator discs. These scenarios cannot be modelled with LMADT because in the first example LMADT is inherently one-dimensional and unable to consider geometric asymmetry, whilst in the second example LMADT can only be used to model multiple discs if the pressure equalises across the flow between successive discs [8].

To explore scenarios like those in Figure 1 and Figure 2 requires a numerical model. In this paper we adopt a point vortex model, described in [9] but adapted here to (i) model flow boundaries using the method of images (Figure 1) and (ii) model multiple discs (Figure 2). The vortex model has advantages over CFD because it is an efficient way to model inviscid flow, in keeping with traditional actuator disc approaches, and it allows naturally for the treatment of unbounded flow (when the method of images is not used). Using the vortex model Figure 1 illustrates how geometric asymmetry can be modelled (with results suggesting an increased power extraction). Figure 2 suggests that staggering leads to more variability in individual turbine power, but it does not increase total power generation. Each of these findings, which maintain the classic assumption that a turbine can be modelled as an actuator disc, suggests optimal strategies for arranging wind and tidal turbines in uniform flow.

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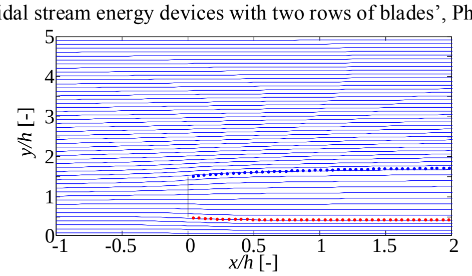

Fig.1 Disc with local resistance k =3.3 placed close to one side of a channel that is 5 times wider than the disc (k is a dimensionless thrust; see [8]). This disc removes more power than centring the turbine in the channel.

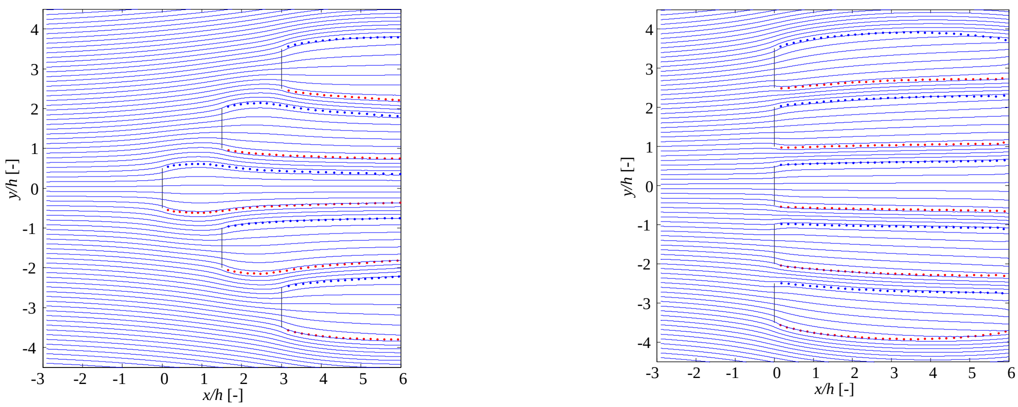

Fig. 2: (Left) 5 staggered discs each with k =2. From top, fraction of kinetic flux removed by disc, 1.03, 0.84, 0.6, 0.84, 1.03 (Right) Identical discs placed in a line. From top, fraction of kinetic flux removed by disc: 0.87, 0.88, 0.89, 0.88, 0.87.