Method for the generation of arbitrary complex vector wave fronts

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We describe an extremely versatile method that permits the accurate generation of arbitrary complex vector wave fields. We implement the scheme using a reconfigurable binary optical element that also permits additional fine tuning, such as aberration correction, to be performed. As examples we demonstrate the generation of both azimuthally and radially polarized beams. © 2002 Optical Society of America

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The ability to generate an arbitrary complex scalar optical wave front accurately is important in fields as diverse as optical shop testing and adaptive optics. In the latter case it is often desirable to be able to reconfigure these wave-front shapes as quickly as possible.[1,2] Although the ability to generate scalar beams is useful, there are an increasingly large number of cases in which it is desirable to create arbitrary vectorial beams. Examples range from studies of singularities in optical fields[3] to tuning the strength of the longitudinal component of an electric field in the focal region of a microscope objective for single molecular experiments.[4,5] These applications require, for example, the use of radially and azimuthally polarized beams that have been obtained by interferometric means[6–8] or by the use of subwavelength gratings.[9] In this Letter we show how we can extend a technique from computer-generated holography that has been used previously to produce arbitrary reconfigurable scalar wave fronts[1,2,10] to produce arbitrary reconfigurable vector beams.

In the scalar case the formal intention is to produce an arbitrary complex wave field $u(\mathbf{r})$. The procedure to achieve this is to binarize this field together with a linear phase tilt, $u(\mathbf{r})\exp(j\tau \cdot \mathbf{r})$. Figure 1 indicates a scheme[2] for mapping these complex values into the required binary transmissivities of $+1$ or -1. When this binarized field passes through the first lens of a 4-f optical system,[11] a number of diffracted orders appear in the intermediate (Fourier) plane. In particular the $+1$ order centered on tilt position τ is an exact Fourier transform of the desired field $U(\rho - \tau)$. A suitably positioned spatial filter (pinhole) selects this order, which is then retransformed by the second lens to produce a spatially inverted form of the desired output field together with the added tilt. The output field can be rendered upright and the tilt removed trivially, for example, with a mirror.

Since the binary optical element at the heart of this device is not a polarization-sensitive element, it is necessary to modify this strategy to apply the method to the vector case. In this situation the aim is to produce an optical field of the form

$$\mathbf{E} = \begin{bmatrix} a(\mathbf{r}) \\ b(\mathbf{r}) \end{bmatrix}. \qquad (1)$$

If we follow the same binarizing scheme that we have just described (see Fig. 1) and construct a binary optical element whose transmissivity is given by binarizing the function

$$f(\mathbf{r}) = a(\mathbf{r})\exp(j\tau_a \cdot \mathbf{r}) + b(\mathbf{r})\exp(j\tau_b \cdot \mathbf{r}), \qquad (2)$$

we have now introduced arbitrary (and different) phase tilts τ_a and τ_b to the two functions. Among other orders, we then create in the intermediate Fourier plane the Fourier transforms $A(\boldsymbol{\rho})$ of $a(\mathbf{r})$ and $B(\boldsymbol{\rho})$ of $b(\mathbf{r})$ at positions determined by their respective phase tilts. If instead of illuminating the binary element with a plane wave we now elect to use a beam of the form

$$\mathbf{E}_0 = \begin{bmatrix} \exp(j\tau \cdot \mathbf{r}) \\ \exp(-j\tau \cdot \mathbf{r}) \end{bmatrix}, \qquad (3)$$

we introduce additional equal and opposite phase tilts to the x and y polarized components of the incident field, respectively. The first-order diffracted components of the field of the Fourier plane are given by

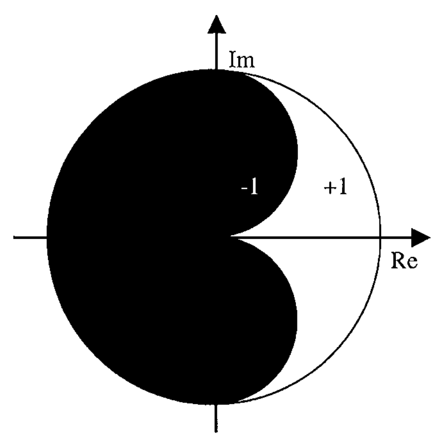

Fig. 1. Mapping of a normalized complex field within a unit circle into the required $+1$ or -1 modulation of the binary optical element.

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$$\mathbf{E}_1 = \begin{bmatrix} A(\rho - \tau_a - \tau) + B(\rho - \tau_b - \tau) \\ A(\rho - \tau_a - \tau) + B(\rho - \tau_b + \tau) \end{bmatrix}. \quad (4)$$

Since our aim is to produce the field of Eq. (1), we need to be able to isolate A from the diffracted x-polarized field and B from the diffracted y-polarized field. This is readily achievable if applied tilts τ, τ_a, and τ_b are chosen such that $2\tau = \tau_a - \tau_b$. The two field components are then centered on the same position in Fourier plane $\tau_a + \tau$ so that a pinhole spatial filter can again be used to isolate these fields, which, after further Fourier transformation by the second lens, produce the desired field of Eq. (1).

To demonstrate the versatility of our approach to generate vector beams we constructed the optical system shown in Fig. 2. Since the operation of the system relies on the ability to choose the relative tilts so that the appropriate diffraction patterns overlap in the Fourier plane of the 4-f system, we elect to image the field in this plane using a microscope objective as shown in Fig. 2. We used a rotating diffuser to remove coherent artifacts when recording the images with a CCD camera. The binary phase modulation was achieved by use of a reconfigurable ferroelectric liquid-crystal spatial light modulator (FLCSLM) in an off-axis configuration to avoid the use of polarization elements but still achieve binary phase modulation regardless of the polarization of input light.[12] The FLCSLM that we used was a Displaytech Model SLM-256 with a Peripheral Component Interface card driver and a 256×256 array of pixels on a 15-μm pitch. The input beam required by Eq. (3) was produced by the use of polarized light and a Wollaston prism. A beam expander then imaged the Wollaston prism onto the FLCSLM. Since the lateral position of the Wollaston prism introduces a phase shift between the two polarization components, we decided not to try to remove this by repositioning the prism but rather by introducing a compensating phase shift into the binarization process. Likewise we tuned the overlap of the desired diffracted components in the Fourier plane by adjusting tilts $\boldsymbol{\tau}_a$ and $\boldsymbol{\tau}_b$ in the computer prior to binarization. We further corrected the optical system by applying aberration correction with the FLCSLM, which we achieved simply by subtracting the premeasured residual (small) aberration from the desired fields before binarization. We measured the aberration using the FLCSLM as a wave-front sensor in the fashion described previously by Neil *et al.*[13] The ability to perform all these fine-tuning operations was made trivial by the reconfigurable nature of our binary optical element.

Important practical examples of applications of vector beams[5,8] are those that require purely azimuthal or purely radial electric fields in the pupil plane of a lens. An azimuthally polarized beam is one for which the electric field takes the form $(-\sin\phi, \cos\phi)$, say, where ϕ is the angular coordinate in the plane $\tan\phi = y/x$. The field in the focal plane of the lens, for the low-angle geometry considered here, could be conveniently written as the Fourier transform of the desired azimuthal field

$$E \sim \begin{bmatrix} -f(v)\sin\phi_p \\ f(v)\cos\phi_p \\ 0 \end{bmatrix}, \quad (5)$$

where the angle ϕ_p is the polar angle with respect to the x axis. The function $f(v)$ can be written in this low-angle case as

$$f(v) = \int_0^1 tJ_1(vt)\mathrm{d}t\,, \quad (6)$$

where $v = (2\pi/\lambda)\sin(\alpha)r_p$ and λ is the wavelength and $\sin(\alpha)$ denotes the numerical aperture in air. We note that the z component of the electric field is always zero even in the high-aperture case. Figure 3(a) compares

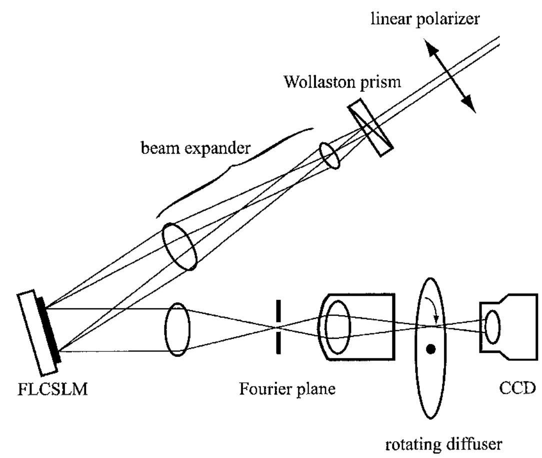

Fig. 2. Schematic diagram of the optical system.

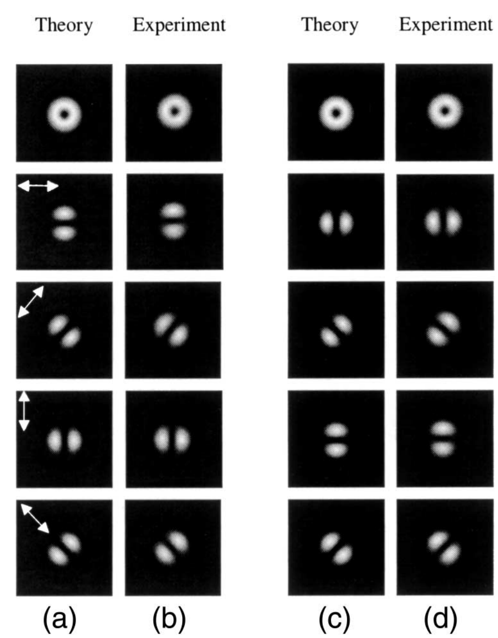

Fig. 3. Intensities in the focal plane comparing theory, (a) and (c), with experiment, (b) and (d), for both azimuthally, (a) and (b), and radially, (c) and (d), polarized illumination. The top row shows the full intensity, whereas the rows below reveal the vectorial nature of the field by showing the intensity passed by an analyzer oriented as indicated by the arrows.

these theoretical predictions with the experimentally observed images of Fig. 3(b). To explore the polarization structure of the field Fig. 3 also shows images taken through an analyzer oriented at 0°, 45°, 90°, and 135° with respect to the x axis.

Having demonstrated that it is straightforward to generate an azimuthally polarized beam, we now show that a radially polarized beam, whose electric field pattern can be written as ($\cos\phi$, $\sin\phi$), can also be produced. The field in the Fourier plane now takes the form

$$E \sim \begin{bmatrix} f(v)\cos\phi_p \\ f(v)\sin\phi_p \\ 0 \end{bmatrix}, \tag{7}$$

where $f(v)$ is again given by Eq. (6). We note, of course, that the z component of the electric field is negligible only in this low-angle experimental situation. Figure 3 shows theoretical and experimental images of this polarized field and again remarkable agreement was found between theory and practice.

The approach we have just described is extremely versatile and able to produce accurately any complex vector wave field. The use of a reconfigurable element as the binary phase screen has advantage over previous approaches since it allows for further fine tuning, such as aberration correction, to be performed. Furthermore, the fundamental basis of the approach is digital and hence significant accuracy advantages accrue. However, both the binarization method employed as well as the particular reconfigurable element (FLCSLM) used lead to a relatively inefficient use of light. The effects of the latter have been discussed in detail elsewhere.[1] The binarization process leads to a maximum efficiency in the first diffracted order of 40.5% ($4/\pi^2$) and the use of only half of the light in the first diffracted order for each polarization reduces this to 20.25%. In addition, the particular form of the desired complex field (amplitude and phase of the components) could lead to an additional reduction in efficiency. For the target fields $\cos\phi$ and $\sin\phi$ described here, additional losses of 50% are incurred.

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