Drained Behaviour of Suction Caisson Foundations on Very Dense Sand

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Abstract

Alternatives to piled foundations, such as suction caissons, are becoming increasingly viable options for fixed platforms, particularly in the development of marginal fields. Whilst pile design procedures evolved smoothly from onshore experience and theory, design of shallow foundation systems has had to be re-examined in light of the intense offshore loading conditions. This is currently being undertaken through experimental investigations, which indicate the possibility of defining foundation response using plasticity theory.

This paper reports the response of circular foundations on very dense sand within the context of plasticity theory. The load and displacement paths are applied using a sophisticated three-degree-of-freedom loading rig, designed specifically to explore plasticity related concepts. The tests are on dry sand to ensure drained behaviour. Eight tests are performed on each sand sample, thus minimising material variations between tests, which is particularly important at the high peak friction angles being explored.

The tests were at 1-g, and concentrate on four embedment ratios (0, 0.16, 0.33 and 0.66) on a sand of 95% relative density. The research indicates a change in shape of a yield surface with increase in embedment ratio. Whilst the tests concentrate primarily on the plastic deformations, the behaviour within the yield surface is also examined. A conceptual model, based on plasticity, is presented.

The findings of the research are particularly relevant to the offshore industry in the development of shallow and skirted foundations for dense sand deposits.

Introduction

Historically, offshore structures for oil and gas production have been anchored to the sea floor by large diameter piles. Recently, a novel foundation type has been used in place of these piles, best described as upturned buckets, typically 15m diameter and 5m deep, they are usually called suction caissons. These foundations are installed by pumping out water to cause a slight suction to the inside of the caisson, once it has made a sufficient seal within the soil. In the North Sea, where a large part of the seabed is covered by a layer of very dense sand, the suction assists penetration by temporarily reducing the resistance of the sand as well as increasing the net downward force on the caisson.

Offshore structures are often exposed to severe loading due to wave, current and wind forces. Their foundations are exposed to higher horizontal and moment loading than are encountered onshore. Whilst pile design procedures have evolved from onshore experience and theory, design guidelines for shallow foundation systems, such as suction caissons, have yet to be devised for the intense loading conditions found offshore. In the particular case of suction caissons, there are no precedents in onshore experience, and at present there is little offshore experience, though this is rapidly changing. This means that there are no accepted procedures, such as the API guidelines for piles, nor are there large amounts of published data available, as firms tend to keep any acquired knowledge in-house so as to gain competitive advantage.

One of the critical aspects of the design is the ability to resist tension, which may occur under extreme conditions. The important issue is the rate at which the caisson is loaded, compared to the rate of fluid flow within the soil, as the latter affects the soil response. The typical loading on the foundation will be random variations about some mean value, which results in a complex flow regime within the soil, which in turn dominates the response of the foundation. However, it is necessary to define the long-term drained loading limits of caisson response before the effects of loading rate and load reversal can be explored. This is currently being undertaken through experimental investigations in which the foundation response is defined within the setting of plasticity theory. Adopting this approach allows a mathematical representation of the foundation, which can be incorporated within typical numerical analyses used in the design of offshore structures.

Suction caisson foundations are the focus of much current research, primarily as they offer substantial cost savings in both materials and installation (both time and money). In this paper selected results are presented that can be used to...
establish the long term, or drained, behaviour of foundations with skirts on very dense sands. These very dense sand layers, with a relative density up to 100%, are typically 10-20m deep, much larger than the probable depths of the caisson skirts, and thus dominate the drained response. The impetus for this investigation is the use of suction caisson technology for the development of marginal fields, particularly in the North Sea, where small lightweight structures may become feasible.

A Review of Bearing Capacity and Load Interaction. During the passage of wind, waves and current through and past an offshore structure it is usual to assume that the loading becomes unidirectional, particularly during the more extreme environmental conditions. Consequently the loading on the foundations can be represented as the three degree-of-freedom problem illustrated in Figure 1. Early solutions of this problem dealt with establishing the vertical load that would cause ‘failure’ of the foundation, whilst more recent work is also concerned with describing the load-deformation behaviour.

The problem of the bearing capacity of a foundation under combined vertical, horizontal and moment loads has received considerable attention. The generally accepted procedures for use in the offshore industry are those established by Hansen, Vesic, and Butterfield and Ticof. These deal with the application of horizontal load by introducing inclination factors, which reduce the vertical bearing capacity depending on the ratio of horizontal to vertical load. Moment loading is dealt with by examining a statically equivalent vertical load acting at the centre of a reduced footing determined by the eccentricity (e = M/V). For a fully combined V:M:H load both processes are used, that is an inclined load acting on a foundation of reduced area. Further factors are introduced to account for the effects of footing embedment. These relationships were developed initially as reduction factors on the allowable vertical load, rather than for the development of interaction diagrams.

An alternative approach in recent years is to view the combined load problem in terms of a three-dimensional yield surface. Roscoe and Schofield first used the concept of a yield surface for this application, and Butterfield and Ticof were amongst the first to investigate footing behaviour on sand in this manner, examining strip footings on dense sand. From large numbers of load controlled ‘probe’ tests, they were able to map out a yield surface. They suggested that a ‘cigar shaped’ yield surface (Figure 2) would capture the data and sized it by dimensionless peak loads M/BV\text{peak} = 0.1, and H/V\text{peak} = 0.12, both occurring at V/V\text{peak} of 0.5. Other investigations have validated the general shape of this surface in H/V\text{peak}-V/V\text{peak} and M/BV\text{peak}-V/V\text{peak} space, whilst showing that in M/BV\text{peak}-H/V\text{peak} space the failure surface can be approximated by a rotated ellipse. Dean et al. pointed out that the rotation depends on the position of the load reference point.

Tan mapped out the behaviour of spudcan footings in V:H space on saturated sand using centrifuge tests. The majority of tests were ‘sideswipe’ tests, in which the footing was loaded to a certain level, before being moved horizontally whilst keeping the vertical displacement locked. By considering the foundation to be elastically incompressible under vertical load, then it can be argued that the sideswipe paths are close approximations to paths across the yield surface. Tan suggested that (H/V)\text{max} occurred at a value of V/V\text{a} = 0.46, where V\text{a} was the maximum pure vertical load experienced by the foundation. Tan also pointed out that a ‘parallel point’ (analogous to the critical state) existed, at which stage the plastic displacement increment was purely horizontal. Tan conducted investigations into the effect of embedment, finding that the shape remained similar, with the maximum H/V\text{a} ordinate increasing slightly with increase of depth. The influence of cone angle and roughness was also examined.

Extensive use of swipe testing was made by Martin, who mapped out the behaviour of spudcan footings on a clay with increasing strength with depth. Martin found that the shape of yield surface could be normalised with respect to the vertical load capacity at any depth, and obtained a similar shape to that of Butterfield and Ticof. Martin used a mathematical function to describe the yield surface, defined by the maximum ordinates of H/V\text{a} (M = 0) and M/2RV\text{a} (H = 0), as well as the curvature at high and low load, and the eccentricity of the ellipse. Martin developed a three-dimensional work hardening plasticity theory based on the experimental results. He used theoretical stiffness coefficients for elastic behaviour, and theoretical lower bound bearing capacity solutions to define the vertical bearing capacity with penetration. Martin incorporated this plasticity model in a structural analysis of a jack-up. Thompson used this model for dynamic analyses of a jack-up rig.

Recent experimental work by Gottardi and Houlsby followed a programme designed to develop a three-dimensional plasticity model for circular footings on dense sand. A numerical model, ‘Model C’, has since been created based on these results and shown to provide accurate simulations of selected test results. This model assumes that the yield surface does not change shape as V\text{a} changes. The tests were conducted from a high ratio of vertical load to the peak bearing capacity sustainable by the soil. An empirically derived hardening law was used. The surface was again similar in shape to that found by previous researchers, and is shown in Figure 2. The peak value of H/V\text{a} was 0.116 and of M/2RV\text{a} was 0.086. This model is being used as part of a complex dynamic analysis package using state-of-the-art wave loading techniques to represent extreme events.

Experimental Apparatus
To test the plasticity model it is essential to establish:
(i) The shape of a yield surface.
(ii) The mechanism of hardening of the surface.
(iii) A flow rule or plastic potential.
(iv) The elastic behaviour within the surface.

Studies of this kind normally use eccentric inclined loads, or examine the two-dimensional cases (V, H), and (V, M/2R).
At the University of Oxford a loading rig was developed, initially to explore the behaviour of spudcan footings on clay. It has undergone several modifications, and it is adaptable to any soil. The unique feature of this apparatus is that any combination of displacement paths can be applied to the model footing using computer controlled stepper motors. The response of the footing is determined by measuring the resultant loads using a 'Cambridge' load cell, whilst accurate foundation displacements are measured using a system of LVDTs. A primary advantage of using this displacement controlled apparatus is the ability to explore work softening behaviour. Figure 3 shows the loading rig, further mechanical details of which are given by Martin and Gottardi and Houlsby. The footing behaviour was assumed to be rate independent in the dry sand. However, the rates used never exceeded 0.01mm/s or 0.01º/s such that, if any existed, the effects of rate should be negligible. The co-ordinate systems, sign conventions and notations for the loads and displacements are shown in Figure 2, following Butterfield et al. The load reference point was at the base of the footing (i.e. mudline), consistent with previous work conducted on flat footings at Oxford University.

Extensive modification has also been undertaken on the data acquisition and control side of the apparatus. Whilst the prime motivation for upgrading the electronics was the need for better control and acquisition during transient and cyclic testing on saturated samples, the increased performance allowed for rather sophisticated testing on dry samples of sand.

The control program allows the control of each of the three stepper motors independently. The motors can either be moved through specified amounts at specified speeds, or a PID control loop can be used in which the velocity and direction of the stepper motor is used as the control signal. Using this method it is possible to provide independent force or displacement control on all three axes.

The displacements of the footing are measured via three small LVDTs (individual ranges of 10 mm), from which the rotation of the footing, and are resolved to ±1µm. The loads are processed depending on the rotation of the footing, and are resolved to ±0.5 N.

Four footings were used to explore the effects of skirt penetration. The footing diameter was 100mm. It is useful to define the non-dimensional embedment ratio (y), as the skirt length (L) divided by the diameter (D);

\[ y = \frac{L}{D}. \]

(1)

to compare different suction caissons. The two platforms that have used the suction caissons on dense sand both used caissons with an embedment ratio of 0.33. Previous testing at Oxford and NGI aimed at resolving design aspects of the second of these platforms (Sleipner Vest) and therefore also used this ratio (the results of the tests are confidential). It is necessary to be able to compare the results with a variety of different skirt depths so ratios of 0, 0.166, 0.33 and 0.66 were used. The results from an embedment ratio of 0 can be compared to other data on flat footings, particularly the series of tests by Gottardi and Houlsby.

The sand used throughout the testing was produced using a cyclone separation system. The choice of sand was dictated by the proposed use of the same sand for modelling rate effects when saturated. It will be saturated by silicone oil, to model drainage times appropriate to the loading of offshore foundations. Figure 4 shows the grading curve of the sand and others that have been used for 1-g modelling of footings. Also shown are the grading curve boundaries of the sands at the sites of the Europipe and Sleipner Vest platforms. Table 1 gives the typical properties of the sand.

The sand was contained in tanks of internal diameter 1100mm, and depth 250mm, to provide a single sample large enough to accommodate multiple tests. The samples were prepared by loosely placing sand within the container. A smooth wooden lid was placed upon the very loose sand, surcharged with a very small weight (to apply a slight confining pressure), and the sample was vibrated until no further downward movement of the lid was observed. This method, also used by Lau, was able to produce very dense samples with typical characteristics shown in Table 2.

**Typical Results**

The tests are shown schematically in Figure 5. Initial testing indicated that the behaviour depends on the ratio of the maximum vertical load previously experienced by the footing to the peak bearing capacity sustainable by the soil (V/Vpeak). The programme was therefore designed to explore this effect as well as other aspects of the plasticity model. The tests included:

(a) **Characterisation Tests:** Characterisation of the samples was essential so that a comparison across different sample tanks could be made. The initial characterisation was made in terms of relative density, of which a mean of 97.8% and a standard deviation of 1.81% was obtained. A more important characterisation was made by determining a bearing capacity factor (and hence friction angle) of the sample by conducting a vertical loading test in the centre of the sample, used a 50mm diameter footing, which was loaded to a displacement of about 10mm. On several samples multiple bearing capacity tests were conducted at other locations to examine the consistency of the samples. If the peak vertical bearing capacity of a flat footing is defined as:

\[ V_{peak} = \frac{1}{2} DN_y^* \left( \frac{\pi D^2}{4} \right) \]

(2)

then it is possible to determine a value for the bearing capacity factor, N_y*. Similarly, by following the procedures of Bolton it is possible to deduce a value for the angle of friction (either triaxial or plane strain) based on the relative density of the sand. The triaxial angle of friction has been plotted against N_y* in Figure 6, and compared to a theoretical curve determined by Bolton and Lau using the method of characteristics. There is a
reasonable correlation between the experimental and theoretical results. The scatter within the experimental results can be explained by the fact that the bearing capacity factor can vary significantly for small changes in the relative density, particularly at the high friction angles being explored.

(b) Swipe Tests: The majority of the tests were swipe tests. Each test involves loading the footing to a given load \( V_o \), and then rotating it or displacing it horizontally (or any combination of these two) whilst keeping the vertical displacement constant. Tan\textsuperscript{37} and Martin\textsuperscript{32} argue that the load path traced under these displacement paths gives a direct indication of the shape of the yield surface - assuming that the ratio of vertical elastic stiffness to vertical plastic stiffness is large. A correction can be made for the expansion of the yield surface in other cases. Since the stiffness of the rig is finite, a correction must usually be made for the expansion of the yield surface as a result of the increased penetration of the footing associated with the relaxation of the rig as the vertical load level falls. In these tests this correction was avoided by using feedback to control the vertical displacement. By conducting swipe tests at different starting ratios of \( V/V_o \), the behaviour within the yield surface could also be examined. Swipe tests at very low \( V/V_o \) ratios give an indication of the yield surface at low vertical load.

(c) Constant Load Tests: The expansion of the yield surface (that is the hardening behaviour) can be examined by conducting tests similar to swipe tests, but at constant vertical load rather than constant vertical displacement.

(d) Vertical Loading Tests: These tests involve loading the footing vertically, usually to 2400N (the limit of the rig) and unload/reload loops at 1000N and 2000N. At the very simplest level the hardening of the yield surface is linked solely to the vertical plastic displacement, thus these tests give a measure of the hardening relationship.

(e) Elasticity Tests: Whilst the prime objective of the testing was to obtain information about the plastic response of the footing, several tests were undertaken to evaluate the elastic behaviour within the yield surface. The results from these can be compared to finite element results\textsuperscript{2}. In these tests one load on the footing was typically cycled while keeping other loads constant. The cyclic amplitude was increased to examine the boundary between elastic and plastic behaviour.

(f) Loop Events: These events consist of completing a circle in \( u:2R \theta \) space by control of all three displacements. The vertical load is kept constant by a feedback system. The resulting load path gives an indication of the shape of the yield surface in \( H:M/2R \) space.

(g) Radial Loading Tests: These involve applying a linear path in load space to the footing and examining the resulting path in displacement space. Typically the horizontal and/or moment load is specified as a fraction of the vertical load. These tests give further information about the hardening relationship for the yield surface.

Overall ten sample tanks were prepared with sand in a very dense state, enabling 80 test sites, and 250 individual test events to be completed. The interpretation of these results will enable the construction of a plasticity model for skirted and flat footings on dense sand. Selected data will be used in the following sections to illustrate behaviour.

Analysis of Results

This section includes some re-interpretation of previously published data specifically relating to flat footings, as well as analysis of the new data on skirted footings.

Vertical Loading Behaviour and Interpretation of Hardening Law. An explicit indication of the hardening of the yield surface can be observed in the vertical loading tests, in which the horizontal and moment loads are kept to a minimum whilst the vertical load is increased. For the tests analysed in this paper a relatively simple representation of the hardening law can be used. The tests in question probed only very low values of the ratio \( V/V_o \), at which stage the vertical loading relationship is essentially linear. The vertical displacement, \( \delta w \), is defined as the sum of elastic and plastic displacements:

\[
\delta w = \delta w_e + \delta w_p
\]

where \( \delta w_e \) is the elastic vertical displacement and \( \delta w_p \) is the plastic vertical displacement. Any vertical loading behaviour inside the yield surface can be represented by:

\[
V = k_e w_e
\]

where \( k_e \) is the elastic stiffness of the foundation on the sand. The hardening of the surface can then be linked to the plastic vertical displacements such that the apex of the yield surface, \( V_o \), is defined as:

\[
V_o = k_p w_p
\]

where \( k_p \) is the plastic stiffness of the foundation on the sand. Adopting this idealisation proves to be most convenient when interpreting the swipe tests. During the swipe test the vertical penetration remains constant while the vertical load reduces. The elastic vertical displacement is negative, and is balanced by an equal positive plastic vertical displacement. This implies an increase in \( V_o \), which must be taken into account in subsequent analysis. The modified value \( V_{oN} \) can be calculated as:

\[
V_{oN} = \frac{V_o k_e - V_p k_p}{k_e - k_p}
\]

where \( V_o \) is the value for \( V_o \) at the start of the swipe and \( V \) is the instantaneous value of vertical load. Figure 7 shows an accumulation of vertical loading tests, for various footings, taken up to a vertical load of 2400N, the limit of the loading rig. During these tests two unload-reload loops were undertaken to determine the vertical elastic stiffness. An accumulation of unload-reload loops is also shown in Figure 7, showing good repeatability between sample tanks, and little effect of footing skirt depth. In general there is little to
distinguish between the vertical loading tests for the different skirt depths. It appears that the effect of the overburden on the foundation response is not significant at these low values of \(V/V_{\text{peak}}\). It is also likely that the slight disturbance to the sample immediately around the skirt, caused during the penetration, may mean that the skirt itself contributes little to vertical capacity at small penetrations. From the elastic unloading/loading results it is clear that, whilst there is some hysteresis evident, 3000N/mm would be a representative elastic stiffness. A value of 600N/mm is representative of the plastic stiffness, in the range of loads from 500N to 2000N.

Throughout the following analysis \(V_{\text{peak}}\) refers to the peak vertical bearing capacity of a flat surface footing (unless otherwise specified by the subscript \(\text{skirted}\)). In places the skirted footings are referenced to a bearing capacity of a skirted footing, so as to account for overburden effects. This value could be estimated by:

\[
V_{\text{peak(}\text{skirted)\text{)}} = \gamma LN_q^{*} \left( \frac{\pi \sigma^2}{4} \right) + \gamma \frac{D}{2} N_l^{*} \left( \frac{\pi \sigma^2}{4} \right)
\]

\[\text{(7)}\]

using values for the bearing capacity factors taken from Bolton and Lau\(^4\). The frictional component is typically small for the short skirt depths being studied, but the overburden effect can add significantly to the bearing capacity. The value for \(N_q^{*}\) can be obtained by interpolating between values from Bolton and Lau\(^4\), based on \(N_q^{*}\) determined from the bearing capacity tests. A typical value for \(K\tan\delta\) is 0.25, and it is recognised that \(V_{\text{peak(}\text{skirted)\text{)}}\) is insensitive to slight changes in this value. From above it is clear that:

\[
\frac{V_{\text{peak(}\text{skirted)\text{)}}}{V_{\text{peak(}\text{flat)\text{)}}}} = 1 + \frac{L}{D} \left( \frac{2}{N_q^{*}} + \frac{L}{2D} \tan \delta \right)
\]

\[\text{(8)}\]

Equation 8 can be approximated for the values of \(L/D\) being explored in this study, combined with the appropriate bearing capacity factors, by the following linear relationship:

\[
\frac{V_{\text{peak(}\text{skirted)\text{)}}}{V_{\text{peak(}\text{flat)\text{)}}}} = 1 + 0.89 \frac{L}{D}
\]

\[\text{(9)}\]

If the peak bearing capacity for the flat footing is scaled directly from the results of the 50mm diameter footing bearing capacity test used for characterisation, then it is straightforward to estimate the value for the skirted footing bearing capacity. Unfortunately, none of the skirted footings was taken to bearing capacity failure, so the values for \(K\tan\delta\) and \(N_q^{*}\) were not verified. However, it is clear that the capacity increases significantly with skirt depth.

**Swipe Testing of Flat Footings.** Before examining the behaviour of skirted footings it is necessary to explore fully the response of a flat footing, as that provides the baseline for examining the skirted footing performance. The results from the study by Gottardi and Houlshby\(^{15}\), interpreted by Gottardi et al.\(^{17}\), provide a convenient starting point.

**A Conceptual Understanding of Results.** Gottardi et al.\(^{17}\) assumed that the yield surface remained constant in shape, while expanding according to the work-hardening rule. The yield surface was based on the results of the main group of swipe tests conducted from \(V_o = 1600\)N, at a ratio of \(V/V_{\text{peak}}\) of 0.78. Several additional swipe tests (in tests GG09, GG24, GG13 and GG27) started from different vertical loads. Examining firstly the behaviour of the footing in the \(\text{V:H}\) plane, Figure 8(a) shows a series of tests, all starting from \(V/V_o =1\), with forces normalised by the \(V_o\) value. Figure 8(b) shows the same tests but with forces normalised by \(V_{\text{peak}}\). It is clear that the shape of the yield surface changes in a consistent way with the ratio \(V_o / V_{\text{peak}}\). It appears that Tan’s\(^{27}\) ‘parallel point’ (analogous to a critical state) is close to the origin, as the load on the foundation does not tend to a finite limiting value, as would be expected if the parallel point existed at some distance from the origin. This observation is confirmed by Gottardi and Houlshby’s horizontal swipe test on a sample starting at low \(V/V_o\).

This last observation conflicts with those of Tan\(^{27}\), who noted that a parallel point for horizontal movement occurred at \(V/V_o\) of 0.152. It is quite probable that his swipe tests involved a slight moment loading, which was not measured during the testing. It may well be that the parallel point moves significantly away from the origin, towards the location for pure moment loading (as detailed below), for quite small moments (as may have been applied in Tan’s tests).

Examining the behaviour in the \(\text{V:M/2R}\) plane, a slightly different behaviour appears. Again a dependence of the shape of the yield surface on the ratio of \(V/V_{\text{peak}}\) is indicated, as shown in Figures 9(a) and 9(b). The parallel point appears in this case to be located at some distance from the origin. It can be defined more consistently in the plot normalised with respect to \(V_{\text{peak}}\), and occurs at about \(V/V_{\text{peak}}\) of 0.26 and \(M/2RV_{\text{peak}}\) of 0.061. Using these values it is clear that the value for \(M/2RV\) tends to about 0.25 for large rotations. By simple rearrangement this leads to a maximum load eccentricity, \(e\), of \(R/2\), given that \(M = V_e\).

Whilst a model with a fixed shape of the yield surface captures approximately the behaviour of the foundation, a more subtle modelling is necessary to capture the features mentioned above. This is particularly important given that the typical ratios of \(V/V_{\text{peak}}\) applicable to the design of offshore structures would often be below about 0.25. Figure 10 shows a new understanding of the yield surface, in which the shape depends on the \(V/V_{\text{peak}}\) ratio. Precise mathematical forms have not yet been derived for the component parts of this new yield surface. In the following the surface through \(V_{\text{peak}}\) will be referred to as the outer surface, and those through lower \(V_o\) values the inner surfaces (of variable shape). The inner surfaces each appear to intersect the outer surface, so that at low \(V/V_o\) all yield points fall on the outer surface.
Behaviour of Flat Footings in the H:V Plane and the M/2R:V Plane (DM2 and DM3). The first two series of tests on very dense sand were used to further the understanding of the flat footing behaviour as illustrated in the Gottardi and Houlsby tests. Under horizontal loading, four swipe tests were carried out from $V/V_o = 1$, but at different ratios of $V/V_{peak}$ including values much lower than those examined by Gottardi and Houlsby. Figures 11(a) and (b) show the results normalised with respect to $V_o$ and $V_{peak}$. For the tests at very low stresses there is a significant change in the shape of the yield surface. After an initial reduction of $V$, the vertical load increases again as the footing slides at approximately constant $H/V$. This is indicative of dilatant behaviour for the tests at lower stress levels. At large displacements sliding failure continues with a reduction of $V$ again occurring. The tests confirm that the ‘parallel point’ for horizontal loading is either at or very close to the origin. Two further tests (shown as thin lines on Figure 11) are for swipe tests starting at $V/V_o < 1$. The filled square points indicate approximately the transition from ‘elastic’ to ‘plastic’ behaviour for these tests. The test starting from $V/V_o = 0.5$, shows significant elastic behaviour until the load point encounters the yield surface, as shown for the test from $V/V_{peak} = V/V_o = 0.22$ (the corresponding test at $V/V_o = 1$). The other test, at a significantly lower $V/V_o$, suggests that a transition is reached slightly within the outer yield surface (sliding line). Both tests reach the sliding line and move towards the ‘parallel point’ at or close to the origin. A further example of the location of the boundary between elastic and plastic behaviour can be seen in Figure 12. This shows the load and displacement paths for three tests. The first test (A) is a swipe test from $V/V_o = 1$ and $V/V_{peak} = 0.16$. It shows slightly dilatant behaviour as the sliding failure line is reached, after which the load point moves towards the origin. The point at which the sliding line is reached is given by a solid square point on both plots. As expected there is significant plastic horizontal displacement during the swipe test. The second test (B) is a constant vertical load test, from $V/V_{peak} = 0.1$, with again a value $V/V_{peak} = 0.16$. Test (A) would be expected to give an indication of the position of the yield surface that existed prior to the constant $V$ test. The open square on the graphs indicates the location of the boundary between ‘elastic’ and ‘plastic’ behaviour, whilst the solid square indicates when the sliding line is reached.

It is clear that there is minimal displacement until after the yield surface is intersected, after which there are large combined displacements, which continue after the sliding line is reached. There is substantial vertical penetration during the plastic displacement, which is consistent with the continual loss of vertical load during the horizontal swipe. The third test (C) shown on Figure 12 is a radial loading test, which again was preloaded to $V/V_{peak}$ of 0.16. A similar range of ‘elastic’ behaviour, in load space, compared to the constant load swipe test can be observed.

Under rotational loading it is clear that a ‘parallel point’ may exist, as observed in the Gottardi et al results. However, the location of this point was beyond the limits of the loading rig. Seven swipe tests from $V/V_o = 1$ were performed as shown in Figure 13, showing clearly that the load paths follow a yield surface before moving upward to a parallel point. Figure 13(a) shows that the shape of the yield surface remains approximately constant, except for the tests performed at very low values of $V/V_{peak}$, also shown on Figure 13 are five tests (shown as thin lines) from $V/V_o < 1$. One test (D) shows some ‘elastic’ behaviour before following the appropriate yield surface to the rotational boundary. Upon reaching the rotational boundary the load path moves towards the ‘parallel’ point. The other swipe tests all move towards the ‘parallel’ point directly, but show only small horizontal displacements until close to the sliding line.

Three tests similar to those shown in Figure 12 were performed to assess the elastic boundary for moment loading, and are presented in Figure 14 in the same format as Figure 12. The closed square symbol shows where the swipe test encountered the rotational boundary, and a change of load path follows. The second test (B) is the constant load test, which was preloaded similarly swipe test (preloaded to $V/V_{peak}$ of 0.16). The open square symbol on both plots shows the transition from ‘elastic’ behaviour to ‘plastic’ behaviour, which occurs at some distance from the yield surface anticipated from the swipe test. Upon reaching the rotational boundary significant heave is apparent as the soil beneath the foundation dilates strongly. This is consistent with the movement towards the parallel point in the swipe test. The third test (C) is a radial loading test, again on a foundation which has been preloaded to $V/V_{peak}$ of 0.16. The elastic/plastic transition point is marked by an open square symbol, and the position is consistent with the constant $V$ test.

Current Results and Conventional Bearing Capacity Theory. Conventional theories of bearing capacity, such as that proposed by Meyerhof and Hansen are usually as regarded as conservative when compared to experimental evidence. These theories use reduction factors on the vertical bearing capacity to account for shape, load inclination and load eccentricity amongst other effects. These factors are largely derived from empirical observations, together with some theoretical input. By manipulation of the reduction factor expressions it is quite simple to derive an interaction envelope for say $H:V$ or $H:M$ or even $V:M:H$ space. For example the vertical bearing capacity of a surface strip footing is:

$$V_{peak} = A(0.5\gamma B N_{1\gamma})$$ ..............................................(10)

If an inclined load is applied then the sustainable vertical load is reduced by a factor $i$, such that:

$$V = A(0.5\gamma B N_{1\gamma})$$ ..............................................(11)

where Meyerhof suggests:
i_T = \left(1 - \frac{\Omega}{\Phi}\right)^2 \text{ and } \Phi = \arctan\left(\frac{H}{V}\right) \quad \text{(12)}

By simple rearrangement it is possible to obtain:

\[
\frac{H}{V_{\text{peak}}} = \frac{V}{V_{\text{peak}}} \tan\left[1 - \left(\frac{V}{V_{\text{peak}}}\right)^{0.5}\right] \quad \text{(13)}
\]

which is an expression of interaction between \(H\) and \(V\). An alternative expression for \(i_T\) as suggested by Hansen\(^8\) is:

\[
i_T = \left(1 - \frac{H}{V}\right)^4 \quad \text{(14)}
\]

which leads to an interaction solution of:

\[
\frac{H}{V_{\text{peak}}} = \frac{V}{V_{\text{peak}}} \left[1 - \left(\frac{V}{V_{\text{peak}}}\right)^{0.25}\right] \quad \text{(15)}
\]

Surprisingly the Hansen solution makes no use of the angle of internal friction of the soil, which intuitively needs to be included if horizontal sliding of the foundation is incorporated at low ratios of \(V/V_{\text{peak}}\). These relationships can be compared with the outer yield surface suggested in Figure 10.

A similar relationship can be obtained for the interaction between moment load and vertical load. Meyerhof\(^2\) suggested that a footing under moment loading be considered as the statically equivalent vertical load acting at an eccentricity \(e\) from the centre of the footing. The footing then has reduced width, called an effective width, of \((B - 2e)\), and the appropriate interaction relationship can be derived:

\[
\frac{M}{BV_{\text{Peak}}} = 1 \frac{V}{2V_{\text{Peak}}} \left[1 - \left(\frac{V}{V_{\text{Peak}}}\right)^{0.5}\right] \quad \text{(16)}
\]

Hansen (1961) also adopts this approach. A similar approach can be found in the procedures recommended by the American Petroleum Institute\(^1\) for the drained design of shallow foundations. Care should be taken when using these expressions to interpret the results of experimental investigations. Typically it is suggested that the expressions provide a conservative estimate to the capacity interaction, but they also provide quite a close approximation to the experimentally observed outer yield surfaces. This can be seen in Figures 15 and 16 which show the appropriate curves with the experimental data reported here as well as that obtained by Gottardi and Houlsby\(^3\). Clearly the conventional theories would provide a conservative estimate if they were to be used in terms of \(V_o\) instead of \(V_{\text{peak}}\) and then applied to results from tests conducted at ratios of \(V/V_{\text{peak}} < 1\).

**Pre-Peak V:M:H Flat Footing Behaviour (DM9).** Further testing was carried out to determine the shape of the yield surface at different ratios of \(V_o/V_{\text{peak}}\) and \(2RH/M\) for a flat footing. During the testing, in tank DM9, the footing was moved not more than 0.75mm, such that the outer yield surface was not intercepted. By conducting the testing in this manner it was possible to carry out multiple tests on individual sites, to build a complete three-dimensional picture of the shape of the yield surface. Swipe tests were conducted at several ratios of \(2RH/M\), to provide sufficient data for a simple surface fitting exercise to be carried out. By assuming that the inner yield surfaces can be described by a three-dimensional shape, it is possible to determine parameters with which to assess the development of the yield surfaces with the increase of \(V_o/V_{\text{peak}}\). The simplest mathematical form of such a shape (a rotated parabolic ellipsoid) is:

\[
\left(\frac{H}{h_oV_o}\right)^2 + \left(\frac{M}{m_o2R V_o}\right)^2 - \frac{2aHM}{h_o m_o 2 R V_o^2} = 0
\]

\[
-16\left(\frac{V}{V_o}\right)^2 \left[1 - \left(\frac{V}{V_o}\right)^{0.25}\right] = 0
\]

where the values of \(m_o, h_o\), and \(a\) can be determined from a least squares analysis of the data. A simpler assessment of the data can be obtained by normalising individual data points \((H, M/2R)\) using the expression:

\[
q = \sqrt{\left(\frac{H}{h_o V_o}\right)^2 + \left(\frac{M}{m_o 2 R V_o}\right)^2 - \frac{2aHM}{h_o m_o 2 R V_o^2}}
\]

which collapses the data onto a single plane in \(q:V/V_o\) space.

In this space the yield surface becomes the parabola \(q - 4\nu(1 - \nu) = 0\), where \(\nu = V/V_o\). Figure 17 shows such a normalisation for the data from \(V_o/V_{\text{peak}} = 0.28\), showing swipes conducted from \(V=1800N\).

**Accumulation of Test Results for Flat Footings.** The accumulation of test results reported here and those by Gottardi and Houlsby is shown in Figure 18. In both instances the footing diameter was 100mm, bearing on dense to very dense sand. The \(h_o\) and \(m_o\) values are mainly derived by modelling individual swipe tests with a simple parabola. There is a strong correlation in the data with trend lines given by:

\[
h_o = h_{o\text{peak}} \left[1 - 0.36\ln\left(\frac{V_o}{V_{\text{peak}}}\right)\right] \quad \text{(19)}
\]

\[
m_o = m_{o\text{peak}} \left[1 - 0.36\ln\left(\frac{V_o}{V_{\text{peak}}}\right)\right] \quad \text{(20)}
\]

where \(h_{o\text{peak}} = 0.11\) and \(m_{o\text{peak}} = 0.08\), corresponding to the yield surface at the peak bearing capacity. These expressions are validated only for \(V_o/V_{\text{peak}} > 0.025\). The peak values compare well with the interaction loci that Meyerhof\(^3\) suggests for \(\Phi = 43^\circ\), where \(h_{o\text{peak}} = 0.113\) and \(m_{o\text{peak}} = 0.074\) (occurring at \(V/V_{\text{peak}} = 0.44\).
**Swipe Testing of Skirted Footings.** A large proportion of the testing was to study the effect of a skirt on the drained behaviour of the foundation. Broadly speaking the behaviour of a flat footing will give indications of the general behaviour of the skirted foundation. The effects of different ratios of \( V/V_{\text{peak skir}} \) was not therefore examined in detail for the skirted footings. The testing at individual test sites was restricted to not more than two swipes, since the site was then sufficiently disturbed to affect the results.

**Behaviour of Footings in the H:V Plane and the M/2R:V Plane for Different Ratios of L/D (DM4 and DM5).** Several tests were conducted in which the only parameter varied was the embedment ratio, \( y = L/D \). Swipe tests were performed at similar ratios of \( V/V_{\text{peak}} \) for different footings to ascertain the effect of the skirt depth. For each set of tests the value for \( 2RH/M \) was fixed. In tank DM4 the value for \( H \) was set to zero, whilst in DM5 the value for \( M/2R \) was set to zero. Figure 19 shows the results from tank DM5, the horizontal swipe tests at \( V/V_o \) = 1 were conducted from starting values of \( V/V_{\text{peak}} \) of 0.08 and 0.21. The flat footing tests show behaviour typical of swipe tests, in which the load path traces around a yield surface, before tracking down the sliding line, towards the origin. At the low stress level the strongly dilatant sand causes a slight increase in capacity as it expands, before allowing the footing to follow the sliding line as the soil beneath the footing begins to contract. Looking at displacement paths, it is clear that, if associated flow is assumed in the \( H:M/2R \) plane, the yield surface for the flat footing is almost perpendicular to the \( H \) axis. A very slightly different displacement path is followed once the sliding line is reached.

Three skirted footings were used during these tests, with \( y \) ratios of 0.16, 0.33 and 0.66. There are minimal differences in the load paths tracked by these three footings, though it is clear that there is a significant difference between the shallowest of the skirted foundations and the flat footing. In all three cases strongly dilatant behaviour is observed as the load paths track slightly up the sliding line. If the tests had been taken to larger displacements it is expected that contractant behaviour would have been observed, as for the flat footing. There are, however, significant differences in the behaviour in displacement space, but for the deeper embedment depths these differences become progressively smaller. For the embedment ratio of 0.16 the yield surface crosses the horizontal axis at an angle of approximately 67° to the horizontal axis (assuming associated flow). At the embedment ratios of 0.33 and 0.66 it crosses at approximately 47° and 43° respectively. Clearly, for the skirted footings, the displacement path is such that both horizontal load and moment load are developed as the footing is displaced horizontally. To reduce the negative moment load developed the footing would have to rotate positively. The larger the skirt depth the larger are the positive rotations required to keep the moment to zero.

Similar observations can be made about the results of tank DM4, in which rotational swipes at \( V/V_o = 1 \) were carried out with horizontal loading set to zero (shown in Figure 20). There is no significant difference in the load paths traversed by the footing, but in displacement space there is a significant difference. This is to be expected because as the footing rotates positively to create positive moments, a negative horizontal load is induced on the skirted footing. To reduce this horizontal load to zero, the footing must experience positive horizontal displacements, the magnitude of which depends on the skirt depth of the footing. There is a well-defined relationship between the skirt depth and the ratios of rotational to horizontal footing movement in this case. Again assuming normality of flow in this plane it is clear that the yield surface crosses the moment axis at approximately 9° (\( y = 0 \)), 14° (\( y = 0.16 \)), 22° (\( y = 0.33 \)) and 34° (\( y = 0.66 \)).

The surprising aspect of these tests is that the drained capacity for pure moment loading is not significantly enhanced by the skirt. It is clear though that as the skirt depth increases that the size of the yield surface in the horizontal loading direction becomes larger. This ties in with the variation of \( h_o \) with \( V/V_{\text{peak}} \) (evident in flat footing tests), given that \( V_o \) was constant for all tests, but \( V_{\text{peak skir}} \) increases for the skirted footings.

**Swipe Testing at Different Ratios of 2RH/M for L/D = 0.33 (DM6 and DM7).** To evaluate the changes of shape of the yield surface, two sets of testing were carried out at different ratios of \( V/V_{\text{peak skir}} \) and consisted of swipe tests from \( V/V_o = 1 \) at different ratios of \( 2RH/M \). Test series DM6 consisted of 16 swipes at \( V/V_{\text{peak skir}} = 0.16 \), whilst DM7 consisted of seven swipe tests at \( V/V_{\text{peak skir}} = 0.1 \). Both sets of tests used a footing of diameter 100mm and embedment ratio of 0.33.

Examining only test series DM6 (as DM7 shows similar trends). Figure 21 shows the load paths in the \( H:M/2R \) plane. The solid squares on the graph indicate the estimated transition points from inner yield surfaces to the outer envelope. They serve as an indication of the shape of the outer yield surface. The open symbols on the graph are points at selected values of \( V/V_o \), and indicate possible shapes of the inner yield surface. It appears that both inner and outer surfaces can be approximated as ellipses. Also included on Figure 21 is an indication of the yield surface shape, determined during a loop event at a constant load of 800N and \( V/V_o \) of 0.33.

An assumption made previously was that normality applied in the deviator plane (that is \( H:2R/M \) of load space, this has been indicated by other studies (for example Gottardi et al.)). It is possible to check whether this is true for these tests by plotting the force ratios against the plastic displacement ratios as in Figure 22. If normality holds then plasticity theory gives the plastic strain rate ratio as:

\[
\sigma_{up} = \frac{2RH_{m_o}}{(h_o - aM)/M - a2RH}
\]

This relationship has been plotted on Figure 22 using
parameters derived from least squares fitting of the experimental yield surface shape. The data support an associated flow rule, although there is some scatter, particularly in the negative quadrant in which the curvature of the yield surface is greatest.

**Determination of Shape of Outer Yield Surface (DM8).** It is clear that there is a significant change in the yield surface shape with increasing skirt depth. The most noticeable changes occur in the -H:+M and the +H:-M quadrants, which are not typically encountered in normal loading regimes. The yield surface shape in the positive quadrant becomes flatter with increasing embedment.

To determine the shape of both yield surfaces, inner and outer, would require a large number of swipe tests. It is possible to determine the shape of the outer yield surface at low vertical loads by performing loop events at constant load. These are performed by imposing a circular displacement path in deviator displacement space whilst keeping the vertical load constant. If the footing is undergoing plastic deformations the load path traced should be across the yield surface. If the footing is at low \( V/V_{peak} \) then small deformations will be required to achieve plastic deformations.

By using constant load loop events it is possible to map out the probable yield surface shape for different embedment depths on the same sample of sand. This is important, as it simplifies interpretation by minimising the possible effects of sample differences. Each site on the sample was subjected to several loop events at different levels of vertical load. An accumulation of these results, normalised by the parameters defined below, is shown in Figure 23. The normalised variables are defined as:

\[
h = \frac{H}{V_{peak(skirted)}h_0C_v^\beta_1(1-\nu)^\beta_2} \tag{22}
\]

\[
m = \frac{M}{2RV_{peak(skirted)}m_nC_v^\beta_1(1-\nu)^\beta_2} \tag{23}
\]

where \( \nu = V/V_{peak(skirted)} \) and \( C \) is defined as:

\[
C = \frac{(\beta_1 + \beta_2)(\beta_1 + \beta_2)}{\beta_1^\beta_1 \beta_2^\beta_2} \tag{24}
\]

The constants used in the normalisation are separately defined by least squares fitting of the yield surface for each skirt depth. It is clear that as the skirt depth increases the shape of the ellipse, as indicated in normalised deviator load space by the pronounced elongation of the surface.

**Implications for the Offshore Design Engineer**

The results presented in this paper have provided information that enables an improved understanding of the behaviour of skirted footings under combined loading, particularly on very dense sand. These results are particularly relevant to the design of small shallow foundations for offshore structures, in which the drained behaviour will be the limiting design case. The drained behaviour also provides a lower bound to the partially drained performance of foundations, and as such the results must also be considered in the design of much larger foundations.

**Effects of Scale.** The objective of this research was to establish the general behaviour of a foundation subjected to combined loading. The effects of scale are not significant, as the generic behaviour of the foundation at model scale is not expected to differ significantly from one at prototype scale. However, it is worthwhile making some comments on the scaling of results, as the numerical values of parameters may change with scale. It is assumed that the model and prototype sands are the same, and deposited at the same density. The vertical capacity is a function of the loaded area and the soil strength. If \( N \) is a geometric scaling factor then:

\[
\text{Area}_{\text{prototype}} = N^2 \text{Area}_{\text{model}} \tag{25}
\]

Soil strength is a function of the stress level in the soil, and these stress levels will factor by \( N \). Soil strength does not scale linearly, because at the low stress levels (in the model) the peak friction angle and dilation rate of the sand are higher than at the high stress levels at prototype scale. Thus soil strength will scale only with approximate order of magnitude \( N \). Combining the two leads to all forces (that is \( V: \frac{M}{2R:H} \)) scaling with approximate order \( N^3 \). Bolton\(^3\) gives an indication of the dependence of strength of a granular material on the stress level, acknowledging the non-linearities involved. To examine further how these parameters may differ with scaling to prototype dimensions it would be necessary to perform key one gravity tests at either field scale or in a geotechnical centrifuge.

**Positive Quadrant Negligibly Affected by Change in Skirt Depth.** One surprising aspect of the results, as discussed above, is that the yield surface in the quadrant of positive loading is relatively unaffected by the depth of the skirt. The loading that would typically be applied to an offshore foundation under normal service conditions falls in this quadrant. Whilst the overall shape of the positive quadrant remains similar, it is clear that for the same diameter footings a skirted footing will have a larger compressive horizontal capacity. Given that the non-dimensional sizes of the yield surface are similar, the maximum horizontal and rotational capacities will factor approximately with with the increase in \( V/\text{peak}\text{skirted} \), due to the increased embedment. By adopting a deeper skirt it might be possible to reduce the diameter of the foundation and still achieve suitable performance. The depth of the skirt is limited by the ability to install the foundation by suction and dead weight.

**The Concept of Safety Factors.** The research indicates that two possible approaches can be made for defining safety factors for a footing on sand. It is possible to define a safety factor against significant or plastic movements, as would be required for serviceability requirements. Load combinations
crossing the internal yield surfaces can be sustained by the sand, but at the expense of some limited plastic footing displacements. If load combinations are such that the outer yield surfaces is reached then more substantial footing displacements, and foundation 'failure', will be observed. These outer yield surfaces, such as those defined by Meyerhof24 and Hansen18 in their early studies, indicate the maximum combinations of load that can be sustained by the sand, regardless of displacements. Safety factors could alternatively be defined in terms of these outer surfaces. For maximum performance of the foundation it would be preferable to preload to a large percentage of the bearing capacity, before reducing to a working load of \( V/N_{\text{peak}} \approx 0.5 \) (which should lead to the largest safety factors against failure for typical loading patterns). In most situations it may not be possible to apply preload to the foundations, however, it may still be preferable to design the footing for operation at \( V/N_{\text{peak}} = 0.5 \).

**Development of Numerical Models for Use in Structural Analyses Packages.** Numerical models based on work hardening plasticity theory can be used to model skirted foundations on dense sand as 'macro' elements within structural analyses packages. The behaviour of the soil-foundation combination is highly complex, as illustrated in this paper, and needs to be appropriately represented within such analyses so that realistic results can be obtained. This is gradually being undertaken, though primarily at a research level, rather than as part of a typical analysis of an offshore structure. This is in part due to the lack of data available for the derivation of parameters to define the numerical model, although this is being addressed23,15,9. The model development has so far been focussed on the drained behaviour of granular soil, in response to loading on the foundation. It is clear from this paper that the understanding of the drained response has improved significantly over the past few years, and it is now appropriate to consider the partially drained behaviour of foundations on granular materials.

**Conclusions**

In this paper we present the results of a series of loading tests of model footings on very dense sand. A sophisticated loading rig was used to complete over 250 individual tests to determine the behaviour of footings with different skirt depths under a variety of conditions. These footings are representative of a novel offshore foundation, called a 'suction caisson', which has become a viable alternative in the development of marginal offshore resource fields. The results represent the long-term (drained) response to general loading regimes. The tests were specifically designed to allow derivation of models of foundation behaviour based on work-hardening plasticity theory. This investigation should be of interest to the oil and gas industry as the understanding of load-transfer mechanisms for these foundations are still in their infancy.

Previous studies have indicated that plasticity theory may be able to provide a rational framework for understanding this complex non-linear geotechnical problem. This paper has shown that the response of a skirted footing can be successfully described within a plasticity context. The yield surfaces in \( \{V:M/2R:H\} \) space are shown to be well described by parabolic ellipsoids, as indicated by several previous studies16,22,17,9. The study indicated that the shape of these yield surfaces, during pre-peak bearing capacity behaviour, was a function of the ratio \( V_{\text{r}}/V_{\text{peak}} \). The study also indicated the change in shape of the yield surface with increasing embedment ratio. In all cases the use of an associated flow rule in the deviatoric \( (M/2R:H) \) plane was supported by the tests.

The results will provide the basis for the development of simple numerical solutions to various aspects of the combined loading problem. They provide evidence of trends and relationships, which were previously not quantified. The development of numerical solutions is important as they make the design process simpler for the engineer, particularly during the conceptual stages of a project. The results also provide the basis for the development of more detailed models, which would be used during the design stage of a project.

**Acknowledgements**

The first author would like to acknowledge the generous support of the Rhodes Trust during the course of the research. The support and funding from the EPSRC (GR/L/64478 entitled "model tests of suction caissons in dense sand") for the experimental equipment and technician time is also gratefully acknowledged.

**Nomenclature**

- \( V_{\text{r}} \): Maximum vertical load previously applied to the footing (equal to the apex of yield surface)
- \( V_{\text{peak}} \): Peak vertical bearing capacity of a flat footing
- \( N_{\gamma} \): Bearing capacity factor for strip footing (friction)
- \( N_{q} \): Bearing capacity factor for strip footing (overburder)
- \( N_{c} \): Bearing capacity factor for strip footing (cohesion)
- \( N_{f} \): Bearing capacity factor for circular footing (friction)
- \( i_{r} \): Inclination factor
- \( V \): Vertical load on footing
- \( H \): Horizontal load on footing
- \( M \): Moment load on footing
- \( v, h, m \): Non-dimensional load parameters
- \( e \): Eccentricity of vertical load causing moment
- \( R \): Radius of footing
- \( D \): Diameter of footing
- \( B \): Width of strip footing
- \( L \): Skirt depth of footing
- \( y \): Embedment ratio
- \( w, u \): Vertical displacement, Horizontal displacement
- \( 2R\theta \): Rotational displacement
- \( k \): Vertical stiffness coefficient
- \( \gamma \): Effective unit weight of soil
\[ R_D = \text{Relative density of soil} \]
\[ \phi = \text{Angle of friction of soil} \]
\[ h_y = \text{Horizontal dimension of yield surface} \]
\[ m_e = \text{Moment dimension of yield surface} \]
\[ a = \text{Eccentricity of yield surface ellipse} \]

References

Coefficient of Uniformity, $C_u$ 3.64
Specific Gravity, $G_s$ 2.69
Minimum density, $\gamma_{\text{min}}$ 12.72 kN/m$^3$
Maximum density, $\gamma_{\text{max}}$ 16.85 kN/m$^3$
Critical state friction angle, $\phi_{\text{cs}}$ 32.5°
Permeability (80% Rd with water), $k$ $7 \times 10^{-6}$ m/s

Table 1 - Properties of the Baskarp Cyclone Sand used throughout the testing

<table>
<thead>
<tr>
<th>Sample</th>
<th>Test</th>
<th>$\gamma$ kN/m$^3$</th>
<th>$R_D$</th>
<th>$V_{\text{max}(50)}$ kN</th>
<th>$q_{\text{fail}}$ kPa</th>
<th>$N^*_f$</th>
<th>$\phi$ deduced from $R_D$ (Bolton$^3$)</th>
<th>$\phi$ deduced from $N_f^*$ (Bolton and Lau$^4$)</th>
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<tr>
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<td>1093.07</td>
<td>44.29</td>
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Table 2 - Summary of Bearing Capacity Tests carried out during testing series

Fig.1 - Sign convention and notation

Fig. 2 - Cigar shaped yield surface in $V:M/2R:H$ space
Fig. 3 - Combined loading rig at Oxford.

Fig. 4 - Particle size distribution for experimental sand.

Fig. 5 - Typical tests required to explore plasticity concepts.

Fig. 6 - Bearing capacity test results.
Fig. 7 - Assembly of vertical loading curves for different footings (unload-reload loops are shown inset).

Fig. 8 - $V: H$ results from Gottardi and Houlsby normalised by (a) $V_{\text{peak}}$, and, (b) $V_o$.

Fig. 9 - $V: M/2R$ results from Gottardi and Houlsby normalised by (a) $V_{\text{peak}}$, and, (b) $V_o$. 
Fig. 10 - New framework for understanding of yield surfaces for combined loading response of foundations.

Fig. 11 - \( V:H \) results from current study for flat footings normalised by (a) \( V_{\text{peak}} \), and, (b) \( V_o \).

Fig. 12 - Different loading tests under horizontal loading shown in (a) load space, and (b) displacement space.
Fig. 13 - $V/M/2R$ results from current study for flat footings shown normalised by (a) $V_{\text{peak}}$, and (b) $V_o$.

Fig. 14 - Different tests under moment loading shown in (a) load space, and (b) displacement space.

Fig. 15 - Conventional theory for horizontal loading compared to recent results.

Fig. 16 - Conventional theory for moment loading compared to recent results.
Fig. 17 - Data from test series DM9 normalised and plotted in $q : V/V_o$ space.

Fig. 18 - Accumulation of results from various studies of flat footings showing dependence of parameters on $V_o / V_{peak}$.

Fig. 19 - Results from horizontal swipe tests (DM5) showing test paths in (a) load space, and, (b) displacement space.

Fig. 20 - Results from moment swipe tests (DM4) showing test paths in (a) load space, and, (b) displacement space.
Theoretical relationship

Experimental Values

Fig. 21 - Accumulation of swipe tests from $V/V_o = 1$ for series DM6 ($y = 0.33$).

Fig. 22 - Assessment of associated flow rule in the deviatoric plane for series DM6.

Increase in $L/D$ ratio

Fig. 23 - Accumulation of results of loop events for different skirt depths.