High Speed Compressors

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ABSTRACT

There is a requirement for small cryocooler compressors with a high power density, and one method of achieving this is to increase the operating frequency of the compressor. The 'Oxford' type clearance seal/flexure bearing compressors are typically operated close to their resonant frequency, which can theoretically be increased by either reducing the moving mass, increasing the spring stiffness or both.

There are potential improvements to be made in the design of the flexures – traditional designs have a poor method of clamping the springs, and the use of high strength alloys and improvements in surface finish can give a significant improvement in spring stiffness. However, in a typical compressor the mechanical spring stiffness is typically about a third or a quarter of the total stiffness, with the remainder coming from the gas spring effect of the compression process. If the mechanical spring rate is doubled, the frequency will only increase by about 12%.

A large increase in spring stiffness can be obtained by use of an auxiliary gas spring, which could be located at the opposite end of the compressor to the main piston. Use of such a gas spring has some disadvantages: there is a second piston/cylinder assembly to manufacture, assemble and align, together with extra thermodynamic losses.

This paper describes how the losses from a gas spring can be evaluated in order to optimize the size and stiffness of such a spring. An example is given, based on an existing Stirling cycle compressor, of how the power output of a compressor can be doubled with a 40% increase in size.

Though this describes the theoretical increase in power of an existing machine, it is reasonable to assume that these techniques can used in the design of small compact cryocooler compressors.

INTRODUCTION

The "Oxford" type of flexure bearing/clearance seal cryocooler has a heritage that dates back to about 1980. These early cryocoolers operated at about 40 Hz, and machines to this basic design are still being manufactured. Since then improvements in design have produced a significant increase in operating frequency, and a parallel improvement in specific mass (Watts of cooling per kg). Recent developments have led to a requirement for smaller, lower cost cryocoolers that can be manufactured in higher quantities and deployed with short lead times.

The size and mass of a cryocooler is determined largely by the compressor, and the primary function of this is to convert electrical power into the Pressure-Volume (*P-V*) power required by

the thermodynamic processes in the cold head. The power delivered by a compressor piston to the gas can be approximated by

$$Power = k\,f\,\Delta P\,\Delta V \tag{1}$$

Where f is the operating frequency, ΔP is the pressure swing, and ΔV is the swept volume and k is a constant which represents the shape of the "P-V" loop.

If a reduction in overall size is required, the following paths should be followed:
- Design for the maximum ΔV within the overall envelope.
- If a further reduction in size is needed, then the possibility of increasing ΔP and the operating frequency should be investigated.

The pressure swing in a compressor is largely a function of fill pressure, assuming that the 'working' volumes within the system have already been minimised by good thermodynamic design. Increasing the fill pressure is straightforward, but the price for this is an increase in thickness of the pressure containment, and this is more significant where flanged and bolted vessels are used. Thicker walls may also lead to higher thermal conduction losses.

Increasing the operating frequency is an obvious means of increasing system power. Historically this was limited by unwanted resonances in the arms of the spiral spring (the flexure), but with good design, this limitation can be avoided.

COMPRESSOR RESONANT FREQUENCY

To maximise the motor efficiency, virtually all cryocoolers operate at (or close to) their resonant frequency, which is defined by the spring rate of the system. Typically the spring rate consists of two elements:
- Mechanical Spring stiffness s_m
- Gas Spring Stiffness s_g, which arises due to the effective spring rate when the gas is compressed by the piston in the cylinder.

Hence with an effective moving mass m, the resonant frequency (in Hz) is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{s_m + s_g}{m}} \tag{2}$$

The gas spring rate can be approximated as

$$s_g = A_p^2\,\frac{\Delta P}{\Delta V} \tag{3}$$

where A_p is the frontal area of the piston.

It is worth noting that for most compressors, the gas spring stiffness is typically 3 to 4 times the mechanical spring stiffness; the typical spiral flexure does not have high axial stiffness.

To increase the operating frequency of a resonant compressor, it is necessary to increase the total spring stiffness and/or decrease the moving mass. It has been assumed that any cryocooler already has the moving mass reduced to the minimum that is permitted given the design specification and margins for the application.

MECHANICAL SPRINGS

A typical spiral flexure is shown in Figure 1. These are usually made by photo-etching, though wire EDM and water jet cutting are alternate techniques. The manufacturing method is rarely chosen to maximize the fatigue strength. A critical feature is the method of clamping – the conventional annular clamp areas around the inside and outside edges require 'teardrop' ends to the slots to minimize stress concentrations.

Figure 1. Typical Spiral Flexure.

Many springs used in cryocoolers are often not optimized, and there is potential for improvement in the following areas:-
- Choice of material
- Choice of manufacturing method
- Use of appropriate surface finish
- Optimisation of the clamping method to minimize stress concentration
- Optimisation of the spring arm shape.

Starting with an un-optimized spring, an increase of about 100% in (for example) the axial stiffness of a spring may be possible, but the percentage increase is very dependent on the starting point. Given that the mechanical springs have a minor role in determining the resonant frequency, even a 100% increase in spring stiffness would only lead to an increase in resonant frequency of about 12%.

COMPRESSOR GAS SPRING EFFECT

The gas spring stiffness is given by equation 3 above, and at first glance, the most obvious way to increase the gas spring stiffness is to increase the piston area (i.e. the diameter of the piston) and to decrease the stroke. With regard to spring design, a small stroke is desirable; and for a given diameter, the lower the stroke, the higher the spring stiffness (both radial and axial).

However, from the point of view of motor design, a large stroke is desirable. The motor efficiency is typically higher if the drive current can be reduced, and the current is usually proportional to the force produced by the coil. The power delivered is given by

$$Power = Force_{rms} \times Velocity_{rms} \tag{4}$$

With a small stroke, the velocity will be low, and the force required from the motor will be high, resulting in a higher current and higher motor losses. Hence, conventional designs try to maximise velocity and minimise force, which does not favour having a small stroke. Hence with a conventional design, there is a limit to the maximum stiffness of the gas spring effect that can be obtained from the primary compression process.

AUXILIARY GAS SPRING

A possible means of increasing the overall spring stiffness is with the use of an auxiliary gas spring. Such an arrangement is shown in Figure 2.

Auxiliary gas springs need careful design to be effective:
- For a given swept volume, there is an increase in overall size of the compressor.
- There is an increase in moving mass associated with the extra piston.
- For the auxiliary spring to be effective, there must be an overall increase in spring stiffness per moving mass.
- There will be thermodynamic losses associated with the gas spring arising from two phenomena:
  - A loss due to flow through the clearance seal between piston and cylinder.
  - A 'compression loss' due to irreversible heat transfer in the gas spring.
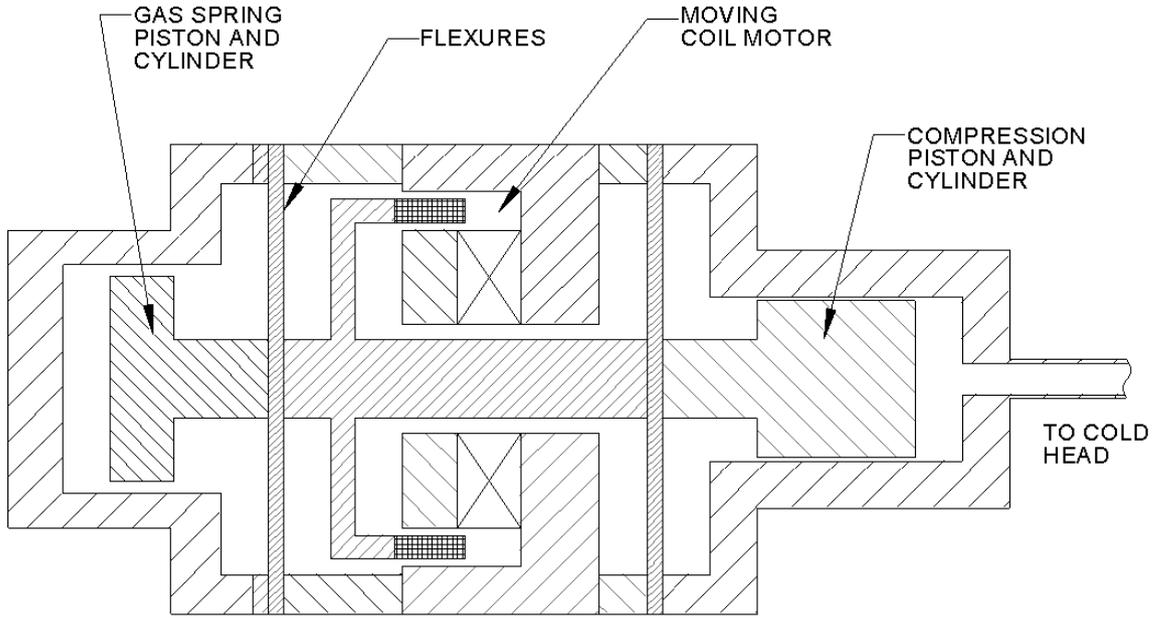
Figure 2. Compressor with Auxiliary Gas Spring

- Some compressors have issues with "DC offset" whereby if the flexures have low stiffness, gas pressures can cause the mid-point of oscillation to shift away from the 'mechanical zero' as defined by the mechanical springs; a process which can lead to an effective reduction in stroke[1,2]. A poorly designed gas spring could exacerbate this, but conversely, a good design could neutralise the "DC Offset" effect.
- The compressor has more components and the assembly process is more complex. With a gas spring there will be two piston/cylinder combinations to be aligned with the axis of motion of the compressor.
- The extra complexity is likely to lead to an increase in cost.
- Unless care is taken with the design, assembly methods and fixturing, it may be difficult to achieve the alignment required for satisfactory operation of the clearance seals.

GAS SPRING LOSSES

The losses in a gas spring were determined by Kornhauser and Smith[3], and their work has been confirmed by Oxford[4] and Lekić[5]. Kornhauser's expression for the gas spring loss is:

$$E_c = \frac{\pi}{2} p_0 V_0 \left(\frac{p_a}{p_0}\right)^2 \frac{(\gamma-1)}{\gamma} \frac{1}{y} \left(\frac{cosh(y)\,sinh(y)-sin(y)\,cos(y)}{cosh^2(y)-\,sin^2(y)}\right) \tag{5}$$

which can be rearranged to give non-dimensional loss term L_{nd} in terms of the Peclet Number Pe (see Figure 3):

$$L_{nd} = \frac{E_c}{\frac{\pi}{2} p_0 V_0 \left(\frac{p_a}{p_0}\right)^2 \frac{(\gamma-1)}{\gamma}} = \frac{1}{y} \left(\frac{cosh(y)\,sinh(y)-sin(y)\,cos(y)}{cosh^2(y)-\,sin^2(y)}\right) \tag{6}$$

Where

$$y = 0.49\, Pe_\omega^{0.43} \tag{7}$$

$$Pe_\omega = \frac{\omega D_h^2}{4\,\alpha_0} \tag{8}$$

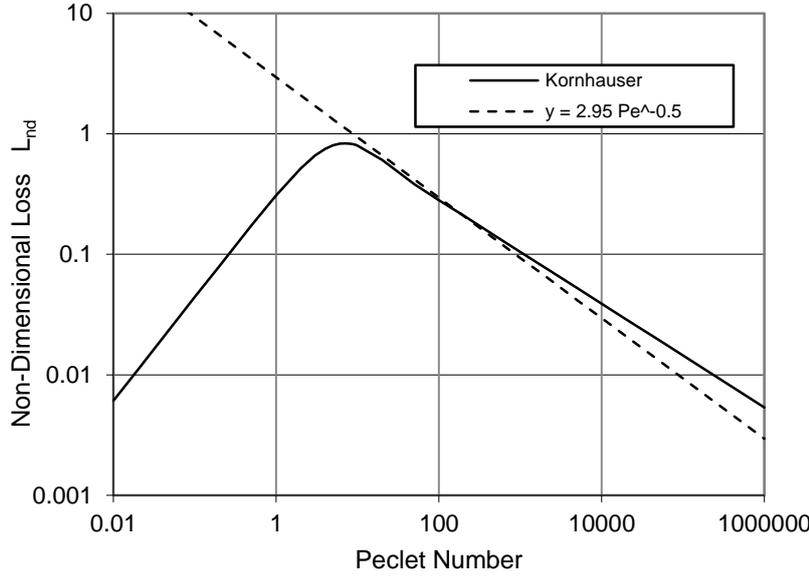$$D_h = \frac{4\,V_0}{A_0} \tag{9}$$

Figure 3. Non dimensional Loss Kornhauser & Smith[3] and equation 5, an inverse square law approximation.

E_c = Compression loss per cycle

$$E_c = \oint p\, dV \tag{10}$$

p_0 = pressure at mid-stroke
V_0 = cylinder volume at mid-stroke
p_a = pressure amplitude
γ = ratio of specific heats
ω = angular frequency
A_0 = cylinder surface area at mid-stroke
α_0 = thermal diffusivity of gas at mid-stroke, given by

$$\alpha = \frac{k}{\rho\, c_p} \tag{11}$$

Where k, ρ and Cp are all gas properties evaluated at mid stroke and are defined as

k = thermal conductivity
ρ = density
C_p = specific heat at constant pressure

For $Pe \gg 10$ the "cosh & sinh" term in the brackets on the right of equation (6) tends to 1, and the non-dimensional loss can be expressed as

$$E_c = 2.04\, Pe^{-0.43} \left\{ \frac{\pi}{2}\, p_0 V_0 \left(\frac{p_a}{p_0} \right)^2 \frac{(\gamma-1)}{\gamma} \right\} \tag{12}$$

An approximation to this can be made by assuming the loss varies with the inverse square root of the Peclet Number, i.e.

$$f(Pe) = K_p\, Pe^{-0.5} \tag{13}$$

Where K_p is a constant to give a good fit over a limited range of Pe (for example, the straight line in Figure 3). With this assumption, the loss, expressed as a power, can be reduced to

$$W_c \approx \frac{K_p}{8} \sqrt{\alpha\, \omega}\, A_0 \left\{ \frac{p_a^2}{p_0} \frac{(\gamma-1)}{\gamma} \right\} \tag{14}$$

The power loss due to leakage in a clearance seal, W_s, can be approximated as follows[6]

$$W_s = \frac{\pi D t^3 p_a^2}{24 \mu L_s} \tag{15}$$

D = Seal diameter
t = Seal radial clearance
p_a = Pressure amplitude
μ = Viscosity
L_s = Seal axial length

Note the following from these approximate loss terms:
- Both the seal loss and compression loss vary with the square of the pressure amplitude
- The seal loss is independent of frequency
- The compression loss varies with the square root of frequency

GAS SPRING GEOMETRY

The geometry of the gas spring can be defined as shown in Figure 4, where L_a is the stroke amplitude and L_e is the 'clearance length' of the gas spring, which determines pressure ratio.

If the pressure in the gas spring obeys a polytropic relationship

$$Pv^n = constant$$

Equation 3 can be written as function of the gas spring geometry as

$$S_g = \frac{\frac{\pi}{4} D^2 p_0 \left[\left(\frac{L_a}{L_e} + 1 \right)^n - 1 \right]}{L_a} \tag{16}$$

WORKED EXAMPLE

Having derived the equations for a gas spring, it is now possible to determine the effect of a gas spring on the design of a real compressor.

To avoid issues of confidentiality, this exercise has been carried out on a relatively old design of compressor, initially developed for use with a Stirling cycle domestic freezer[7], and subsequently used in a series of experiments on Gas Spring losses[4]. Note that this particular compressor is not compact, but does have reasonably high motor efficiency (80%). The method for evaluating the gas spring is as follows:
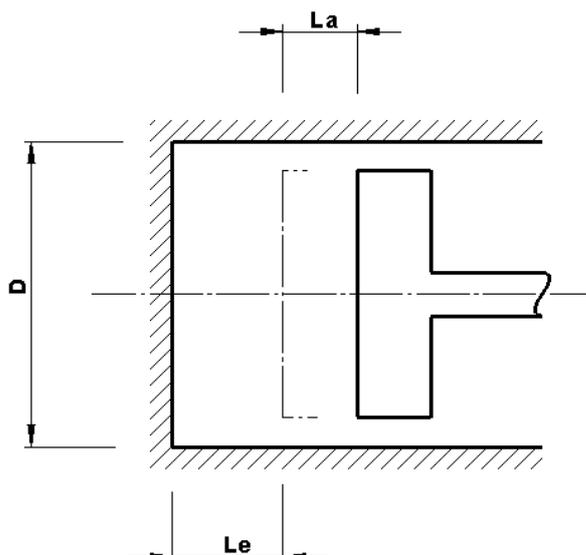

Figure 4. Nomenclature for the cylinder surface area.

1. Choose a diameter for the gas spring ("*D*" in Figure 4).
2. Choose a clearance Length ("*L_e*" in Figure 4).
3. Calculate the expected peak-to-peak pressure in the gas spring, and hence the gas spring stiffness.
4. Calculate the total spring stiffness for the compressor (compression space + auxiliary gas spring + mechanical springs).
5. For the given piston diameter (in mm) and gas spring pressure amplitude (MPa), calculate the piston mass (g) according to the following empirical formula[*]

$$Gas\ Spring\ Piston\ Mass\ =\ 16 +\ 1.7 \times 10^{-6}(0.8 +\ p_a)D^4 \qquad (17)$$

6. Calculate the total moving mass is found, and hence the resonant frequency.
7. Calculate the gas spring compression loss[†].
8. Calculate the gas spring seal loss
9. Assuming the same "*P-V*" work per cycle, calculate the gross "*P-V*" power in the compression space at the revised resonant frequency.
10. By subtracting the gas spring seal and compression losses, calculate the net "*P-V*" work from the compression space which is available for powering a cold head.

   This approach gives an 'Upper Bound' to the increase in power capability that can be achieved with a gas spring.

   This technique was applied to the Ambient Compressor, and the following results were obtained.

   The effect of varying the End Clearance (*L_e*) is shown in Figure 5. With a small end clearance the pressure swing is high and the losses high. For this case, the maximum net work out is with an end clearance of about 60 mm. Figure 6 shows the effect of varying the gas spring
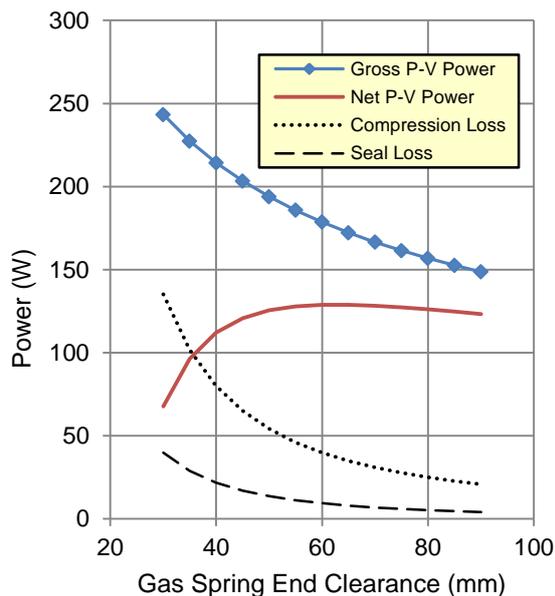

Figure 5. Gas Spring gross and net power, and losses, as a function of End Clearance (gas spring piston diameter = 60 mm).
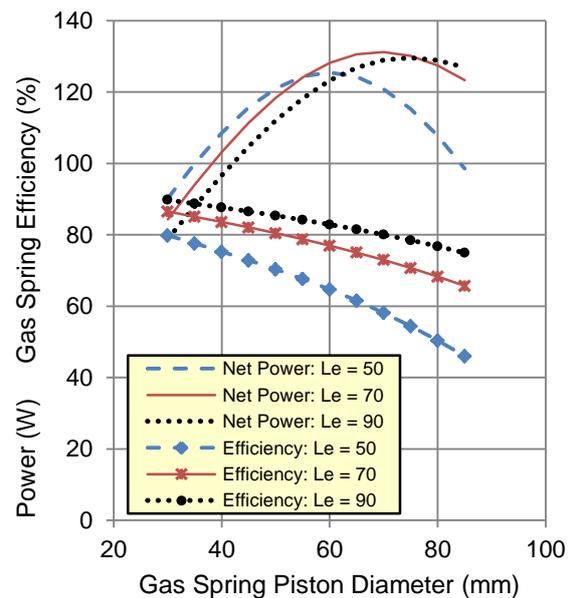

Figure 6. Net Power and Gas Spring Efficiency as a function of Gas Spring Piston Diameter and End Clearance.

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[*] A simple aluminium piston 'cup-shaped' piston of varying diameters was subject to FE analysis with a range of pressure loads, and the wall thickness varied to limit the radial deflection and the maximum stress. The equation given is a best fit to this data.
[†] A separate calculation should also be carried out due to the gas spring effect in the gas that surrounds the springs, motor etc. This volume typically has a very small pressure swing, and a very large surface area (the surfaces of the springs, magnetic circuit, moving coil, etc.). This has not been done in this analysis.

piston diameter. The net power is at a maximum with piston diameters between 60 and 75 mm. It can be seen that there is steady decrease in efficiency as the piston diameter increases. Note that a constant stroke has been assumed for this example.

In this case, it is not obvious what the optimum values of gas spring diameter and clearance length are, as there is a continual trade-off between net power and efficiency. Arbitrarily, a gas spring with a diameter of 40 mm, together with a clearance length of 90 mm have been chosen, and the resulting compressor performance is shown, in comparison with the original values, in Table 1. In this case, the power output for the compressor is increased by a factor of 2.32 with a 38% increase in overall length, and an input power 2.45 times the original.

Table 1. **Ambient Compressor With Gas Spring**

		Original	With Gas Spring
Power Piston Diameter	mm	18	18
Gas Spring Piston Diameter	mm		40
Gas Spring End Clearance	mm		90
Maximum Stroke	mm (pk/pk)	12.8	12.8
Design Frequency	Hz	50.1	131
Effective Moving mass	gram	120	141
Fill Pressure	bar (abs)	35	35
Pressure Swing (power piston)	bar (pk/pk)	5.75	5.75
Pressure Swing (gas spring piston)	bar (pk/pk)		8.5
Mechanical Spring Stiffness (axial; total)	N/m	3900	3900
Compressor Shaft (P-V) power (Gross)	W	42.3	110.4
Compression Loss (gas spring)	W		10.2
Seal Loss (power piston spring)	W	0.67	0.67
Seal Loss (gas spring)	W		2.7
Compressor Shaft (P-V) power (Net)	W	41.6	96.8
Copper Losses	W	9	9
Eddy Current Losses (proportional to freq^2)	W	1.5	10.2
Total Motor Losses	W	10.5	19.2
Electrical Power Input	W	52.8	129.6
Motor Efficiency	%	80	85
Overall Efficiency	%	78.8	74.7
Overall Compressor Length	mm	160	220
Increase in length	%		38%

OTHER CONSIDERATIONS

There are several other loss mechanisms, which are not easy to quantify with a simple model of this nature; detailed analysis is beyond the scope of this work. Most (but not all) of these have a negative impact on overall performance.

Motor Power.

A simplistic view of the power consumption and motor efficiency as frequency increases is as follows, which is discussed in terms of a moving coil motor (though other types of motor will behave in a similar fashion):

- The pressure swing in the compression space remains constant.
- The force delivered by the coil in the air gap remains constant.

- The current in the coil remains constant.
- If the coil resistance and current remain constant, then the Joule heating of the coil (i^2R) is constant.
- The voltage supplied to the coil must increase proportionally with the piston velocity.
- The useful power delivered by the motor increases proportionally with piston velocity.
- The electromagnetic efficiency of the motor increases, as the Joule loss is a constant value, whereas the delivered power increases with frequency.

However, there are two factors which also must be taken into account:
- The increased voltage supplied to the coil may require an increased thickness of electrical insulation on the windings of the coil. This will tend to reduce the amount of copper in the air gap, which will cause a decrease in motor efficiency.
- The higher voltage requirement may lead to added mass and complexity with regard to the compressor power supply and control systems.

Motor Losses.

Given that these are difficult to evaluate, and are very specific to the particular design of motor, these have not been analysed in detail, but should not be ignored in a real design. For a typical moving coil, permanent magnet motor they can be can be split into three categories:
- Magnetic hysteresis. This is typically a constant loss per cycle, so the power loss will be proportional to frequency.
- Eddy Current Losses. These are induced losses in components which are moving or adjacent to the windings. These losses are typically proportional to the square of current in the coil, and vary with the square of the drive frequency.

At higher frequencies, the 'eddy current' losses are likely to be significant unless care is taken in the design of the motor and adjacent components.

Structural Design.

Inertial (acceleration) forces within the moving components vary with the square of the operating frequency. Hence any increase in frequency may require an increase in critical component dimensions in order to keep inertial stresses (or deflections) within acceptable limits. This will result in an increase in moving mass, and a decrease in resonant frequency, though this is not an 'across-the-board' increase in mass, but is likely to be focussed in a few locations on some components.

Flow and Thermodynamic Losses.

- It is assumed that the cold head has been designed for the anticipated flow velocities.
- Within the compression space, the 'gas spring' losses are taken into account elsewhere.
- Within the compression space, the clearance seal loss is also accounted for elsewhere (these are likely to be independent of frequency).
- It is assumed that the exit port from the compression space has been designed for the higher flows expected.

CONCLUSION

This study has investigated the possibilities of increasing the operating frequency of cryocooler compressors, which is dependent on the moving mass and the mechanical and gas spring stiffnesses.

It is assumed that the moving mass is already optimised, and there is little scope for further reduction. Improving the design of the mechanical springs may give a significant increase in their stiffness, but such an improvement is unlikely to have much effect on the resonant frequency.

Provision of an auxiliary gas spring does provide the potential for a large increase in operating frequency, but there are associated losses which can be significant at higher frequencies.

A methodology is demonstrated for the simple evaluation of a gas spring, and this has been applied to an existing design of compressor. There is a wide spectrum of possible designs, but one has been evaluated which shows an increase in net compressor power by a factor of 2.32, while the input power has increased by a factor of 2.45 and the overall size (length) of the compressor increased by a factor of 1.38.

This example has been based on the potential to increase the power of an existing design, but these techniques can be used to design from scratch a compressor. If this is done, then the design goal will not be an increase in power, but an optimisation of the specific power (Power per unit mass or per unit volume).

It must be noted that in the design of compressor, there may be many other factors, such as electromagnetic losses, which might put an upper limit on the operating frequency.

It is hoped that the design techniques outlined in this report can be used to produce smaller, high frequency cryocooler compressors.

ACKNOWLEDGMENT

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