Pseudospectral Methods for Optimal Control: Theory and Practice

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Abstract

In recent years a great deal of research has been done on approximating the solution of continuous-time constrained nonlinear optimal control problems using pseudospectral methods. This body of research has uncovered the computational efficiency and accuracy of solving optimal control problems using pseudospectral methods along with the close connection between the properties of the continuous time optimal control problem and the pseudospectral discrete approximation. This research has led to powerful methods that employ collocation Legendre-Gauss (LG) and Legendre-Gauss-Radau (LGR) points and the advent of open-source software that implement these methods.

The objective of these lectures is to provide an overview of the subject of continuous-time nonlinear optimal control and to develop the mathematical theory associated with pseudospectral methods for solving optimal control problems. In the first part, the optimality conditions that arise from the application of the calculus of variations to optimal control will be developed. The topic of numerical methods for solving optimal control problems will then be discussed. The discussion will then focus more specifically on the mathematical foundation of pseudospectral methods including the topics of quadrature approximation and interpolation. The course will then describe the mathematics associated with so called global pseudospectral methods. Connections between the continuous-time and discrete optimality conditions will be described and transformations will be developed between the Lagrange multipliers of the discrete approximation and the costate of the continuous-time optimal control problem. Finally, the topic of more practical \(hp\)-adaptive pseudospectral methods will be provided.

Agenda

- Day 1
  9 am – 11 am: Calculus of Variations
  11:15 am – 1:15 pm: Optimal Control Theory

- Day 2
  9 am – 11 am: Basic Numerical Methods for Optimal Control
  11:15 am – 1:15 pm:
    - Basics of Nonlinear Optimization
    - Quadrature Approximation and Interpolation

- Day 3
  9 am – 11 am: Pseudospectral Methods Using Gaussian Quadrature Collocation
  11:15 am – 1:15 pm: Practical hp-Adaptive Pseudospectral Methods