Bounding the magnitude of extreme events in nonlinear dynamical systems using convex optimisation

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Abstract:

Predicting the magnitude of extreme events in dynamical systems governed by ordinary and partial differential equations (ODEs and PDEs) is a fundamental problem with a wide range of applications. Questions of practical relevance include estimating the largest load on a structure due to dynamical forcing, or the maximum mixing efficiency achievable by incompressible flows with a given energy budget. From a more theoretical perspective, bounds on the maximum growth of enstrophy or palinstrophy in fluid flows have far-reaching implications regarding the regularity of solutions of the incompressible Navier-Stokes equations. In this talk, I will describe a general convex framework to derive rigorous bounds on the extremal value of observables of nonlinear dynamical systems, evaluated along trajectories that start from a given set of initial conditions. The framework, which applies both to ODE and PDE systems, is closely related to both Lyapunov analysis and relaxation methods for optimal control problems based on occupation measures. It also generalises so-called "energy methods" used in the analysis of PDEs that arise in fluid mechanics. In the ODE case, bounds on the magnitude of extreme events can be optimised numerically using sum-of-squares (SOS) programming. For a large class of ODEs that satisfy mild assumptions, moreover, arbitrarily sharp bounds can be computed by solving a hierarchy of SOS optimisation problems. Analytical and numerical examples that demonstrate both the potential and the generality of the proposed approach will be presented and discussed.