David Clarke and Model Predictive Control

In celebration of David Clarke's contribution to MPC

St Edmunds Hall, Oxford University, January 9, 2009

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David

Congratulations on your many achievements!

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- SOME OF DAVID'S ACHIEVEMENTS
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- A CURRENT ISSUE: ROBUST MPC
- FUTURE CHALLENGES
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SOME OF DAVID'S ACHIEVEMENTS

- Identification
- Adaptive and Self-tuning Control
- GPC
- Smart, self-validating sensors and actuators
- Director of Invensys: University Technology Centre for Advanced Instrumentation

Identification

- Ph.D. Topic (1967)
- Generalised least squares for system identification
- Implemented on a computer ... in 1967!
- 2nd UKAC Convention, Bristol 1967

Adaptive and self-tuning control

- System: $A(\delta)y(t) = B(\delta)u(t) + \xi(t)$
- Hot topic in 1970's
- Many, many proposals
- Revolution: Astrom-Wittenmark 1973
  - Minimum variance control
  - Least squares estimation of parameters
  - Certainty equivalence
  - Magical result:

IF parameters converge, they converge to minimum variance controller!

Clarke's adaptive controller

- Two shortcomings in min var adaptive controller
  - Agressive control (cancels expected error)
  - Stable zero dynamics required but
    - rapid sampling generates u/s zeros
- Clarke-Gawthrop solution (1975):
  - Replace minimum variance control by
  - $\arg\min_{u(t)} E_{I(t)}\{y(t+k)^2 + \lambda u(t)^2\}$
- Costing u reduces control activity with small increase in variance of o/p y
- CL stability requires OL stability and adjustment of λ
- Considerable impact (338 google citations)

Generalised Predictive Control

- Clarke introduced GPC in 1987 as a general method of control for *unconstrained* linear systems that:
  - are non-minimum phase
  - are OL unstable
  - have unknown dead-time
  - have unknown order
- Method:
  - Minimize $E_{I(t)}\{\sum_{j=0}^{N}(y(t+j)^2 + \lambda u(t+j)^2\}$ wrt sequence $\mathbf{u} = \{u(t), u(t+1), \ldots,\}$
  - Apply the first element of the minimizing sequence to the plant
  - This is receding horizon control

Generalised Predictive Control

- Clarke's two 1987 papers on GPC had a substantial impact
- First paper has 854 citations (Google)
- Papers contain rich set of extensions:
  - Tuning knobs (generalised output)
  - Rejection of constant disturbances
  - Adaptation
  - Ability to achieve wide range of control objectives
  - Terminal equality constraint (on y) to ensure cl stability
  - Ability to handle control constaints (1988)

Recent research

- Control loop tuning
- Performance monitoring
- Self-validating sensors and actuators
- Bounds on ultimate performance of sensors, and
- Design of sensors that approach these bounds

WHERE IS MPC NOW?

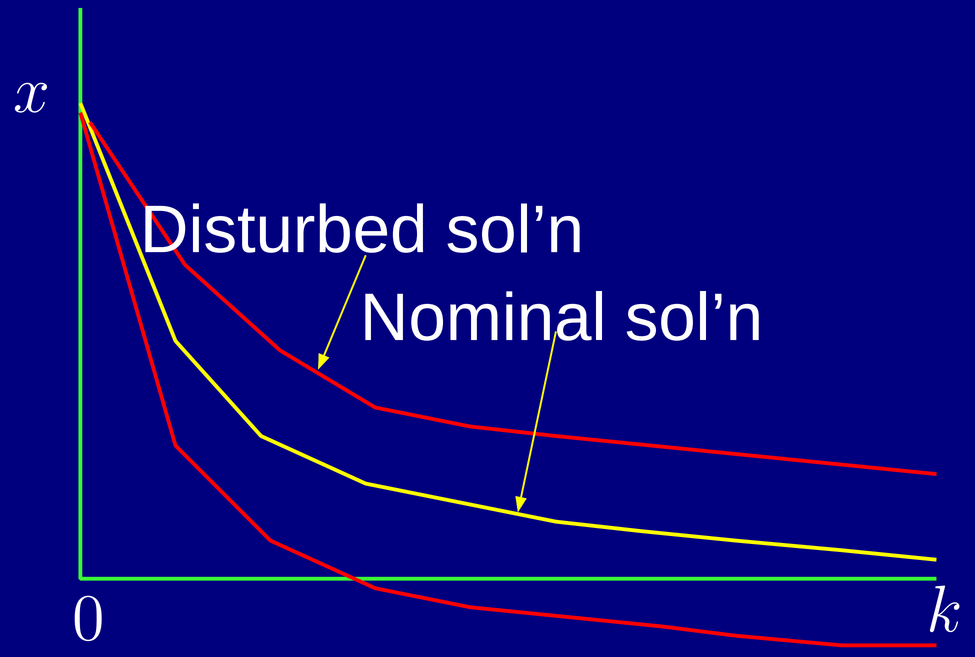
- GPC restricted to linear systems
- In linear context had broad focus
  - eg adaptation, tuning
  - reflecting Clarke's concern for application
- MPC: Constrained linear and nonlinear systems
  - Narrower focus in broader context
  - Sufficient conditions for stability
  - Suboptimal MPC for NL systems
  - Unreachable set points
  - Distributed MPC, etc
  - Improved optimization procedures
- Uncertain systems ... jury still out

A CURRENT ISSUE: ROBUST MPC

- MPC successful for deterministic systems because
  - Solution of OL OC Pb (for given initial state)
  - is same as FB solution (via DP) for same state
  - Feedback not necessary for deterministic system
- To get similar properties for robust MPC
  - requires optimization over control policies: $\pi = \{\mu_0(\cdot), \mu_1(\cdot), \ldots, \mu_{N-1}(\cdot)\}$
  - subject to satisfaction of all constraints by all realizations of state and control trajectories
  - Impossibly complex
  - Implementation requires simplification

Robust MPC

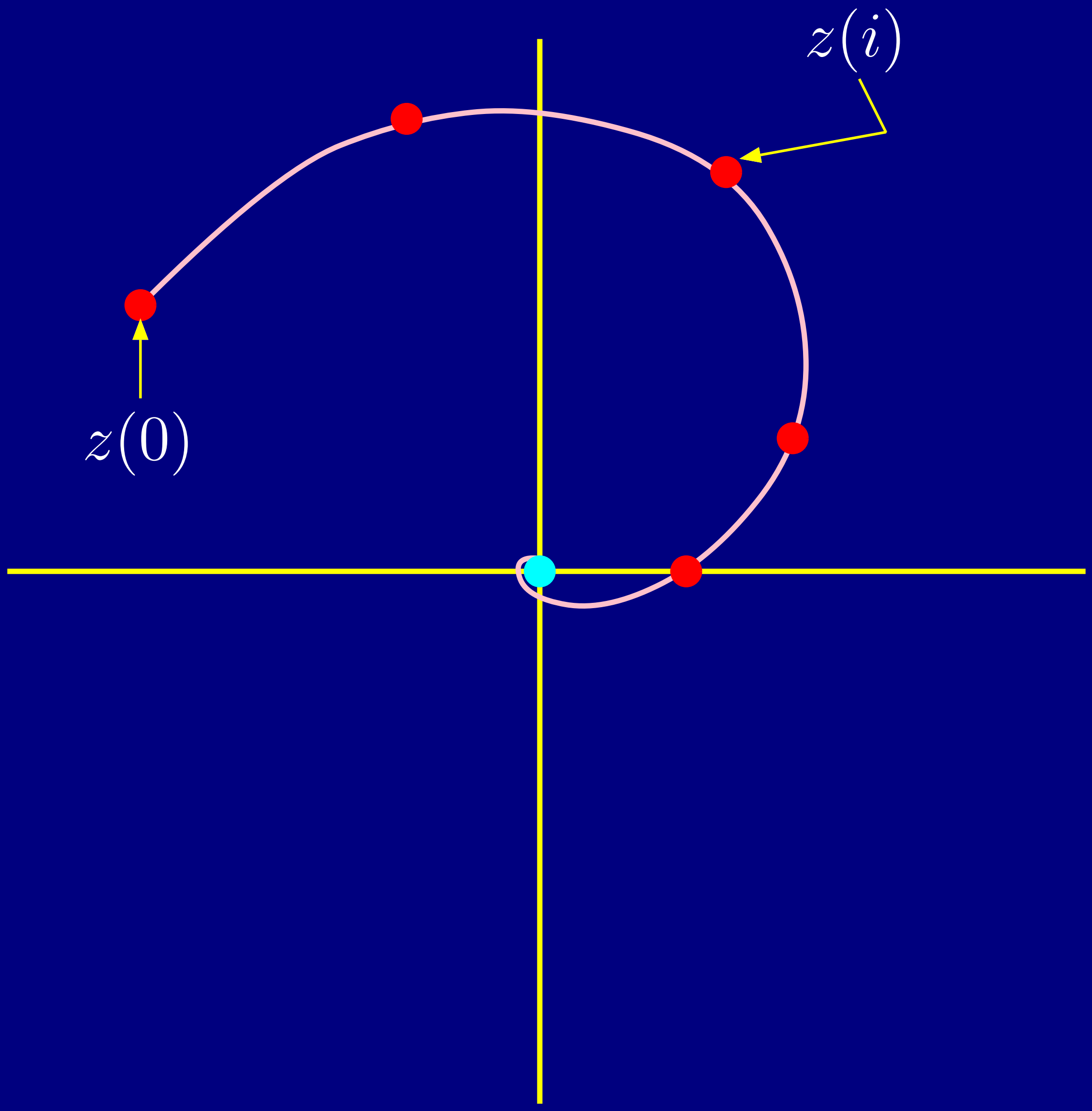
- Pb simple to state – hard to solve
- Need implementable sol'n $\approx$ exact sol'n
- B'ded dist implies approx'n needed only over 'tube'



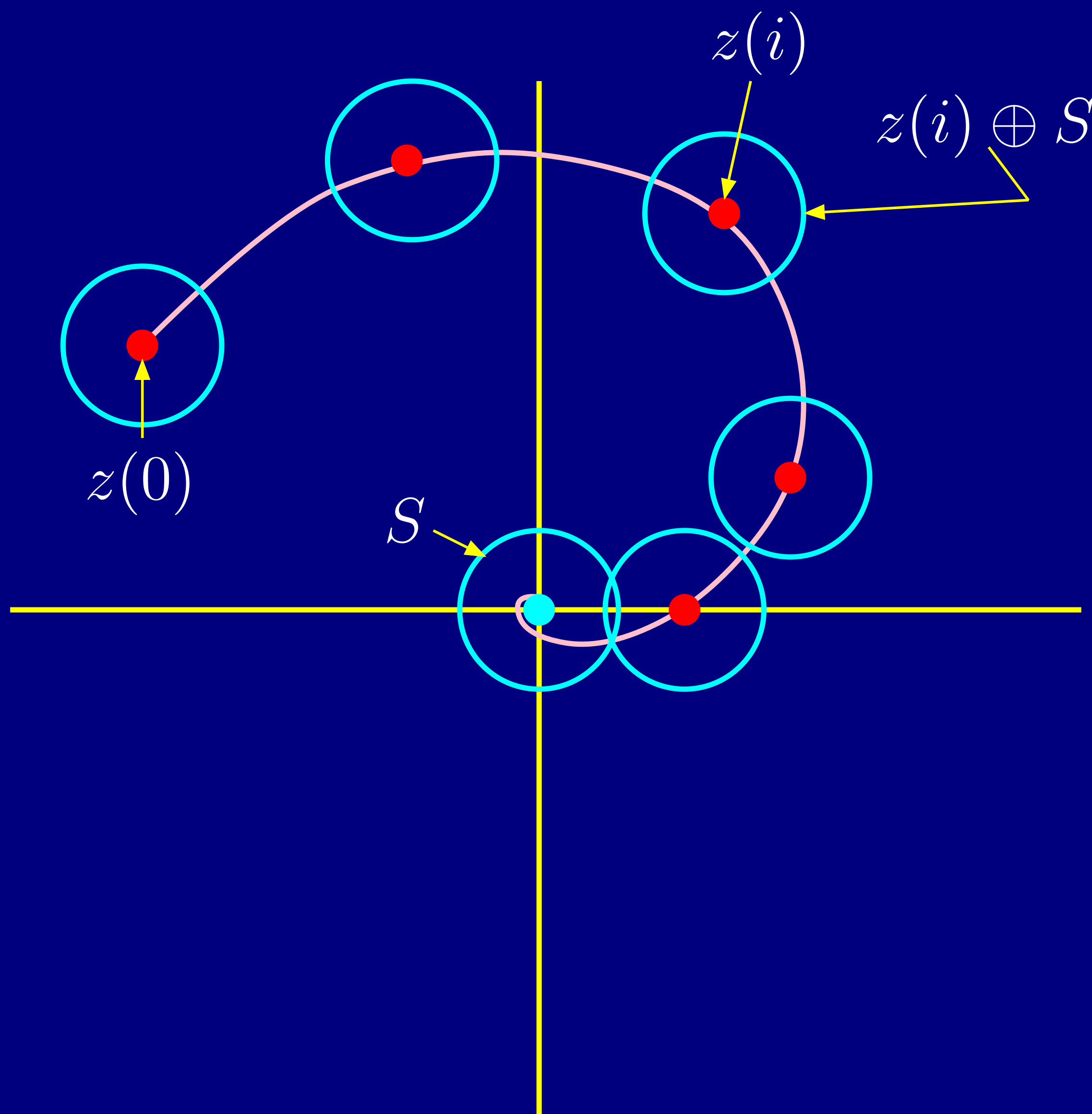
Robust MPC

- Linear approx'n of opt policy over tube seems good
- For system $x^+ = Ax + Bu + w$, quadratic cost, initial state x
  - opt control at (x, i) is $v(i) + K(x - z(i))$
  - $\{v(i)\}$ is opt sol'n to nominal pb, initial state x
  - $\{z(i)\}$ is resultant state sequence, $v(i) = Kz(i)$
- If $w \in W$, W compact, $x(i)$ lies in $z(i) + S$ where S is compact (K any stabilizing controller)
- Basis of tube sol'n for robust MPC for constrained linear systems
- that merely requires solving conventional MPC Pb with tightened constraints

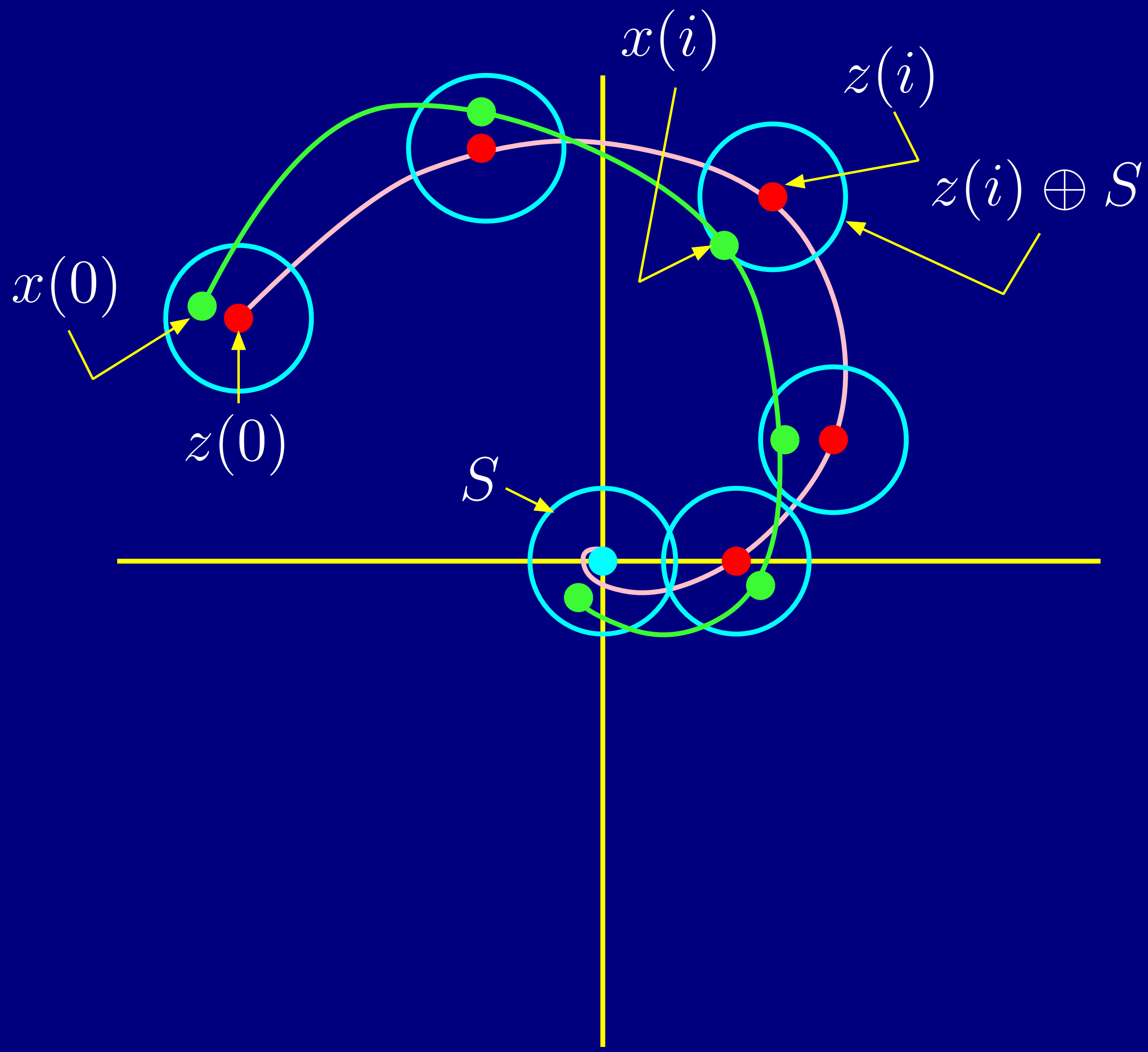
Tube MPC for constrained linear systems



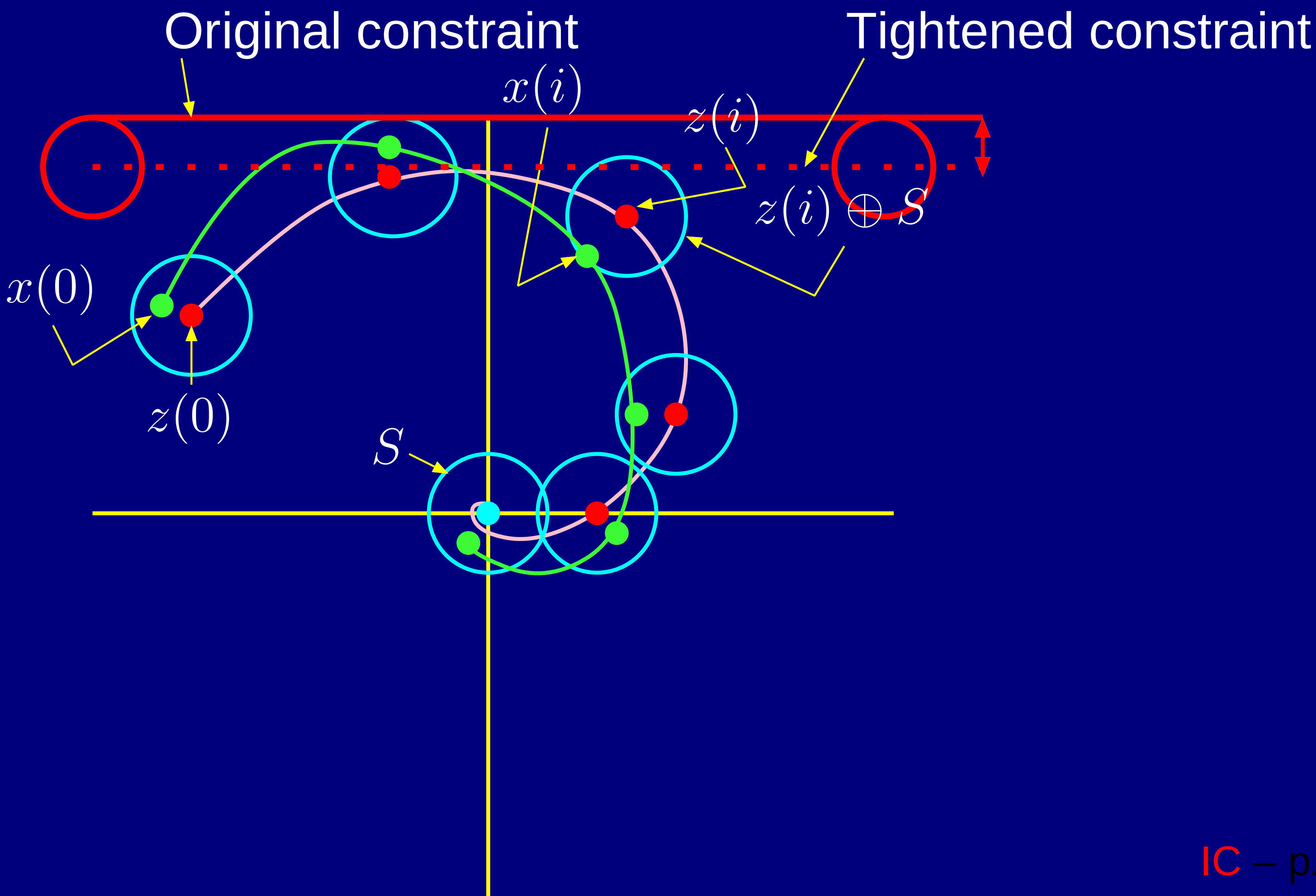
Tube MPC for constrained linear systems



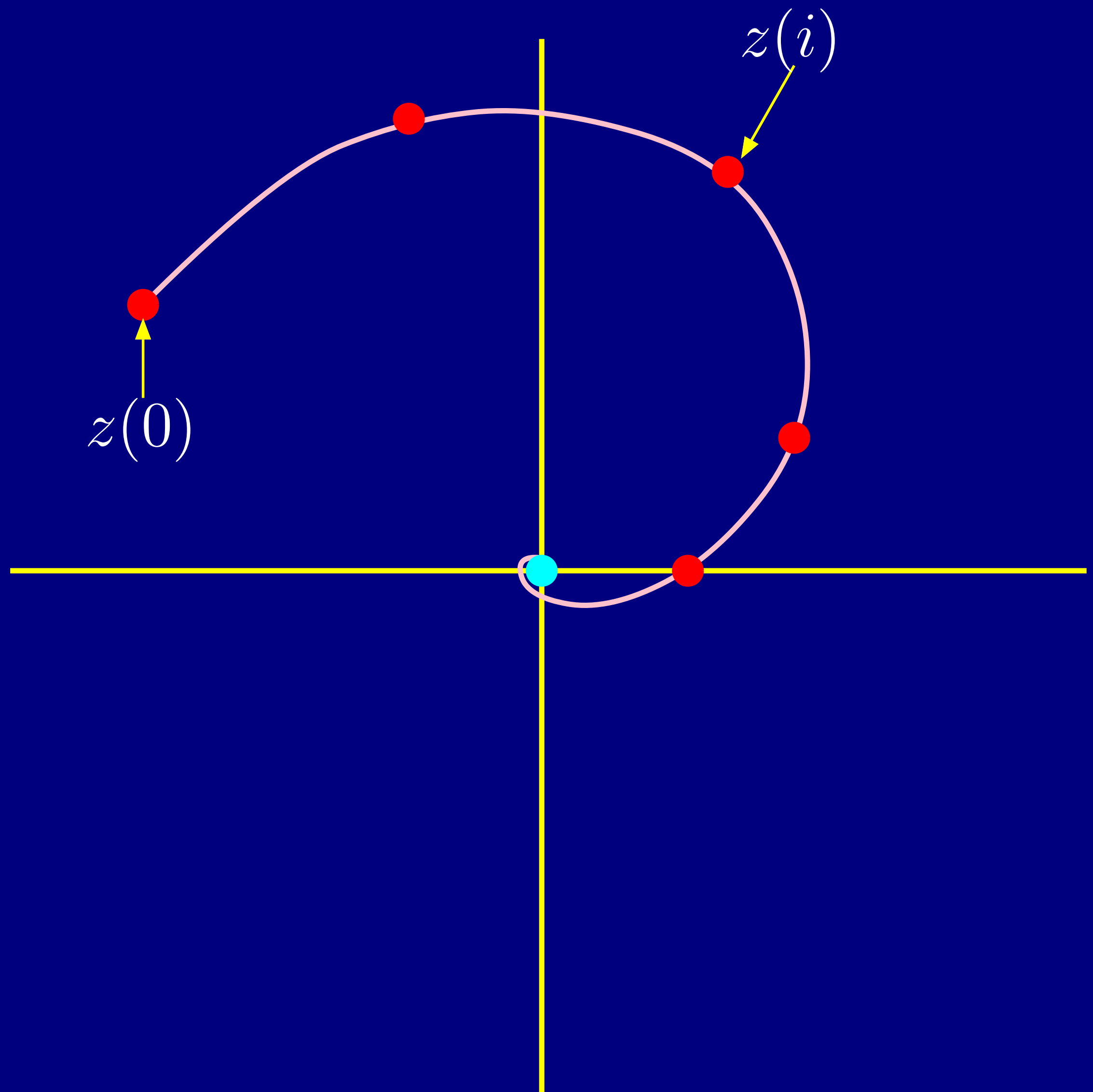
Tube MPC for constrained linear systems



Tube MPC for constrained linear systems



Constrained linear system: output MPC



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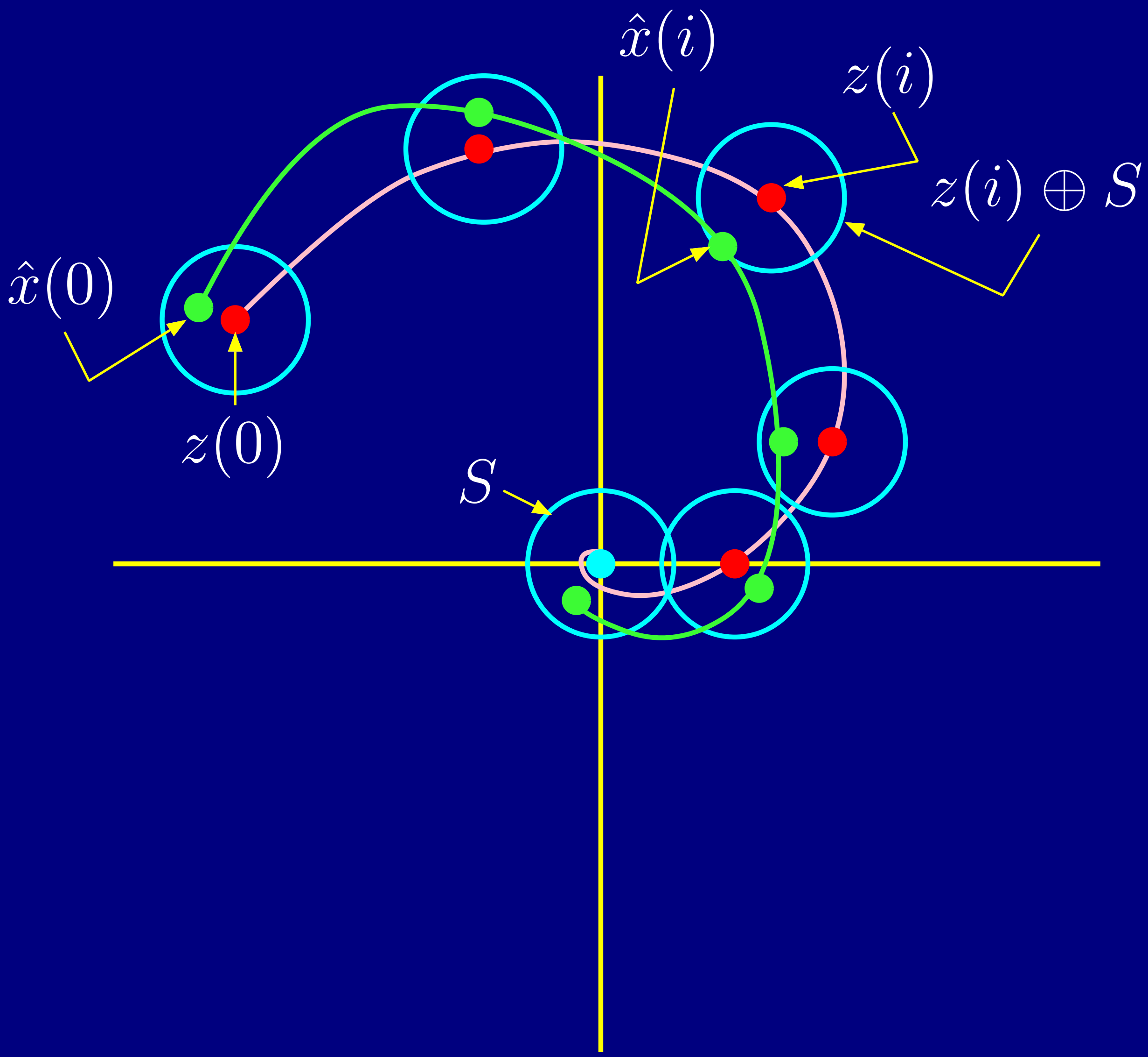

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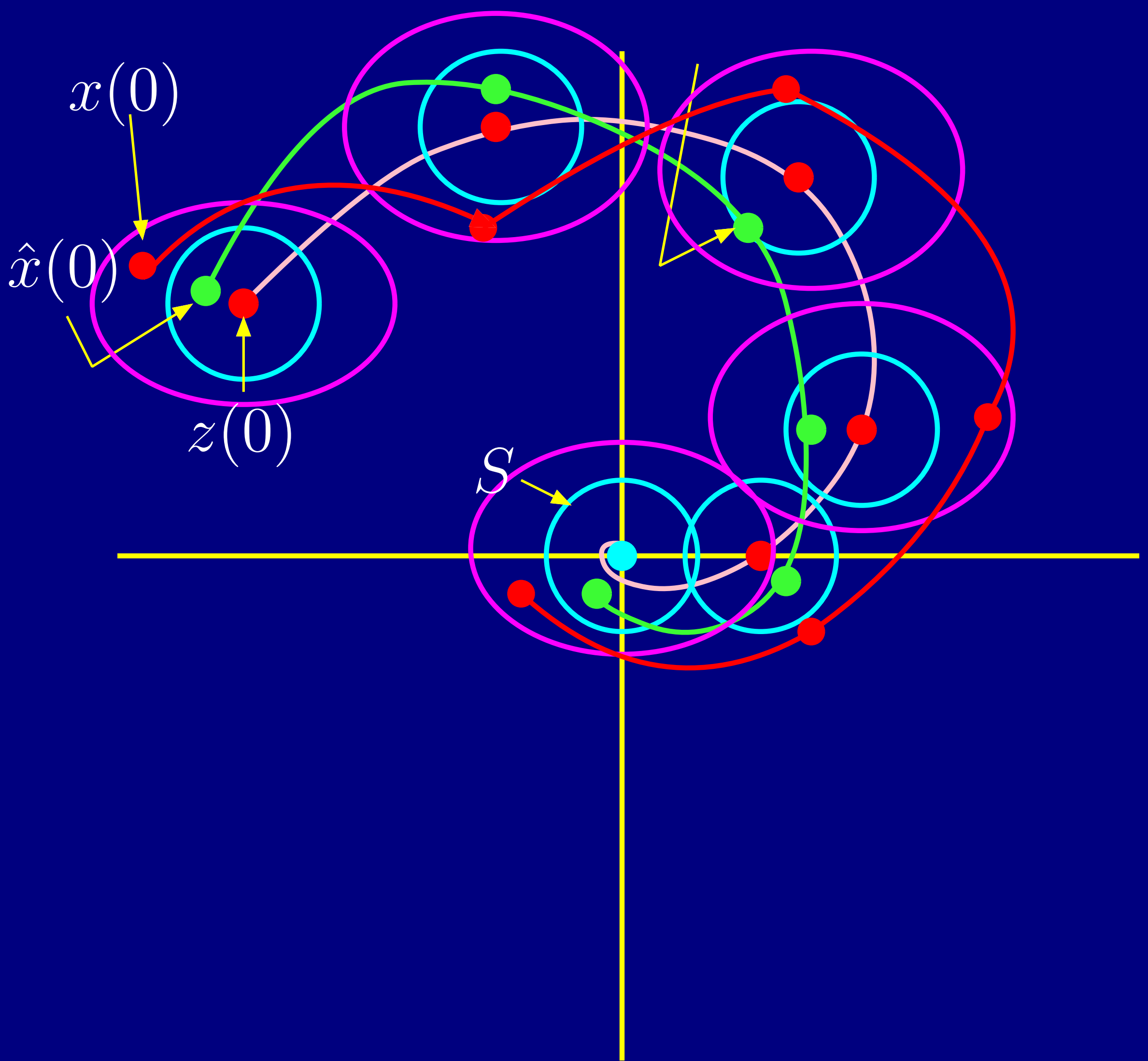

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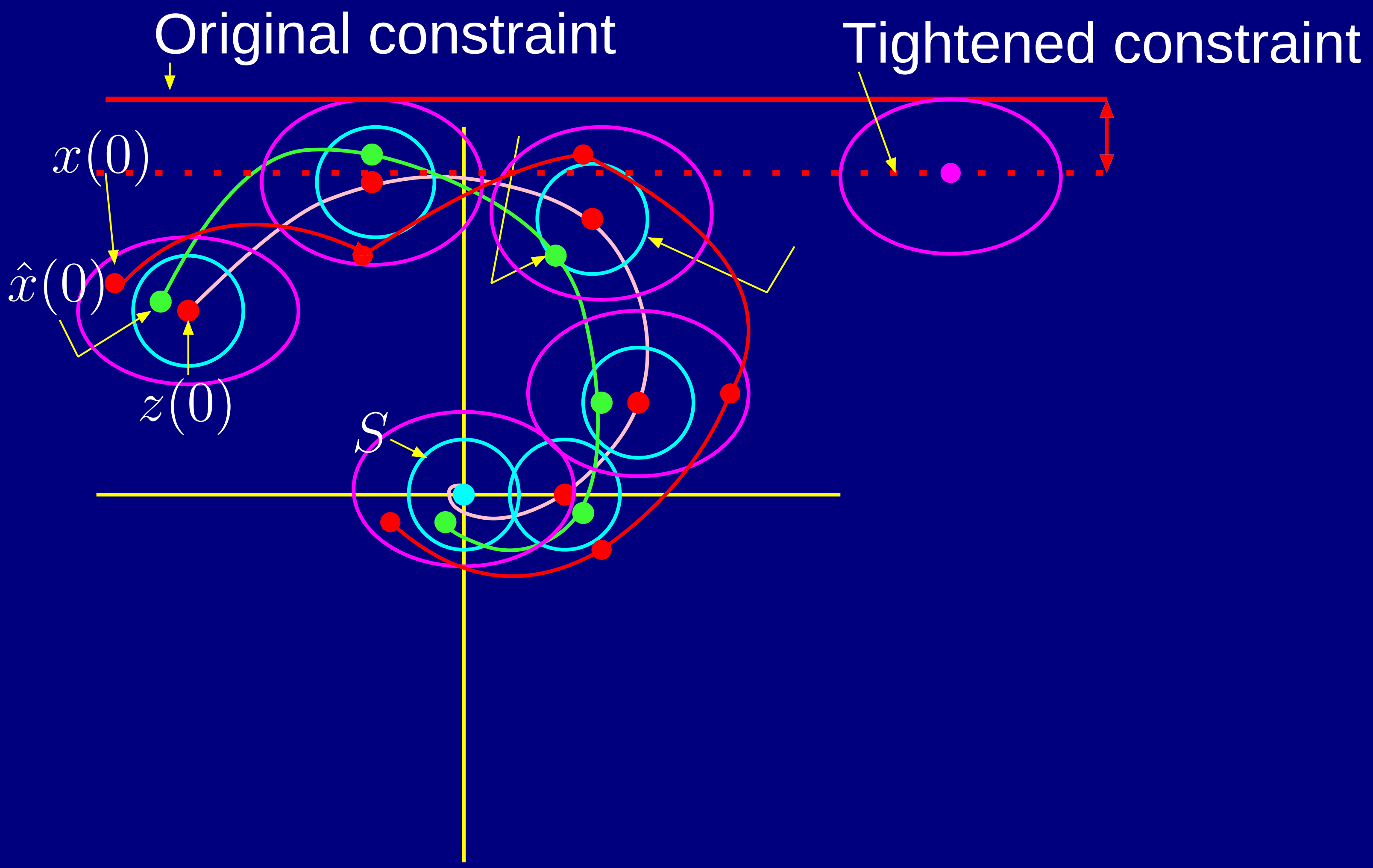


Tube MPC

- Tube MPC applicable to constrained linear systems:
  - With additive bounded disturbance
  - With parametric uncertainty
  - With uncertain state (O/P MPC using observer $+$ certainty equivalence)
- BUT can it be used for constrained NL systems?

Tube MPC: constrained NL systems

- Can tube approach be extended to NL systems?
- System $x^+ = f(x,u) + w$, $w \in \mathbb{W}$
- At first sight, looks very difficult
- Need a control law valid in tube
- For NL systems, determination of control law (in contrast to control sequence) difficult
- In linear case, law is $x \mapsto v + K(x-z)$ where $v = \kappa_N(z)$, z and K easily determined
- NL case?

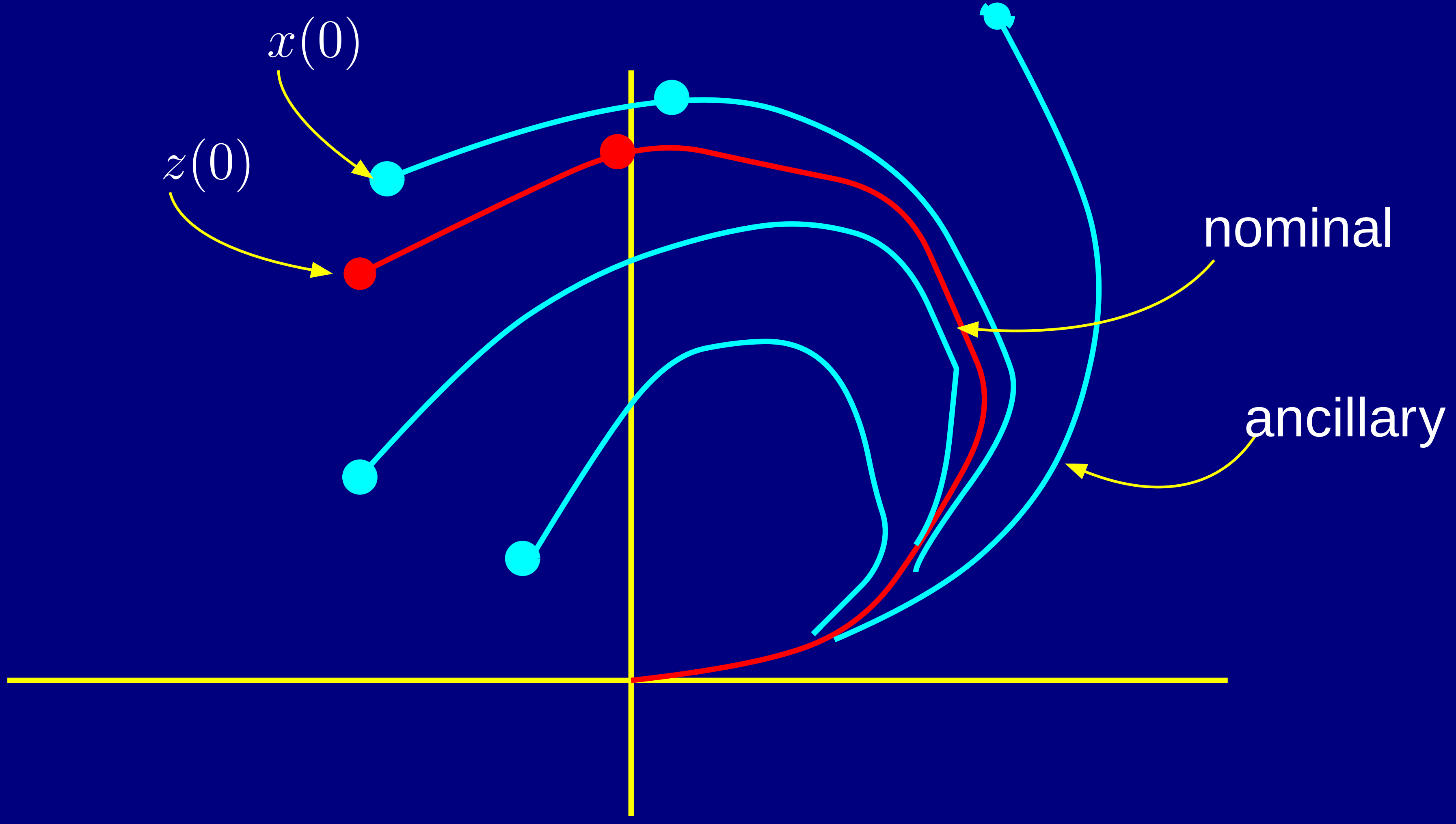
Tube MPC: constrained NL systems

- Proposal: instead of determining control law, use second MP Controller to compute control action for each state
  - Compute $\{v(i)\}$ and $\{z(i)\}$, solution of OC Pb for nominal system $z^+ = f(z, v)$
  - At each (x, z), solve ancillary pb $\mathbb{P}_N(x, z)$ to determine u
- What should ancillary pb $\mathbb{P}_N(x, z)$ be?

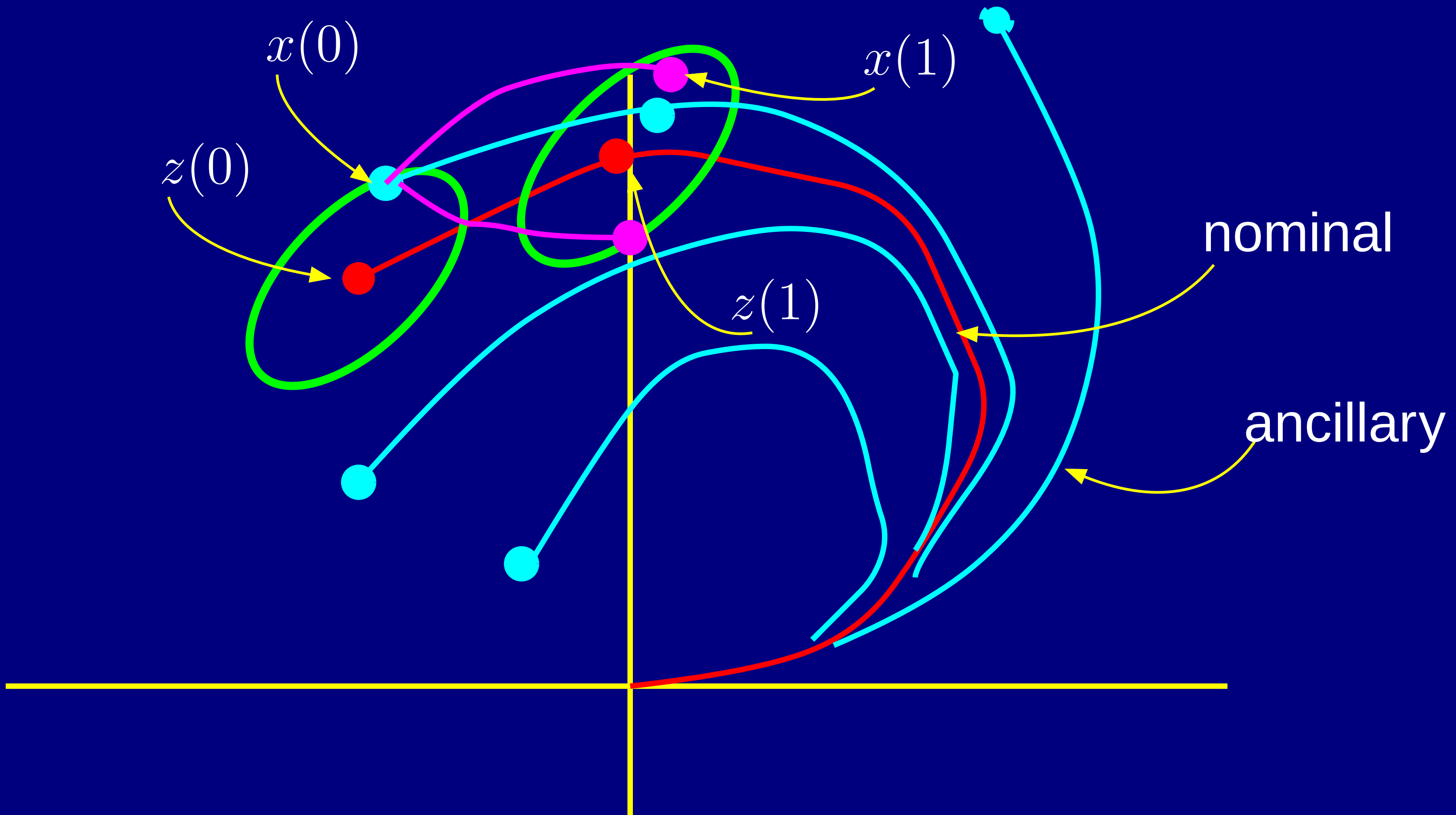
What should ancillary pb be?

- To motivate: look at LQG Pb
- Suppose we have solution $\{z(i)\}$, $\{v(i)\}$ to nominal OC Pb
- Then OC at any x is solution to ancillary nominal Pb
- in which cost is second variation cost (quadratic and zero at solution of nominal OC Pb)
- Ancillary controller steers trajectories towards the nominal solution.
- And bounds their deviation from the optimal nominal trajectory

Solutions of ancillary Pb



Solutions of ancillary Pb



The ancillary problem

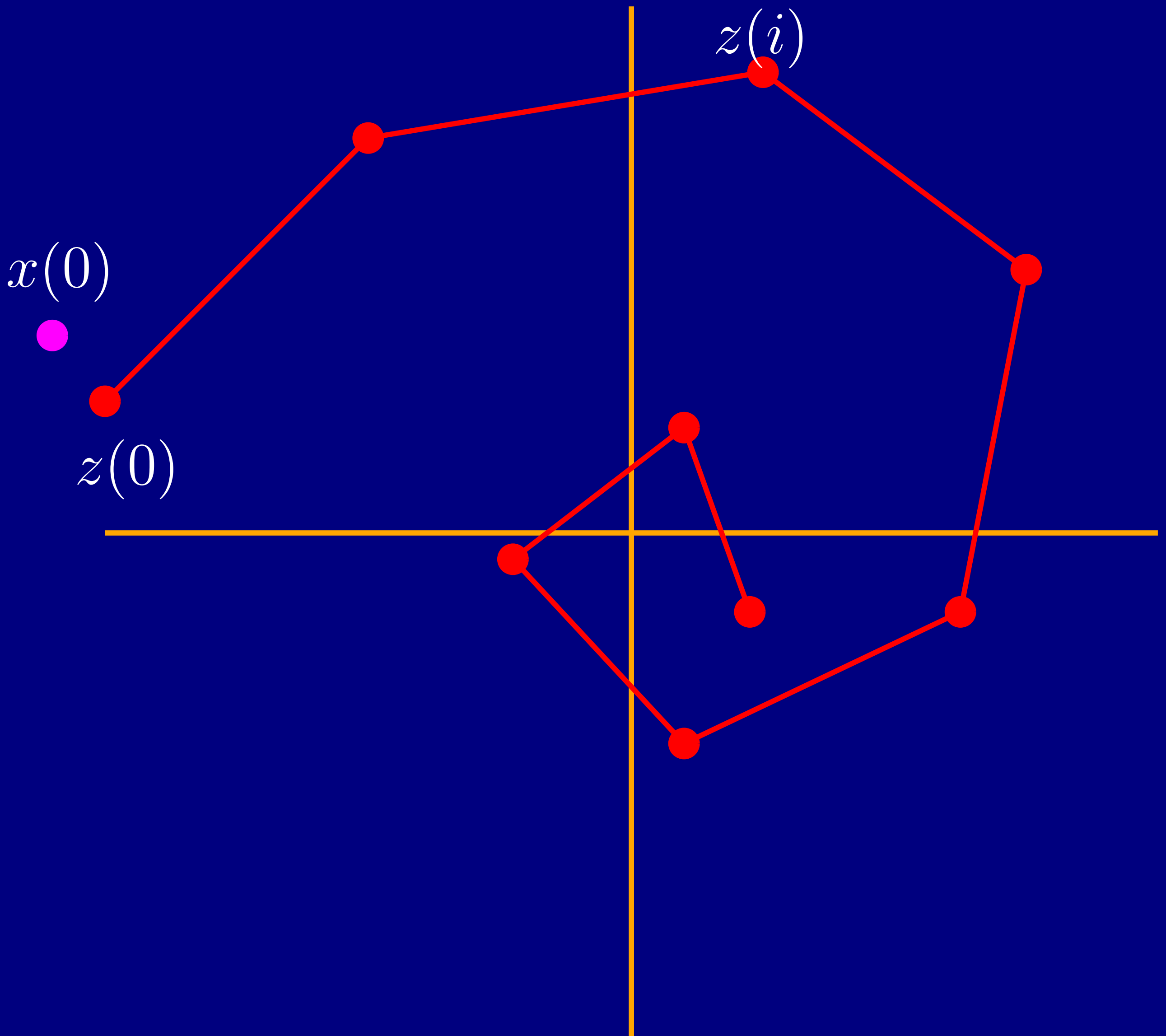
- The ancillary Pb is deterministic
- Uses nominal system $x^+ = f(x, u)$,
- Cost = deviation from optimal nominal trajectory:
- $V_N(x, z, \mathbf{u}) = \sum_{i=0}^{N-1} \ell(x(i) - z(i), u(i) - v(i))$
- $\mathbf{u} = \{u(0), u(1), \ldots, (N-1)\}$
- Ancillary OC Pb:

  $\mathbf{u}^0(x, z) = \arg\min_{\mathbf{u}}\{V_N(x, z, \mathbf{u} \mid \mathbf{u} \in \mathbb{U}^N,\ x(N) = z(N)\}$
- $\kappa_N(x, z)$=first element of sequence $\mathbf{u}^0(x, z)$
- Apply resultant control $u = \kappa_N(x, z)$ to plant.
- $\kappa_N(x, z)$ replaces $v + K(x - z)$

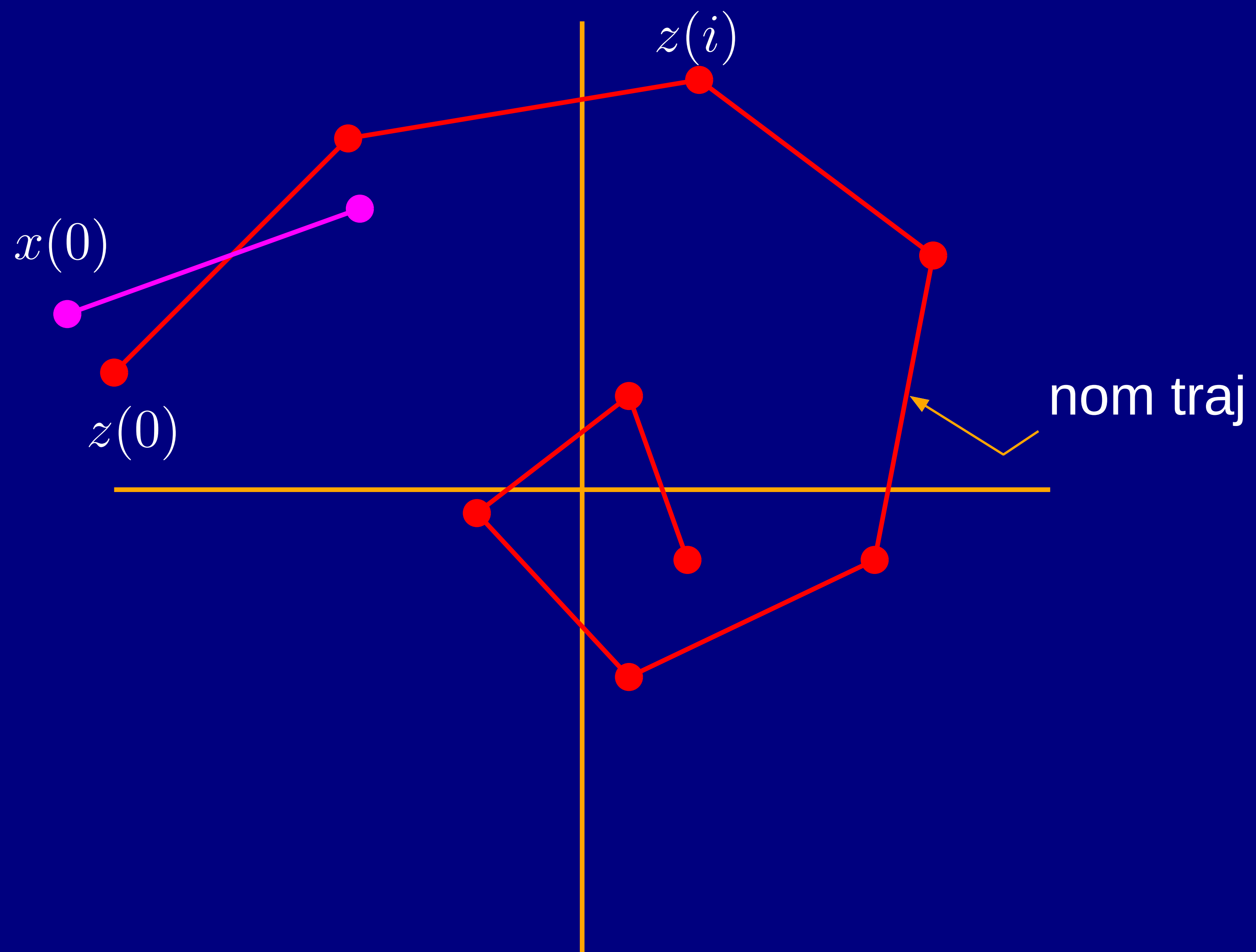
Constrained NL systems

- $\kappa_N(x, z)$, sol'n of ancillary OC Pb
- $V_N^0(x, z)$ is value fn of ancillary Pb)
- Let $S_d(z) \triangleq \{x \mid V_N^0(x, z) \leq d\}$
- $S_d(z)$ is level set of $V_N^0(x, z)$
- There exists a $d > 0$ such that:
- $x(0) \in S_d(z(0)) \implies x(i) \in S_d(z(i))$, $u(i) \in \mathbb{U}\ \forall i$
- Similar to $x(i) \in z(i) + S$ in linear case
- But sets $S_d(z)$ cannot be predetermined
- Choosing tighter constraints for nominal OC Pb hard

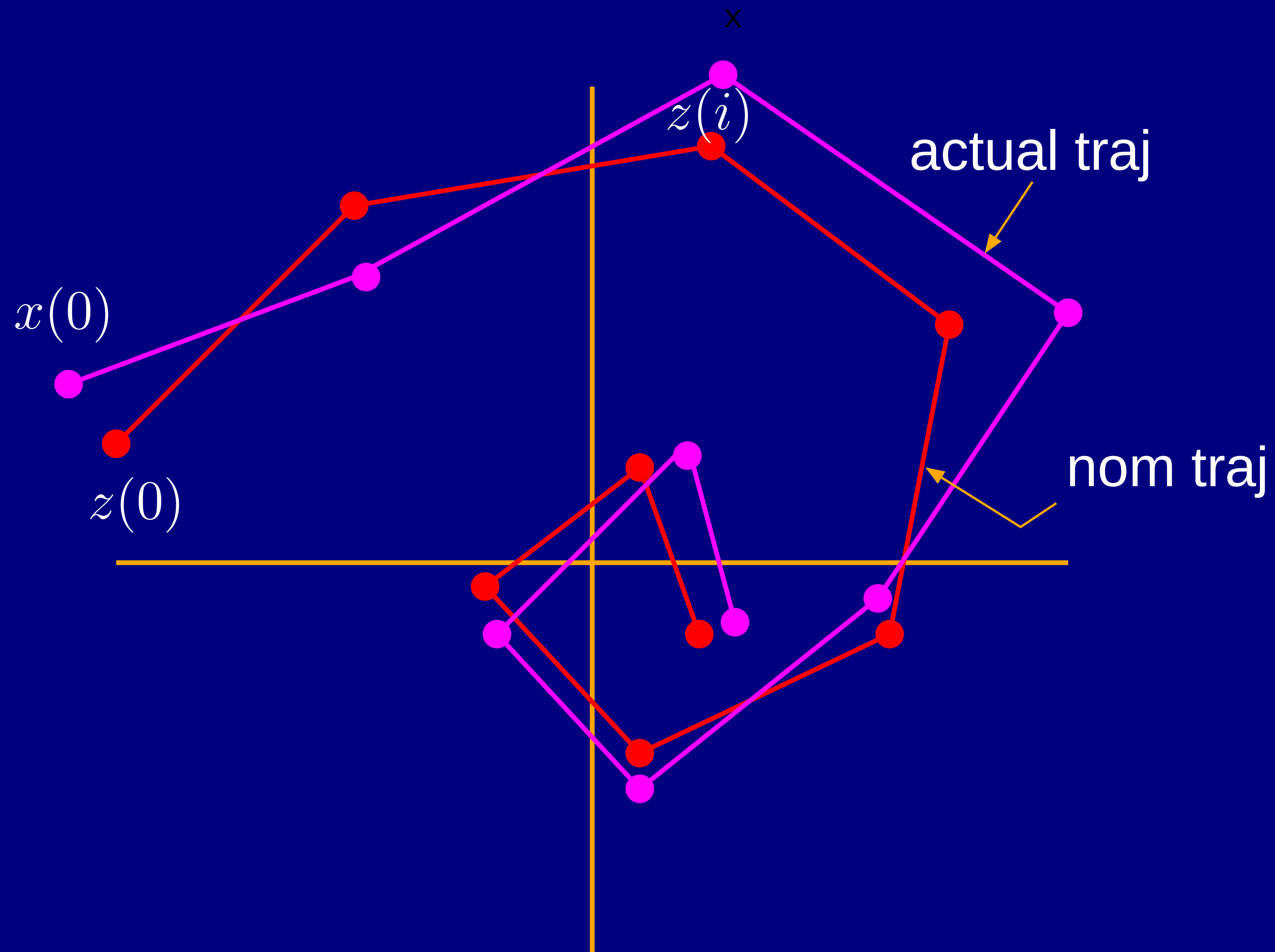
Constrained NL systems



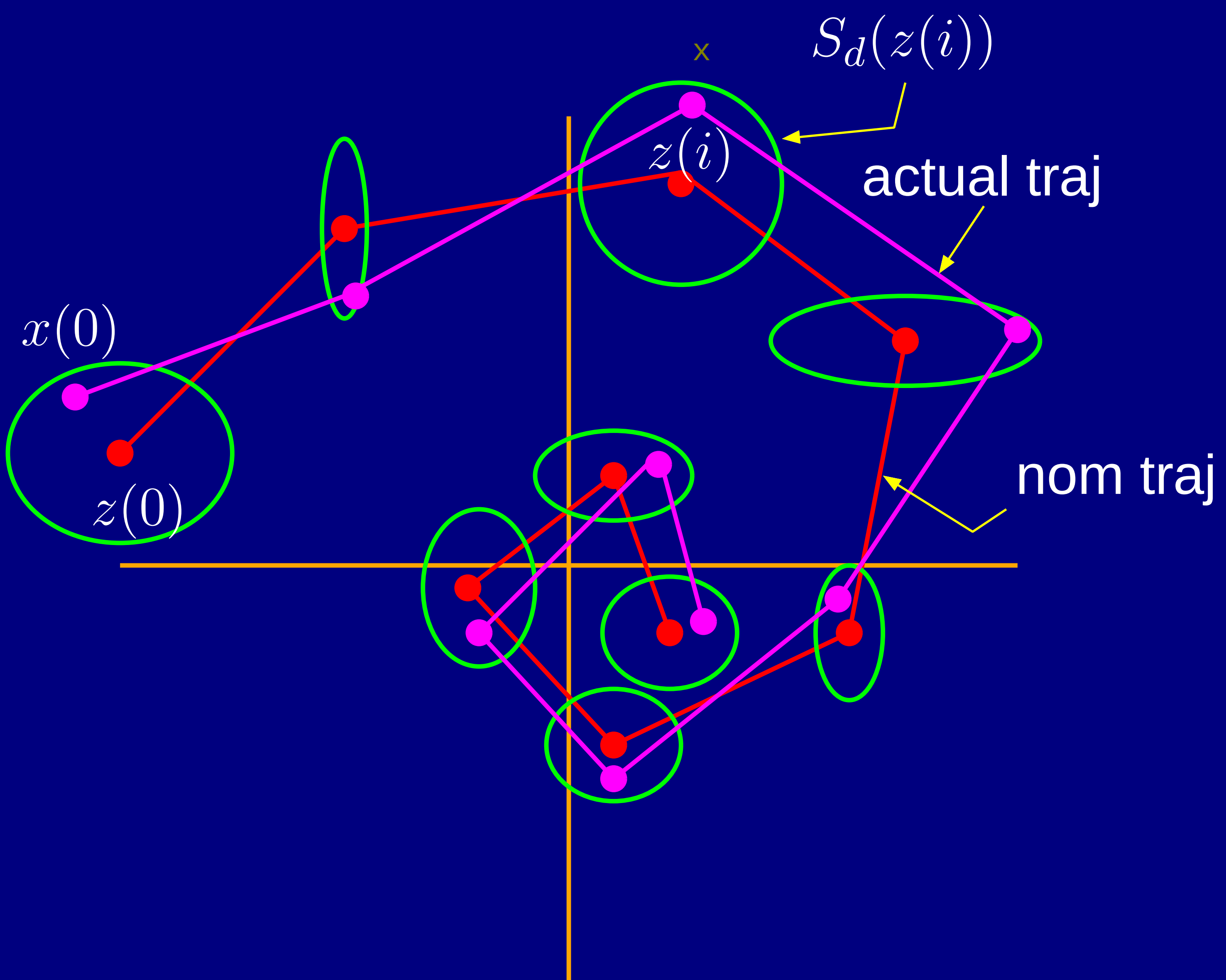
Constrained NL systems



Constrained NL systems



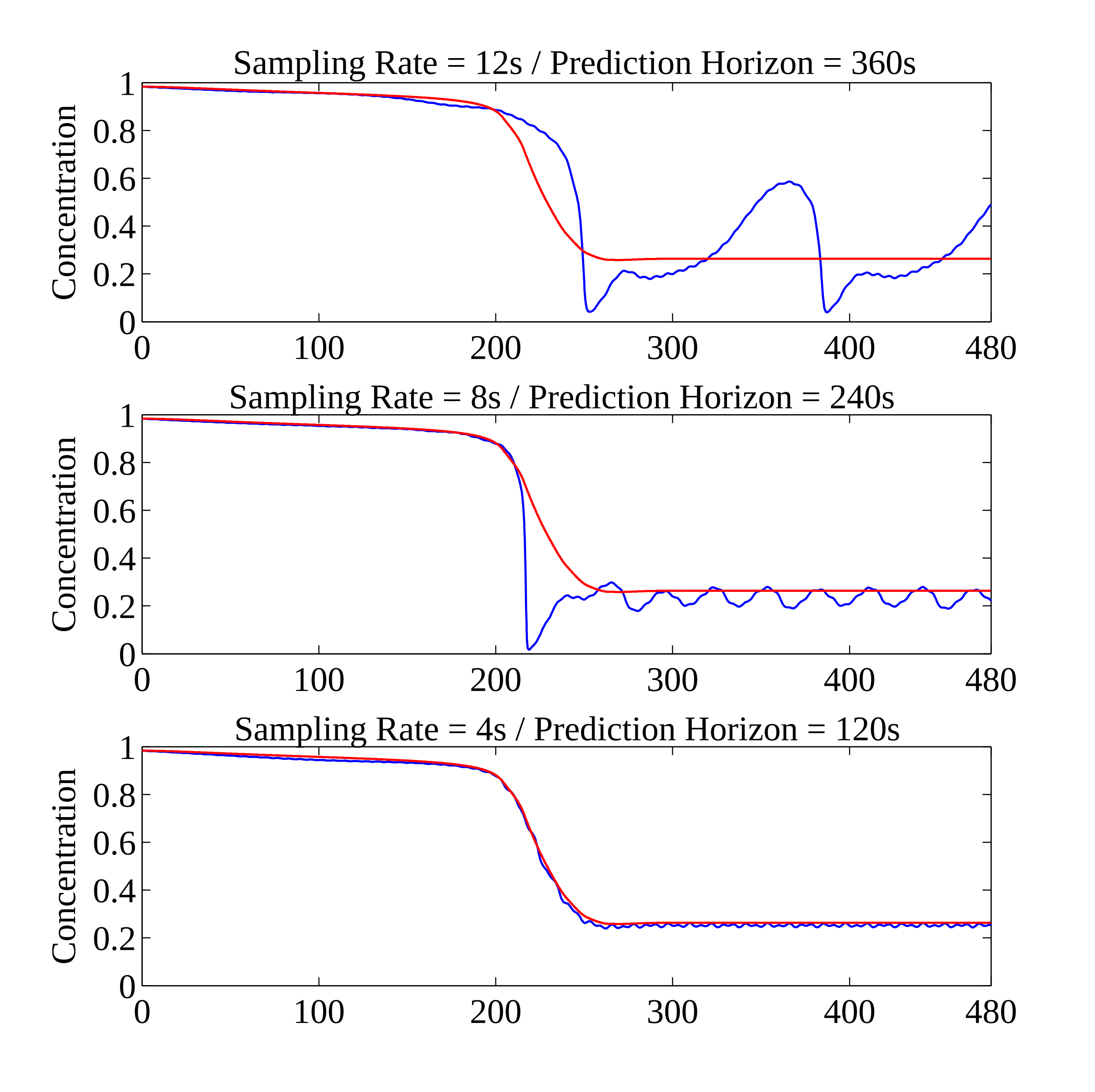
Constrained NL systems



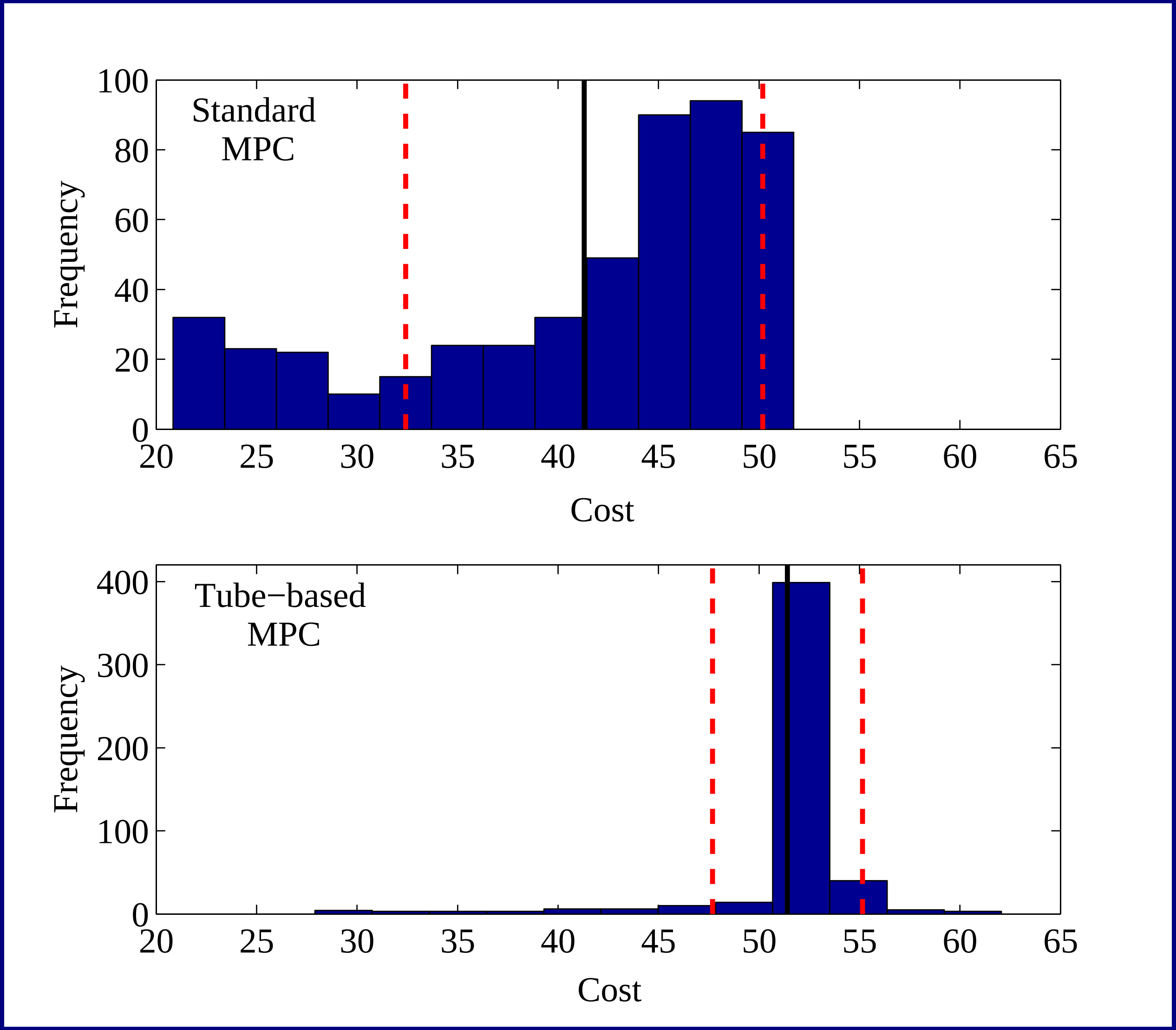
Ancillary controller

- The nominal controller steers initial state to desired state, neglecting disturbances
  - Responds to changed in desired final state
- The ancillary controller reduces effect of disturbances
  - Can be tuned
  - Can have distinct cost function
  - Can have different sampling period
- Analagous to two-degree of freedom controller

Example: Control of CSTR



Control of CSTR



FUTURE CHALLENGES

- Output MPC for nonlinear systems
- Adaptive MPC
  - Difficulty: uncontrollable subsystem modelling unknown parameters
- Distributed MPC
  - Cooperative vs non-cooperative
- Stochastic
  - Constraints?
- Computation
  - Fast systems

CONCLUSION

- MPC can solve a wide range of control problems for deterministic or uncertain, linear or nonlinear, constrained systems
- Because it creates its own Lyapunov function
- There remain big challenges

CONGRATULATIONS

DAVID