Numerical Modelling of Building Response to Tunnelling

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Abstract

The construction of underground tunnels in soft ground in urban areas involves the potential for ground movements caused by the tunnelling to affect existing surface structures. Masonry structures are at particular risk of crack damage. Conventional empirical building assessments do not fully capture all aspects of this soil-structure interaction situation. Numerical methods are increasingly used for such problems. It is common practice in empirical and numerical methods to model a building as an elastic beam in 2D. The objective of this thesis is the development of a new approach to the numerical modelling of masonry buildings using surface beams in 3D.

In phase one of this project, finite element analyses of elastic and masonry facades are undertaken and the traditional beam method of modelling them is assessed. New equivalent elastic surface beams are developed, the properties of which account for the dimensions and openings in facades which were found to influence the response to settlements. Equivalent masonry beams are also developed which have a constitutive model that accounts for the different response of masonry buildings in hogging and sagging. Timoshenko beams are chosen to model the facades and these beams were implemented into the OXFEM finite element program with full 3D capability along with the new constitutive beam models.

Example masonry structures were modelled in 3D using the new surface beams in phase two. Tunnel construction was simulated under the buildings and the response of the beams compared to a full masonry building model. Example analyses included buildings both symmetric and oblique to the tunnel. Results showed that the equivalent elastic beams accurately simulate full masonry building response in sagging regions. Parametric studies confirmed the choice of equivalent beam parameters and the impact of different relative stiffnesses. The equivalent masonry beams displayed the same good agreement in sagging but were less accurate in hogging.

In phase three, finite element models are used to compare ground movements and structural response of buildings using the 3D equivalent masonry beam method and observed data from the construction of the London Underground Jubilee Line Extension. The surface beams showed good agreement with the observed building responses in both sagging, where the building response was essentially rigid and in hogging where a more flexible response was observed.
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List of Notations

$A$  
cross sectional area

$A^*$  
effective cross sectional area

$a$ (1)  
tunnel inner radius

$a$ (2)  
a constant

$B$  
building width

$B$  
strain-displacement matrix

$b$ (1)  
distance to neutral axis

$b$ (2)  
a constant

$C_1$  
a constant

$C_2$  
a constant

$c$ (1)  
residual tensile strength

$c$ (2)  
a constant

$c'_\alpha$  
shear strength parameter

$c_i$  
triaxial yield strength for yield surface $i$

$D$  
tunnel diameter

$D$ (1)  
tangent stiffness matrix

$D$ (2)  
material property matrix

$DR$  
deflection ratio

$d$  
array of nodal degrees of freedom

$E$  
Young’s modulus in compression

$E_r$  
residual modulus

$E_s$  
representative soil stiffness
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EEB</td>
<td>equivalent elastic beam</td>
</tr>
<tr>
<td>EMB</td>
<td>equivalent masonry beam</td>
</tr>
<tr>
<td>e</td>
<td>eccentricity of building with respect to tunnel centreline</td>
</tr>
<tr>
<td>F</td>
<td>transverse applied force</td>
</tr>
<tr>
<td>f</td>
<td>residual stiffness factor</td>
</tr>
<tr>
<td>f</td>
<td>nodal load vector</td>
</tr>
<tr>
<td>f_b</td>
<td>residual bending stiffness factor</td>
</tr>
<tr>
<td>G</td>
<td>shear modulus</td>
</tr>
<tr>
<td>G_i</td>
<td>tangential shear stiffness after yield surface i</td>
</tr>
<tr>
<td>g'_α</td>
<td>shear stiffness parameter</td>
</tr>
<tr>
<td>H</td>
<td>building height</td>
</tr>
<tr>
<td>H_{lim}</td>
<td>limiting height</td>
</tr>
<tr>
<td>H_{ri}</td>
<td>Hermitian polynomial of level r and order j at node i</td>
</tr>
<tr>
<td>h'</td>
<td>effective height</td>
</tr>
<tr>
<td>I</td>
<td>second moment of area</td>
</tr>
<tr>
<td>I^*</td>
<td>effective second moment of area</td>
</tr>
<tr>
<td>i</td>
<td>distance to point of inflexion for a surface settlement trough</td>
</tr>
<tr>
<td>i_z</td>
<td>distance to point of inflexion for a subsurface settlement trough</td>
</tr>
<tr>
<td>J</td>
<td>polar moment of inertia</td>
</tr>
<tr>
<td>K (1)</td>
<td>trough width parameter</td>
</tr>
<tr>
<td>K (2)</td>
<td>stiffness</td>
</tr>
<tr>
<td>K</td>
<td>stiffness matrix</td>
</tr>
<tr>
<td>K_b</td>
<td>beam stiffness</td>
</tr>
<tr>
<td>K_{equiv}</td>
<td>equivalent elastic beam stiffness</td>
</tr>
<tr>
<td>K_{fe}</td>
<td>facade stiffness</td>
</tr>
<tr>
<td>k</td>
<td>shear coefficient</td>
</tr>
<tr>
<td>L</td>
<td>building length</td>
</tr>
<tr>
<td>L_i</td>
<td>Lagrange polynomial at node i</td>
</tr>
</tbody>
</table>
\( L/H \) length to height ratio
\( L/H_{\text{crit}} \) critical length to height ratio
\( M \) bending moment
\( M_{\text{sag}}^{DR} \) deflection ratio modification factor in sagging
\( M_{\text{hog}}^{DR} \) deflection ratio modification factor in hogging
\( M_{\text{sag}}^{\epsilon} \) strain modification factor in sagging
\( M_{\text{hog}}^{\epsilon} \) strain modification factor in hogging
\( m \) soil constant
\( N^* \) stability ratio
\( N_i \) shape function for node \( i \)
\( NSR \) normalised stiffness ratio
\( n \) proportion of overburden stress on tunnel lining
\( n \) vector perpendicular to tunnel axis
\( P \) point load
\( Q \) shear force
\( q \) load distribution
\( R \) radius of curvature
\( RS \) relative stiffness
\( r \) tunnel outer radius
\( S \) shear force
\( S_h \) horizontal displacement
\( S_{\text{max}} \) maximum vertical settlement
\( S_v \) vertical settlement
\( s_u \) undrained shear strength
\( T \) condensation matrix
\( t \) (1) wall thickness
\( t \) (2) distance between neutral axis and a beam edge
\( V_L \) volume loss
\( V_s \) (1) vertical displacement due to shear
\( V_s (2) \) settlement trough volume

\( V_o \) volume required to construct a tunnel

\( w_i \) lateral displacement of node \( i \)

\( w_o \) parabolic load parameter

\( \ddot{x} \) non-dimensional beam position

\( y (1) \) transverse distance from tunnel centre line (2D analyses)

\( y (2) \) longitudinal distance from start of tunnel (3D analyses)

\( z \) depth below ground surface

\( z_1 \) depth depth to the interface between two soil layers

\( z_2 \) distance of tunnel centre line below soil change interface

\( z_o \) depth to tunnel axis

\( \alpha \) angular strain

\( \alpha^* \) relative axial stiffness

\( \beta \) angular distortion

\( \Delta \) relative deflection

\( \Delta/L \) deflection ratio

\( \delta \) differential settlement

\( \delta_r \) radial displacement of tunnel lining

\( \epsilon \) tunnel ground loss ratio

\( \epsilon^{cr}_o \) stiffness reduction ratio

\( \epsilon^{cr} \) cracking strain

\( \epsilon_v \) vertical strain

\( \epsilon_h \) horizontal strain

\( \epsilon_{crit} \) critical tensile strain

\( \epsilon_{lim} \) limiting tensile strain

\( \epsilon_{bmax} \) maximum bending strain

\( \epsilon_{dmax} \) maximum diagonal strain

\( \epsilon_{br} \) limiting bending strain

\( \epsilon_{dr} \) limiting diagonal strain
\( \gamma (1) \) self weight
\( \gamma (2) \) shear strain
\( \gamma_0 \) constant shear strain
\( \kappa \) bending strain
\( \kappa_{crit} \) critical curvature
\( \mu \) increase in undrained shear strength with depth
\( \mu_i \) rotation of beam neutral axis at node \( i \)
\( \nu \) Poisson’s ratio
\( \theta \) rotation or slope
\( \theta_i \) rotation of beam cross section at node \( i \)
\( \rho^* \) relative bending stiffness
\( \sigma_o \) initial total stress
\( \sigma_1, \sigma_2 \) principal stresses
\( \sigma_x, \sigma_y \) stresses on an element
\( \tau \) shear stress
\( \omega (1) \) increase in shear modulus with depth
\( \omega (2) \) building tilt
Chapter 1

Introduction

The construction of new transport and utilities infrastructure in urban environments frequently involves the construction of tunnels under existing surface structures. As increasing population pressures drive the need for more infrastructure while simultaneously leading to the consumption of more surface space for housing and other developments, underground construction will continue to flourish as the preferred solution for infrastructure provision. Economic factors are also contributing to the increase in tunnel construction. Compared to surface developments, tunnels can be significantly cheaper when costs for acquiring land or moving utilities are considered in urban areas. Tunnel construction costs in urban areas are around £50 million per kilometre, although this cost has been falling at around four per cent each year in recent times (Automobile Association, 2001) making the underground option even more attractive.

Tunnel construction, however, particularly in soft ground conditions, can cause ground movements which have the potential to damage existing buildings and other structures. At particular risk are masonry buildings. An increasingly significant portion of the cost of tunnelling in is due to protective measures required to reduce the risk of damage to these structures.

As a result of both the increased physical congestion in urban environments and the increasing involvement of more cost conscious private investors in infrastructure projects,
the assessment of the potential impact of new tunnels on existing structures is increasingly important. The efficient and accurate prediction of damage to structures is an important part of the planning and feasibility stage of any urban tunnelling project. General geotechnical conditions, and more particularly soil-structure interaction considerations, can have a significant impact on the choice of the horizontal and vertical alignment, the design of the works and the contractual arrangements under which the construction ultimately takes place (Attewell, 1988). For example a more circuitous route or deeper tunnel may be required to ameliorate predicted damage to structures or expensive protective measures may be required. If building damage assessment methods are overly conservative this could lead to more expensive tunnelling or excessive and unnecessarily costly protective works. As a result, the construction and operating costs of the tunnel project can increase, threatening the viability of the project (New and Bowers, 1994). With more accurate and efficient methods of assessing tunnel-induced ground movements and the risk of associated building damage, such costs can be minimised and construction operation and contractual arrangements can be more easily made.

Investigating methods of assessing the impact of soft ground tunnelling on buildings is thus the thrust of the research described in this thesis. In particular, this research is concerned with developing improved numerical methods to model the response of masonry structures to soft ground tunnelling in urban areas.

A review of current literature can be found in Chapter 2, followed by a discussion of the gaps in current knowledge, opportunities for research and the aim and scope of this project in Chapter 3. The research undertaken is presented in the Chapters comprising the main body of the thesis and the concluding Chapter provides a summary of the new developments and an assessment of their potential for wider use or further development.
Chapter 2

Prediction of Damage to Buildings due to Soft Ground Tunnelling: A Review of Literature

2.1 Introduction to tunnelling methods

The method chosen to construct a tunnel is dependent firstly on the ground conditions expected on site and secondly on other considerations such as the availability of plant, time and cost constraints and other construction considerations.

Tunnelling in hard rock is generally undertaken by drill and blast, road headers, tunnel boring machines or a combination of methods followed by the installation of tunnel support such as rock bolts, steel sets or concrete lining. As this research is concerned with damage to buildings due to tunnelling in soft ground, hard ground methods are not considered.

The construction of tunnels in soft ground (sands and clays) was historically achieved by hand excavation using shovels and picks with openings supported temporarily by timber and later lined with masonry. Collapses of tunnel excavations were frequent, however, prompting the invention of the protective tunnelling shield, patented by Marc Brunel in 1820. Brunel’s rectangular faced shield was used during the construction of the first
Thames Tunnel between 1825 and 1843 with excavation carried out by hand within the
shield followed by the erection of a brick lining (Sandstrom, 1963). Peter Barlow patented
a cylindrical tunnelling shield in 1865 which was used to construct a foot tunnel under
the Thames at Tower Hill in 1869 using bolted cast iron lining instead of masonry, against
which the shield was jacked forward. The engineer for the works was J. H. Greathead who
made improvements to the shield tunnelling process developing what is now considered the
forerunner of modern tunnelling shields (Sandstrom, 1963). Permanent linings currently
used in shield tunnelling include precast concrete segments, steel or cast iron segments, cast
insitu concrete or reinforced shotcrete (Potts and Zdravkovic, 2001). Tunnelling shields
can be divided into two general categories: open and closed shields.

Open shields have an unsupported face where material is excavated by mechanical means
such as excavators, cutters or road headers within the shield. These can only be used in
conditions such as stiff clays, where the soil is relatively self-supporting. Where ground
conditions are too unstable for open shield tunnelling, closed shield tunnelling is used.

Closed shields, known as Tunnel Boring Machines (TBMs) support the face as the tunnel is
excavated. A rotating cutting head is advanced by jacks reacting on the completed lining,
with the face supported by controlling the applied thrust and rate of removal of excavated
material (Potts and Zdravkovic, 2001). Where the ground is less stable, additional support
can be provided by using slurry shield or Earth Pressure Balance (EPB) TBMs.

Slurry shield TBMs were introduced to the UK in the 1960’s and use bentonite slurry
under pressure to stabilise the working face (Leca et al., 2000). Excavated soil mixes with
the slurry and is pumped back to the surface. The use of slurry shield TBMs is now
common, with recent projects utilising the method including the Sophia railway tunnel
near Rotterdam in the Netherlands (Netzel, 2002).

In an EPB machine the face is supported by retaining excavated spoil in the working
chamber under pressure, thus balancing the earth pressures in the ground (Fujita, 1989).
Recent uses of EPB machines include the construction of the Madrid Metro (Hernandez
et al., 2000) and the Lisbon Underground (Marahna and Marahna das Neves, 2000).
Recent advances in the use of TBMs include large diameter machines such as the 14.2m diameter TBM used for the fourth crossing of the Elbe River in Hamburg, Germany (Leca et al., 2000) and mixed or universal TBMs designed to handle a range of soil and rock conditions by operating in any one of the different modes described above.

Another tunnelling method now used in urban tunnelling projects, including the Heathrow Express rail tunnel (New and Bowers, 1994), involves the use of a sprayed concrete lining. Known as the New Austrian Tunnelling Method, it came to prominence under von Rabcewicz during the construction of the Schwaikheim Tunnel in 1964. The first use of the method in soft ground in an urban area was in 1968 in Frankfurt am Main, Germany (Sauer, 1988). The process involves the excavation of a section of tunnel followed by the application of shotcrete (or other temporary support) to the excavated surface before the installation of a permanent lining. Such lining is usually a second application of reinforced shotcrete (or permanent rock bolts for a hard ground tunnel). For large diameter excavations, the advance is usually undertaken by using headings and side drifts to limit the size of the open excavation face (Potts and Zdravkovic, 2001).

### 2.2 Prediction of settlements - semi-empirical methods

The construction of a tunnel in soft ground results in deformations of the surrounding soil which manifest on the surface as a surface settlement trough. It is commonly accepted that the transverse profile of these surface settlements can be described by a Gaussian curve, shown in figure 2.1 and represented by the formula,

\[ S_v = S_{\text{max}} e^{-\frac{y^2}{2i^2}} \]  \hspace{1cm} (2.1)

where \( S_v \) is the vertical settlement at the surface, \( S_{\text{max}} \) is the maximum vertical settlement over the axis of the tunnel, \( y \) is the transverse distance from the tunnel axis and \( i \) is the transverse distance to the point of inflexion of the curve. This description was first put forward by Martos (1958) and subsequently shown to be a valid approximation for
the shape of the settlement trough above a tunnel in soft ground (Peck, 1969). This formulation assumes that the tunnel is passing under a greenfield site where there are no buildings present. The extent of the surface settlement trough at a greenfield site in three dimensions is shown in figure 2.2.

It is accepted that $i$ is a linear function of the depth of the tunnel axis, $z_0$, below the surface when the assumption is made that all movement of soil occurs along radial paths towards the tunnel axis under constant volume (O’Reilly and New, 1982). Thus,

$$i = Kz_0$$

where $K$ is a trough width parameter which depends on the soil type and condition. Values of trough width parameter $K$ vary in the range of 0.2 to 0.3 for granular materials above the water table and from 0.4 for stiff clays to approximately 0.7 for soft silty clay (O’Reilly and New, 1982; Rankin, 1988; and Mair et al., 1993). Choice of $K$ requires judgment depending on the soil type as well as the level of the water table.

The trough width parameter $K$ can be considered approximately constant for different soil depths when determining surface settlements but varies with depth when considering subsurface settlements (Mair et al., 1993). Various alternative empirical expressions for $i$ and $K$ exist including those suggested by Schmidt (1969), Gunn (1993) and Selby (1988) but equation 2.2 is generally used in practice.

The volume of the settlement trough $V_s$, per unit length of tunnel advance can be evaluated
CHAPTER 2. REVIEW OF LITERATURE

Figure 2.2: 3D surface settlement profile (after Attewell, 1986)

by integrating equation 2.1,

\[
V_s = \int_{-\infty}^{\infty} S_{max} e^{-\frac{y^2}{2\pi^2}} = \sqrt{2\pi} S_{max}
\] (2.3)

The volume loss, \(V_L\), is the volume of the settlement trough per unit length expressed as a percentage of the total excavated volume of the tunnel,

\[
V_L = \frac{V_s}{V_o} \times 100
\] (2.4)

where \(V_o\) is the volume required to construct the tunnel. This is based on the assumption that soil movements occur under constant volume.

Volume loss is caused by the difference in the volume of soil excavated for the tunnel and the volume of the completed lined tunnel taking its place. Soil around the tunnel moves to fill this volume loss, the magnitude of which is also termed the ground loss and is dependent on the tunnelling method, soil type and care taken by the excavation contractor (Potts and Zdravkovic, 2001). Sources of volume loss as shown in figure 2.3 include four
major contributors (Leca et al., 2000):

- Movement of soil towards the tunnel face, *face loss*, due to face stress release;
- Displacements along the tunnelling shield, *shield loss*, due to deviations of the ma-
  chine or shear stresses along the side;
- Ground movements into the tail gap, *tail loss*, from transition to the liner; and
- Permanent liner deformations (much less significant than the previous three).

Volume loss for London clay is likely to be in the range of 1.0-3.0% for shield tunnelling
(O’Reilly and New, 1982) and 1.0-1.5% for NATM tunnelling (New and Bowers, 1994).

![Diagram of sources of ground loss during soft ground tunnelling](image)

**Figure 2.3: Sources of ground loss during soft ground tunnelling**

The vertical settlement at any surface position can thus be found by combining equations
2.1, 2.2 and 2.3 to give,

\[ S_v = \frac{V_s}{\sqrt{2\pi K z_o}} e^{-\frac{y^2}{2Kz_o^2}} \]  

(2.5)

The slope and curvature of the settlement profile can thus be obtained by differentiation,

\[ \frac{dS_v}{dy} = \frac{-V_s y}{\sqrt{2\pi i^3}} e^{-\frac{v^2}{2i^2}} \]  

(2.6)

\[ \frac{d^2 S_v}{dy^2} = \frac{V_s}{\sqrt{2\pi i^3}} \left[ \frac{y^2}{i^2} - 1 \right] e^{-\frac{y^2}{2i^2}} \]  

(2.7)
The vertical ground strain $\epsilon_v$ is thus,

$$\epsilon_v = \frac{dS_v}{dz} = \frac{V_s}{\sqrt{2\pi K z_o^2}} \left[ \frac{y^2}{i^2} - 1 \right] e^{-\frac{y^2}{K z_o^2}} \quad (2.8)$$

In addition to vertical movements the soil undergoes horizontal displacement at the surface. The assumption that all particulate movement of soil occurs along radial paths towards the tunnel axis under incompressible plain strain conditions (New and O’Reilly, 1991; O’Reilly and New, 1982) allows the determination of the horizontal displacement $S_h$,

$$S_h = S_v \frac{y}{z_o} \quad (2.9)$$

Differentiating equation 2.9 with respect to $y$ and including equation 2.3 gives the horizontal strain $\epsilon_h$ as,

$$\epsilon_h = \frac{dS_h}{dy} = \frac{V_s}{\sqrt{2\pi K z_o^2}} \left[ 1 - \frac{y^2}{i^2} \right] e^{-\frac{y^2}{K z_o^2}} \quad (2.10)$$

The plane strain constant volume deformation condition is thus satisfied as $\epsilon_h = -\epsilon_v$.

Longitudinal settlement profiles in the direction of tunnelling are assumed to take the form of a cumulative probability curve (Attewell and Woodman, 1982) which advances with tunnel construction. The settlement directly above the excavation face is assumed to be equal to $0.5S_{max}$.

Where multiple tunnels are present, as occurs with twin tunnels carrying traffic in opposite directions, it is generally assumed that ground movements arising from the construction of each tunnel (calculated using the semi-empirical methods above) can be superimposed. For tunnels that are separated by less than one tunnel diameter, this may be unconservative (Burland, 1997). Settlements at a monitored greenfield reference site at Old Jamaica Road in London show this assumption to be unconservative for two 19.5m deep, 4.85m diameter tunnels separated by 26m in the Lambeth Group (Withers, 2001a). Chapman et al. (2002) use the results of finite element analyses of twin tunnels to demonstrate that twin piggy
back tunnels result in surface settlements that agree well with superimposed semi-empirical predictions but that settlements due to the construction of the second of side by side tunnels exhibit greater settlements on the side of the first tunnel. They propose that this is due to the soil near the first tunnel having been previously strained by its construction. Mecsi (2002), however, finds that for the twin 5.5m diameter tunnels of the Budapest Metro, separated by 22m in Kiscell clay, the superposition of predicted Gaussian settlements from each tunnel compares favourably with measured field data.

The prediction of subsurface displacements has been undertaken in London clay by various researchers. Mair et al. (1993) propose that the relationship for \( i \) for a subsurface settlement trough at a depth \( z \) (equation 2.2) be amended to \( i_z = K(z_o - z) \). New and Bowers (1994) improved their subsurface predictions by assuming that all ground movements are towards a ribbon sink along the longitudinal tunnel axis rather than a line sink at the tunnel centre.

Moh and Hwang (1996) proposed the use of the following formulation for \( i \) for subsurface settlement troughs at depth \( z \) which is based on the expression proposed by Schmidt (1969),

\[
   i_z = \left( \frac{D}{2} \right) \left( \frac{z_o}{D} \right)^{0.8} \left( \frac{z_o - z}{z_o} \right)^m
\]

(2.11)

where \( m \) is a constant based on the soil type and \( D \) is the tunnel diameter. Values of \( m = 0.4 \) for silty sand and \( m = 0.8 \) for silty clays are recommended. The formulation is based on case study data from the construction of the Taipei Rapid Transit System.

The majority of physical modelling of settlements due to tunnelling has been undertaken in laboratory centrifuge tests. Centrifuge tests undertaken by Mair (1979) are referred to by Mair et al. (1993) who use the detailed subsurface measurements to propose the relationship above. Leca et al. (2000) give a summary of centrifuge modelling including tests by Stallebrass et al. (1996) and Grant and Taylor (1996) whose results indicated smaller and thinner settlement troughs than expected. Grant et al. (1999) also describe a centrifuge test of a tunnel heading in kaolin clay where ground movement results were compared to a three-dimensional finite element analysis and found to give reasonable agreement. In general it appears that, as might be expected, field data from real tunnels is more useful
for theoretical model validation or formulation than laboratory data due to the difficulties inherent in physically modelling the complex tunnelling processes at small scale.

The semi-empirical approach described above provides a simple means of estimating surface settlements due to tunnelling in soft ground while ignoring the presence of any structures. The key parameters of the volume loss and trough width parameter have been widely investigated and shown to be sensitive to soil type and condition, tunnel construction method and the care taken by the excavation contractor.

2.3 Prediction of settlements - analytical methods

2.3.1 Closed form solutions

Closed form solutions can at best only provide a rough approximation of ground behaviour as they cannot account for the inherent complexities of tunnel construction methods and the non-linearity and anisotropy evident in tunnelling problems (Mair, 1999). They can, however, provide a useful and quick method of settlement prediction.

Two closed form solutions are described by Chow (1994). The first, by Poulos and Davies (1980), uses the solution for vertical displacements due to a point load in elastic half space. Vertical displacements are obtained by integrating the solution for a line load equal to the magnitude of the weight of material excavated. As this prediction only accounts for unloading, not volume loss, heave is predicted. The Sagaseta (1987) method is also described by Chow. This method accounts for volume loss and is based on incompressible irrotational fluid flow solutions. Chow derives the solution for vertical displacements as,

\[ S = -\frac{\gamma D^2 z_0^2}{4G(y^2 + z_0^2)} \]  

where \( S \) is the vertical settlement, \( \gamma \) is the soil density, \( G \) is the shear modulus and \( D \) is the tunnel diameter.

Predictions using this method are compared to the Gaussian profile and field measurements
from the Caracas Metro and M-40 Motorway in Madrid (Oteo and Sagaseta, 1996) and data from the construction of the Valencia Underground Line 5 (Celma and Izquierdo, 1999). The method is found to produce a wider settlement trough than the Gaussian profile and case study data but similar maximum settlement.

Celma and Izquierdo (1999) also consider the method of Verrujit and Booker (1996). This solution is a generalisation of the Sagaseta method and includes the factors $\epsilon$ and $\delta$ which take into account the ground loss and ovalisation of circular tunnels respectively. The settlement is given by,

$$S = 2\epsilon a^2 \frac{z_o}{(y^2 + z_o^2)} - 2\delta a^2 \frac{y^2 - z_o^2}{(y^2 + z_o^2)^2}$$

where $a$ is the tunnel radius. Predictions based on this method are found to be similar to those predicted using the semi-empirical Gaussian profile.

Loganathan and Poulos (1998) proposed a solution for the surface settlement which also incorporates the ground loss ratio $\epsilon$ as,

$$S = 2\epsilon a^2 \frac{4z_o(1 - v)}{(y^2 + z_o^2)} e\left(\frac{-1.38\sigma^2}{(a+z_o)^2}\right)$$

where $\nu$ is the Poisson’s ratio of the soil. Predictions using this method are compared with data from the New Southern Railway in Sydney, by Loganathan et al. (2000). Predictions gave higher than maximum field settlements, and a wider settlement profile.

Bobet (2001) presents an elastic solution assuming uniform circular radial deformation which is modified by Park (2005) to account for ovalisation. Park compares this method with those described above and with five case studies (including the Heathrow Express and Jubilee Line Extension tunnels in London), concluding that while the closed form elastic solutions are limited in scope, they produce similar displacement profiles which agree reasonably well with case study data and are thus useful for preliminary investigations.

Plasticity solutions are used to predict subsurface settlement profiles by Mair and Taylor (1993). They use the solution for a cylindrical contracting cavity in a linear elastic-perfectly
plastic soil for transverse ground movements and the solution for a spherical cavity for movements ahead of the tunnel advance. For an unloaded spherical cavity the solution is,

$$\frac{\delta}{a} = \frac{s_u}{3G} \left(\frac{a}{r}\right)^2 e^{(0.75N^*-1)}$$

(2.15)

where, $\delta$ is the radial movement at radius $r$, $a$ is the inner radius of the tunnel, $s_u$ is the undrained shear strength, $N^*$ is the stability ratio given by $\sigma_o/s_u$, $\sigma_o$ is the initial total stress at the cavity boundary and $G$ is the shear modulus. An amended version of the spherical cavity equation is presented for lined tunnels. For an unloaded cylinder,

$$\frac{\delta}{a} = \frac{s_u}{2G} \left(\frac{a}{r}\right) e^{(N^*-1)}$$

(2.16)

Grant and Taylor (2000) evaluate these plasticity solutions by comparing them to measured data from centrifuge tests and express confidence in the use of the unloaded cylinder approach for interpreting data from field measurements.

Stochastic methods provide analytical justification for the use of the Gaussian profile according to Attewell and Woodman (1982) who show that for moderate subsidence, the surface trough of a stochastic model in two dimensions follows the Gaussian distribution.

### 2.3.2 Numerical methods

The use of numerical methods for the prediction of settlements due to tunnelling is becoming increasingly common in engineering practice. In particular, finite element methods are commonly used in the analysis of tunnelling problems. There exists a significant number of texts on the subject (for example Zienkiewicz, 1977; Dawe, 1984; and Astley, 1992), including recent texts on the application of finite element analysis in geotechnical engineering (Potts and Zdravkovic, 1999 and 2001). A detailed description of the fundamentals of the method is thus not given here. A good summary of the use of finite element models for tunnelling analyses prior to 1989 is given by Clough and Leca (1989) and a more recent summary is given by Negro and de Queiroz (2000).
The majority of numerical tunnelling models (92% according to Negro and de Queiroz), especially early work, are two-dimensional (2D), with most assuming plane strain conditions. Papers including Mair et al. (1981), Finno and Clough (1985), van Jaarsveld (1999), Karakus and Fowell (2000), Romera et al. (2000), Drakos et al. (2002) and Tolis and Dounias (2002) describe plane strain finite element analyses and compare the settlements predicted with field data. Plane strain analyses are commonly used for the reason that they require less computer resources and time than three-dimensional (3D) analyses (Augarde, 1997). Two-dimensional representations, however, cannot model the effects of the passage of a tunnel in the longitudinal direction, complex 3D geometries such as tunnel joints or other inherent three-dimensional effects as noted by Augarde (1997), Mahranha and Maranha das Neves (2000), Fricker and Alder (2001) and Vermeer (2001).

Three-dimensional numerical analyses are increasingly evident in the literature (8% of reported analyses in the last decade according to Negro and de Queiroz although the percentage in immediately recent years appears to be much higher) due to advances in computer hardware and the increasing availability of appropriate commercial software. It is generally accepted that 3D analyses are required to fully capture all the mechanisms of ground deformation around a tunnel (Burd et al., 1994; Attewell et al., 1986; and Potts, 2003). Three-dimensional analyses are commonly used now both for research, typically utilising in-house finite element software at research institutions (Rowe et al., 1983, Augarde, 1997, Hernandez et al., 2000 and Franzius, 2004) and for commercial design, using software packages such as FLAC 3D (Dias et al., 2000 and Fricker and Alder, 2001), ABAQUS (Guedes de Melo and Santos Pereira, 2002) and PLAXIS 3D Tunnel (Vermeer, 2001 and Schweiger, 2001). The number of papers describing the use of 3D finite element modelling of tunnels is also increasing and includes Lee and Rowe (1989, 1990 and 1990a), Augarde et al. (1999), Maranha and Maranha das Neves (2000), Hernandez et al. (2000), Vermeer (2001), Truty and Zimmermann (2002), Lee and Ng (2002) and Kasper and Meschke (2004).

Three-dimensional analyses are more time consuming to prepare and use considerably more computer resources and time in comparison to 2D analyses, a fact which which continues to limit their use in industry (Fricker, 2001 and Miliziano et al., 2002).
When modelling tunnelling in soft ground using finite elements there are a number of key areas to be considered, which influence the quality of predictions. These include the constitutive soil model, modelling of the tunnel lining and the modelling of excavation.

A range of constitutive models for overconsolidated clays, such as London clay, are reported in the literature. Considering such soil as a linear elastic material has been found to be unsuitable as the predicted displacements involve heave due to unloading effects and stress relief (Rowe et al., 1983, Rankin, 1988 and Chow, 1994). Linear elastic-perfectly plastic models are investigated by Rowe et al. (1983) who find that they give much more realistic surface settlements than elastic models; they are also used by Chow (1994) who does not find any significant improvement. Chow notes that the use of a linear elastic model where stiffness increases linearly with depth provides improved results. Guedes de Melo and Santos Pereira (2002) use a linear elastic-perfectly plastic soil to model the construction of the Shanghai Metro Line 2 and find that it predicts shallower and wider surface settlement troughs than observed during construction. The Modified Cam-clay model is used by Mair et al. (1981) but is also found to produce wider and flatter profiles due to the soil elasticity dominating the surface response. Modified Cam-clay plasticity with non-linear elasticity is found to give reasonable predictions by Karakus and Fowell (2000).

It is generally accepted that simple linear elastic-plastic models lead to the prediction of profiles that are too wide and shallow as they cannot correctly account for the non-linear and inelastic soil behaviour which has been shown to occur at small strains and is an important feature of soil-structure interaction (Calabresi et al., 1999).

Recent work has highlighted the necessity of modelling soil non-linearity at small (pre-failure) strains which occur in overconsolidated clays. The review of reported numerical analyses by Negro and de Queiroz (2000) concluded that maximum surface settlements predicted in finite element analyses were close to the measured field value in 71% of cases but that over half of the analyses gave poor predictions of overall soil movement profiles. The reason was considered to be oversimplification of soil constitutive models and the authors recommended the use of a non-linear pre-yield soil model to overcome this problem. The significance of the non-linear behaviour of soil at small strains was investigated
using laboratory tests by Jardine et al. (1986) who proposed an empirical stress strain relationship which matched their observed non-linear response of undrained clay. This non-linear pre-yield model combined with a Mohr-Coulomb failure criterion and plastic potential was used by Addenbrooke et al. (1997) and compared with a linear elastic model to conclude that modelling non-linear pre-failure stiffness is required to predict reasonable surface settlements. The same model has been used in other reported numerical analyses including those of Potts and Addenbrooke (1997) and Franzius (2004). Gunn (1993) also used a model combining non-linear elasticity at small strains with a Tresca yield criterion which predicted wider troughs than the Gaussian profile but good ground loss values. Kinematic yield hardening models have been developed by Atkinson and Stallebrass (1992) and Houlsby (1999) which model the variation of stiffness at small pre-failure strains. In these models multiple nested von Mises yield surfaces exist inside an outer surface. The surfaces translate in stress space as the stress point moves until the outermost boundary is reached which defines the undrained shear strength of the material. Chow (1994) found that predictions using such a model with ten surfaces gave reasonable results in 2D. The Houlsby (1999) model has been used in 3D analyses by Augarde (1997), Liu (1997), Bloodworth (2002), Wisser (2002) and is also described by Burd et al. (2000). A recent study by Grammatikopoulou et al. (2002) found that both two and three surface models gave good results but slightly shallower and wider settlement troughs than a Gaussian profile.

Soil anisotropy and the choice of $K_o$ are also found to influence surface settlement predictions to varying degrees. Simpson et al. (1996) undertook a range of 2D analyses of the Heathrow Express trial tunnel using linear and non-linear anisotropic soil models and a non-linear isotropic model and concluded that modelling soil anisotropy gave improved settlement predictions. Addenbrooke et al. (1997), however, concluded that the modelling of soil anisotropy did not enhance plane strain predictions for surface settlement if non-linear pre-failure stiffness is included. Potts and Zdravkovic (2001) compare settlement profiles obtained from 2D finite element models using linear isotropic, linear anisotropic and non-linear elastic constitutive models with field data. For all cases stiffness increased with depth and plastic behaviour was modelled using the Mohr-Coulomb model. All three
cases give settlement profiles that are too wide and shallow in comparison with the field data. The role of stiffness anisotropy in 3D analyses was investigated by Lee and Ng (2002) who used an elastic-perfectly plastic model and concluded that anisotropy is less important than an appropriate choice of $K_o$ for the realistic prediction of both transverse and longitudinal settlements and that 3D modelling also leads to improved predictions. Franzius et al. (2005) present a suite of both 2D and 3D analyses using non-linear elastic-plastic soil for both isotropic and anisotropic conditions. They conclude that for the isotropic model, 3D modelling does not significantly improve predictions, nor does the inclusion of anisotropic soil in both 2D and 3D analyses.

Early numerical models of tunnels did not include the effect of a tunnel lining; unlined tunnels were modelled for simplicity. Methods of modelling the tunnel lining are described by Potts and Zdravkovic (2001) who suggest zero thickness curved shell elements in 2D analyses in preference to solid elements. A bedded beam model in 2D is described by Fricker and Alder (2001) where beam elements are used to model the lining. Karakus and Fowell (2000) also describe the use of curved beam elements. Augarde (1997) describes the formulation of overlapping three noded beam elements for use in 2D and overlapping shell elements for use in 3D. Augarde et al. (1999) and Augarde and Burd (2001) compare the use of these overlapping shells with the use of continuum elements for the modelling of the lining in 3D and conclude that while there may be situations in which solid elements are poorly conditioned geometrically, any associated errors are not significant and their use is more robust when compared to the overlapping shell elements which have a tendency to respond in an over-stiff manner when embedded in a continuum mesh. Continuum lining elements are used to good effect by Wisser (2002) who also asserts their benefits over shell elements in contributing to the prediction of realistic surface settlements.

The modelling of a tunnel excavation should ideally be a continuous process to simulate the construction of a real tunnel. An analytical method for solving problems with such a constantly changing domain is presented by Aubry and Modaressi (1989) but its practical numerical application is unclear (Augarde, 1997). Using discrete finite elements, excavation can be modelled by the incremental removal of groups of soil elements in stages. The
unloading due to the excavation is then considered. Previous examples (Gioda and De Donato, 1979; Swoboda et al., 1989; and Chow, 1994) use a method to calculate the nodal loads to be applied to the mesh but ignore the body forces and surface tractions of the elements remaining after excavation has taken place. Augarde et al. (1995) and Augarde (1997) describe a method based on the procedure proposed by Brown and Booker (1985) which gives the correct nodal loads to be applied at the excavation stage. This method has also been used successfully by Rowe and Lee (1991).

Recognising the importance of capturing the full 3D effects of tunnel construction, a number of methods used previously for modelling 3D aspects of excavation in two dimensions are summarised by Potts and Zdravkovic (2001). These include the gap method where a predefined void is inserted into the mesh representing the expected value of the ground loss. This is achieved by keeping the tunnel invert on the soil and specifying a gap parameter at the crown. The movement of soil towards the tunnel lining is limited by the magnitude of the gap. The convergence confinement method involves prescribing the proportion of unloading during excavation and prior to lining construction. A progressive softening method is described which reduces the stiffness of the soil in the heading by a factor $\beta$ before the excavation of the soil and construction of the lining. A volume loss control method is also described where the volume loss on completion is prescribed. This is used by Potts and Addenbrooke (1997) where the volume loss is calculated at the end of each incremental excavation of soil elements. Once it reaches a specified value the calculation is terminated. These methods are proposed for use only in two dimensions and are intended to account for the stress and strain changes ahead of tunnel advancing in the longitudinal direction.

Despite the ability of 2D methods to model some of the 3D aspects of tunnelling, three-dimensional analyses are required to fully capture all necessary facets of tunnel construction and as such a number of different methods have been used for the process of simulating tunnel construction and lining erection methods in 3D. Lee and Rowe (1990 and 1990a) describe an investigation into various simple 3D methods including the staged excavation of a tunnel with no lining and a perfectly rigidly lined tunnel where soil elements to be excavated are removed from the analysis progressively in steps. Rowe and Lee (1992)
used these 3D methods, as well as simplified 2D methods including an axisymmetric and a longitudinal plain strain analysis, to compare the predictions with case study data from the Thunder Bay sewer tunnel. The three-dimensional analysis was found to give much more realistic surface displacements than either of the two simplified 2D approaches.

Two different construction techniques are simulated by Guedes de Melo and Santos Pereira (2000); the NATM method with delayed lining construction and slurry shield tunnelling. For NATM simulation the analysis takes place in stages each involving the removal of a 2m section of tunnel elements followed by the installation of concrete lining after soil displacements have occurred (Vermeer (2001) uses an identical technique for NATM construction). For slurry shield tunnelling, the steps within each phase involve the removal of soil elements and the simultaneous placement of a shield and pressure on the excavation face with a gap allowing the soil to deform. Lining elements are installed in the previous 2m step at this time with a void around them into which a low stiffness grout is inserted and pressure applied. Surface displacements and tunnel lining loads predicted by these methods are compared to 2D predictions using the convergence confinement method and are found to give more realistic results in both cases. It is noted that the 2D analyses only agree with the 3D analyses for certain choices of model parameters. The importance of the choice of parameters is emphasised by Dias et al. (2000) who use a similar approach to modelling TBM tunnelling in 3D and find that on comparison with 2D predictions using the convergence confinement method, reasonable agreement was only reached once appropriate parameters were obtained by back analysis. Modelling of tunnel construction in 3D is becoming more detailed with the model described by Kasper and Meschke (2004) including the simulation of a TBM with frictional soil contact, hydraulic jacks, installation of lining, tail grouting and a control algorithm for tunnelling shield.

Augarde et al. (1998) describe a 3D method where soil elements in the tunnel are removed and lining elements activated simultaneously with no unsupported section. The lining elements are then subjected to uniform hoop shrinkage to develop the required ground loss. This method has been used to effectively by Bloodworth (2002) and Wisser (2002).

A good recent comparison of some previous 3D numerical analyses is given by Franzius and
Potts (2005) who compare physical mesh dimensions and excavation stage lengths used by previous authors. They conclude that a distance of 13 times the tunnel diameter is required in front of the excavation face for the vertical end mesh boundary not to affect settlement results at a location of interest; that no steady state longitudinal settlement was possible for the mesh dimensions compared and that there is a trade off between the longitudinal tunnel stage excavation length and computational efficiency.

2.4 Case studies of greenfield settlement

There is now a significant amount of data available on surface settlements at greenfield sites. Many papers contain case study data for the purpose of validation of models, while there are others which present general data comparing key parameters from a range of tunnelling projects (Rankin, 1988; O’Reilly and New, 1982; and Lee, 1996), and a small number containing detailed data from specifically instrumented greenfield sites.

Many of the papers cited so far in this thesis contain case study data for the validation of empirical or analytical methods of predicting settlement. Data are presented from such projects as the Lisbon Underground (Marahna and Marahna das Neves, 2000), Madrid Metro (Hernandez et al., 2000 and Romera et al., 2000), the London Underground (Mair and Taylor, 1993 and Cooper and Chapman, 2000) and the Budapest Metro (Mecsi, 2002). The number of reported measured parameters is typically small and the number of data points scarce, as mostly monitoring was not conducted for the specific purpose of research.

New and Bowers (1994) present the results of a comparison of empirical and analytical prediction methods with surface settlements specifically measured for research at the Heathrow Express trial tunnel in a uniform strata of London clay overlain by a thin gravel layer. The tunnel was constructed using the sprayed concrete lining technique and instrumented extensively. The measured transverse settlement trough confirmed the assumption of a Gaussian profile and the finite element model described in the paper predicted the surface settlement profiles reasonably well. No details are given regarding the composition of the model. Ground loss values were also found to be in reasonable agreement with
the measured values. The data from the same site were also used by Karakus and Fowell (2000) for validation of a finite element model for sprayed concrete lining construction in soft ground. The soil was modelled using a non-linear elastic model with modified Cam-clay plasticity. Maximum settlements predicted by the model, which simulated nine separate construction stages, were found to be similar to those in the field, but simulations with five stages and one stage resulted in shallower and wider settlement troughs.

As part of a research project coordinated by Imperial College and sponsored by government bodies and industry centred on the construction of the Jubilee Line extension in London, four greenfield sites were instrumented to measure surface displacements due to tunnelling. The sites were located at St James’s Park, Westminster (London Clay) (Nyren et al., 2001) and Southwark Park, Old Jamaica Road and Niagara Court, Berdmonsey (Lambeth Group beds) (Withers, 2001a). At each site, instrumentation comprising surface survey points, rod extensometers and electrolevel inclinometers was installed transverse to the direction of tunnel advance. Vertical surface and subsurface movements as well as horizontal surface movements were recorded from prior to the construction of the tunnels.

Vertical settlements at the St James’s Park site for the 4.9m diameter, 20.5m deep East-bound tunnel are shown in figure 2.4. The settlement profile can be seen to be approximately Gaussian from the inset plot of $\ln S/S_{\text{max}}$ versus the square of the distance from the centre line, which is almost straight. Volume loss was found to be 2.8% with a maximum settlement of 23.4mm (Nyren et al., 2001) for the eastbound tunnel and 3.3% and 20.4mm for the west bound tunnel (Standing and Burland, 2006). This level of volume loss was unexpected; what was thought to be a conservative value of 2% having been used for design. Standing and Burland (2006) attribute the larger than expected ground losses to the tunnelling technique (length of unsupported heading) and the particular geological conditions at this location and tunnel depth (sand and silt partings in the London Clay).

Settlement at the Southwark Park site for both tunnels is shown in figure 2.5. The transverse settlement is reasonably close to a Gaussian curve. Much smaller volume losses of the order of 0.4% and settlements of the order of 3.5mm were recorded for both the 21m deep west and east bound tunnels constructed in Glauconitic sands of the Lambeth group.
Transverse settlement profiles at the Niagara Court and Old Jamaica Road sites were found to give reasonable agreement with the Gaussian profile (Withers, 2001a). For all the reference sites, longitudinal settlement profiles were found to have a shape similar to a cumulative probability curve. The magnitudes of the horizontal strains induced at the surface differed between the Southwark and St James's Park sites with the former having around a third of the tensile strain magnitude of the latter. Horizontal strains were
determined by measuring horizontal ground movements between adjacent survey stations with a micrometer stick. Values determined for the trough width parameter $K$ were around 0.4 for the London clay and 0.5 for the Lambeth Group sites.

Measurements of the St James’s Park site confirm the assertions of Burland (1997) and the finite element predictions of Chapman et al. (2002) that simple superimposition of Gaussian settlements for multiple tunnels can be unconservative. Ground movements associated with the second of the driven tunnels were found to be asymmetrical with a larger zone of movement and greater volume loss on the side of the existing tunnel. Shear strains in the soil, caused by the construction of the first tunnel, which reduce the soil stiffness were found (with the help of numerical predictions), to be the cause of this phenomenon.

Numerical predictions of the ground movements at these greenfield sites, described by Addenbrooke and Potts (2001) for St James’s Park and Kovacevic et al. (2001) for the other sites, predicted surface settlement troughs which were wider and shallower than the measured values described above. The maximum settlement predicted at the St James’s Park site for each tunnel was 12mm (eastbound) and 11mm (westbound) as compared to the measured values of 23mm and 20mm respectively.

2.5 Prediction of damage to buildings

2.5.1 Early empirical methods and definitions

Early work relating to building damage due to settlements was based on an empirical approach which was not specific to the cause of the settlements. The studies by Skempton and Macdonald (1956) and Polshin and Tokar (1957) led to recommendations on the allowable settlement of structures. Skempton and MacDonald used as their criterion for damage the *angular distortion*, $\beta$ defined as the ratio of the differential settlement $\delta$ and the distance $l$ between two points. They found that cracking of walls and partitions would commence when $\beta > 1/300$ and structural damage would occur when $\beta > 1/150$. The recommendation was made to limit $\beta$ to a value of 1/500. Polshin and Tokar used three
criteria including the *slope*, defined as the difference in settlement of two adjacent supports relative to the distance between them. Maximum slopes were recommended as 1/500 for steel and concrete frame buildings or 1/200 where there is no infill. These recommendations agreed well with those of Skempton and MacDonald.

Current physical definitions relating to the prediction of damage, as given by Burland and Wroth (1974) and updated by Burland (1997), are given below and shown in figure 2.6:

- *Rotation* or *slope*, $\theta$, is the change in gradient of a line joining two points;
- *Angular strain*, $\alpha$, is the change in angle between adjacent straight lines joining two points on the building base;
- *Relative deflection*, $\Delta$, is the displacement of a point relative to a line connecting two reference points on either side;
- *Deflection ratio* is given by $\Delta/L$ where $L$ is the distance between reference points that define $\Delta$;
- *Tilt*, $\omega$, defines the rigid body rotation of the structure;
- *Angular distortion* or *relative rotation*, $\beta$, is the rotation of the line joining two points relative to the tilt; and
- *Average horizontal strain*, $\epsilon_h$, is the change in length $\delta_L$ over the length $L$.

A system of classifying building damage for masonry structures was first proposed by Burland et al. (1977) and is summarised in table 2.1.

Damage to structures usually occurs initially as visible cracking caused by tensile strains induced in the building. Polshin and Tokar (1957) introduced this concept and suggested a critical tensile strain, $\epsilon_{\text{crit}}$, which when reached would result in cracking. The critical value they suggested was 0.05%. Burland and Wroth (1974) further investigated tensile strain and suggested that $\epsilon_{\text{crit}}$ for the onset of cracking was in the range of 0.05-0.1% for masonry structures and 0.03-0.05% for reinforced concrete beams. They also noted that the onset
Table 2.1: Classification of building damage (after Burland et al., 1977)

<table>
<thead>
<tr>
<th>Damage Category</th>
<th>Degree of severity</th>
<th>Description of typical damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Negligible</td>
<td>Hairline cracks less than about 0.1mm wide.</td>
</tr>
<tr>
<td>1</td>
<td>Very slight</td>
<td>Fine cracks easily treated during normal decoration. Crack width up to 1mm.</td>
</tr>
<tr>
<td>2</td>
<td>Slight</td>
<td>Cracks are easily filled. Redecoration probably required. Crack width up to 5mm.</td>
</tr>
<tr>
<td>3</td>
<td>Moderate</td>
<td>Cracks can be patched by a mason. Repointing and possibly replacement of some brickwork. Crack width from 5-15mm.</td>
</tr>
<tr>
<td>4</td>
<td>Severe</td>
<td>Extensive repair work involving replacement. Crack widths from 15-25mm.</td>
</tr>
<tr>
<td>5</td>
<td>Very severe</td>
<td>Major repairs required including partial or complete re-building. Crack width typically greater than 25mm.</td>
</tr>
</tbody>
</table>
Table 2.2: Damage categories (after Boscardin and Cording, 1989)

<table>
<thead>
<tr>
<th>Category of damage</th>
<th>Normal degree of severity</th>
<th>Limiting tensile strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Negligible</td>
<td>0.000 - 0.050</td>
</tr>
<tr>
<td>1</td>
<td>Very slight</td>
<td>0.050 - 0.075</td>
</tr>
<tr>
<td>2</td>
<td>Slight</td>
<td>0.075 - 0.150</td>
</tr>
<tr>
<td>3</td>
<td>Moderate</td>
<td>0.150 - 0.300</td>
</tr>
<tr>
<td>4 to 5</td>
<td>Severe to very severe</td>
<td>&gt;0.300</td>
</tr>
</tbody>
</table>

of cracking does not necessarily compromise the serviceability of the structure. Burland et al. (1977) replaced the concept of critical tensile strain with that of limiting tensile strain \( \epsilon_{lim} \) used as a serviceability parameter.

Boscardin and Cording (1989) further developed the use of limiting tensile strain as a damage criterion by examining case studies and linking the damage category directly with \( \epsilon_{lim} \) as shown in table 2.2.

2.5.2 Calculation of building strains

An analytical method of calculating the tensile strains caused by ground settlement that can then be used to predict damage is given in the key paper by Burland and Wroth (1974). The building is treated as a weightless, uniform, deep elastic beam of length, \( L \), and height, \( H \), with unit thickness. Relationships are derived to determine the deflection ratio \( \Delta/L \) in hogging and sagging at which cracking is initiated, from the calculated tensile strain in the building. Tensile strains can occur either due to bending, with vertical cracks due to direct tensile strain, or shear, with diagonal cracks due to diagonal tensile strain. In most cases both modes of deformation will occur at the same time. The deflection of a simply supported deep beam under a central point load flexing in both bending and shear is given by Timoshenko (1957) as,

\[
\Delta = \frac{PL^3}{48EI} \left[ 1 + \frac{18EI}{L^2HG} \right] \tag{2.17}
\]

where \( E \) is Young’s Modulus, \( G \) is the shear modulus, \( I \) is the second moment of area, \( H \) is the height of the beam and \( P \) is the central point load. The use of the point load
equation is justified by Burland and Wroth (1974) on the basis that other load cases give a similar result. Equation 2.17 can be rewritten in terms of the maximum bending strain in the extreme fibre $\epsilon_{b_{max}}$ and the deflection ratio $\Delta/L$,

$$\frac{\Delta}{L} = \left[ \frac{L}{12t} + \frac{3EI}{2tLHG} \right] \epsilon_{b_{max}} \quad (2.18)$$

where $t$ is the distance between the neutral axis and the edge of the beam in tension. For beams in sagging it is assumed that the neutral axis is in the middle of the beam as the ground foundation provides no restraint. For beams in hogging, however it is assumed that the foundation provides restraint and that the neutral axis lies along the base. The maximum diagonal strain $\epsilon_{d_{max}}$ can be written in the same way,

$$\frac{\Delta}{L} = \left[ 1 + \frac{HL^2G}{18EI} \right] \epsilon_{d_{max}} \quad (2.19)$$

Using these equations the maximum tensile strain can be calculated from a given deflection ratio by setting $\epsilon_{\text{max}}$ equal to $\epsilon_{\text{lim}}$.

Boscardin and Cording (1989) extended the above work by considering lateral ground movements induced by tunnelling rather than just vertical settlement. From case studies, they found that the component of horizontal strain was significant and previously unaccounted for. They included the horizontal strain by assuming that under the influence of lateral ground movements, the beam uniformly extended over its full depth. In the bending region the limiting tensile strain thus becomes,

$$\epsilon_{br} = \epsilon_{b_{max}} + \epsilon_h \quad (2.20)$$

In the diagonal strain due to shearing region, the horizontal strain can be combined using Mohr’s circle of strain giving,

$$\epsilon_{dr} = \epsilon \left( \frac{1 - v}{2} \right) + \sqrt{\epsilon_h^2 \left( \frac{1 - v}{2} \right)^2 + \epsilon_{d_{max}}^2} \quad (2.21)$$
where \( v \) is Poisson’s ratio. The maximum tensile strain is then the greater of \( \epsilon_{br} \) and \( \epsilon_{dr} \). This maximum tensile strain can be used in table 2.2 to predict the damage category. Boscardin and Cording also developed the chart shown in figure 2.7 for predicting potential damage by relating the horizontal strain to the angular distortion \( \beta \).

Burland (1997) proposed the use of a similar interaction chart by adapting the values of \( \epsilon_{lim} \) associated with the various damage categories in table 2.2 and using deflection ratio rather than angular distortion. Such a diagram for \( L/H=1 \) is shown in figure 2.8.

---

**Figure 2.7:** Relationship of damage to angular distortion and horizontal strain (after Boscardin and Cording, 1989)

---

**Figure 2.8:** Relationship of damage category to deflection ratio and horizontal strain for \( L/H=1 \) (after Burland, 1997)
Son and Cording (2005) propose an updated generalised damage criterion. This is similar to the Boscarding and Cording (1989) approach but is not dependent on $L/H$ or $E/G$ ratios. It is based on the strain at a point or the average strain across a building and uses the relationship between angular distortion and lateral strain. Updated damage categories, based on building damage observations are proposed as shown in table 2.3.

### Table 2.3: Damage categories (after Son and Cording, 2005)

<table>
<thead>
<tr>
<th>Damage level</th>
<th>Critical tensile strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negligible</td>
<td>0.000 - 0.050</td>
</tr>
<tr>
<td>Very slight</td>
<td>0.050 - 0.075</td>
</tr>
<tr>
<td>Slight</td>
<td>0.075 - 0.167</td>
</tr>
<tr>
<td>Moderate</td>
<td>0.167 - 0.333</td>
</tr>
<tr>
<td>Severe to very severe</td>
<td>$&gt;0.333$</td>
</tr>
</tbody>
</table>

#### 2.5.3 Influence of surface structure on settlement profile

The conventional building damage prediction methods described above are based on the assumption that the building has no stiffness or weight and deforms according to the greenfield settlement profile. In reality, buildings have been shown to influence the shape and magnitude of the greenfield settlement trough. This section contains a description of the issues associated with the influence of surface structures on settlements while section 2.5.4 contains a discussion of numerical modelling approaches.

A study of movements predicted at the Mansion House in London due the construction of the Docklands Light Railway (DLR) is presented by Frischmann et al. (1994). The Mansion House is a five-storey structure constructed of load bearing masonry walls and suspended timber floors. As part of the DLR extension, three separate tunnel sections were planned under the building in ground comprising alluvium overlying gravel and London clay. The results of monitoring the first constructed tunnel indicated the ground loss to be as predicted by empirical methods, but the shape of the settlement profile to be very different. Figure 2.9 displays the predicted greenfield and measured settlements showing clearly the building influence. A two-dimensional finite element analysis was undertaken.
to establish the influence of the stiffness of the building with the settlements predicted by the model agreeing well with the measured settlements after optimisation. Discussion of the methods used in the finite element analysis is presented in section 2.5.4.

![Settlements at the Mansion House](image)

Figure 2.9: **Settlements at the Mansion House (after Frischmann et al., 1994)**

Potts and Addenbrooke (1997) carried out a parametric study of the effect of building stiffness on settlement profiles using 2D finite element methods. The building was represented as a beam with bending stiffness $EI$ and axial stiffness $EA$ with the soil modelled as non-linear elastic perfectly plastic. A discussion of the finite element analysis is given in section 2.5.4. The geometry considered in the analysis included the building half width $H$, its eccentricity with respect to the tunnel centre line, $e$, and the tunnel depth. The tunnel diameter was fixed. Two relative soil-structure stiffness parameters were defined, the relative axial stiffness $\alpha^*$ and the relative bending stiffness $\rho^*$ given as,

\[
\rho^* = \frac{EI}{E_s H^4} \tag{2.22}
\]

\[
\alpha^* = \frac{EA}{E_s H} \tag{2.23}
\]

where $E_s$ is the soil stiffness. A comprehensive range of finite element analyses was performed. The effect of relative bending stiffness as measured by $\rho^*$ is shown in figure 2.10. For each analysis, a settlement profile and a horizontal displacement profile were generated.
and the building distortion parameters of deflection ratio in sagging and hogging ($DR_{sag}$ and $DR_{hog}$ respectively) and horizontal strain in tension and compression ($\epsilon_{ht}$ and $\epsilon_{hc}$) interpreted from the results. For each building size and location the greenfield ground movements were used to obtain the greenfield values of deflection ratio and horizontal strain. The greenfield values were then compared to the values including the building, and modification factors derived to relate the two. The modification factors are defined as

$$M^{DR_{sag}} = \frac{DR_{sag}}{DR_{sag}^g}, \quad M^{DR_{hog}} = \frac{DR_{hog}}{DR_{hog}^g}$$

(2.24)

$$M^{\epsilon_{ht}} = \frac{\epsilon_{ht}}{\epsilon_{ht}^g}, \quad M^{\epsilon_{hc}} = \frac{\epsilon_{hc}}{\epsilon_{hc}^g}$$

(2.25)

where $DR_{sag}^g$ and $DR_{hog}^g$ are the deflection ratios for the greenfield settlement trough beneath the building and $\epsilon_{ht}^g$ and $\epsilon_{hc}^g$ are the maximum horizontal tensile and compressive strains of a greenfield trough beneath the building.

The modification factors were then plotted against $\rho^*$ and $\alpha^*$ respectively for each $e/B$, where $B$ is the building width. Empirical design curves were fitted through the data and
charts presented. Modification factors read from the charts can be used by designers to modify the empirically obtained greenfield parameters of horizontal strain and deflection ratio to account for the relative building stiffness before imposing these on the structure and assessing any potential damage. The damage assessment method proposed is the use of the chart similar to that given as figure 2.8 in this thesis.

An evaluation of the relative stiffness approach using centrifuge modelling was undertaken by Taylor and Grant (1998). A rubber pad placed on the surface of the soil model was used to simulate a building and surface and sub-surface ground movements were observed. Results indicated that the relative stiffness of the building (although quite flexible) influenced the settlement profile by reducing the curvature. Modification factors for deflection ratios estimated from the observed settlements agreed reasonably well with those suggested by Potts and Addenbrooke (1997).

The relative stiffness method proposed by Potts and Addenbrooke is a significant improvement on the empirical methods without the building, but in its original form does not include 3D effects or the effect of vertical loads imposed by the building. Three-dimensional effects not considered by the 2D analyses include the transitory effect of the longitudinal settlement trough or geometrical considerations when a tunnel is constructed obliquely under a building. Consideration of the impact of including the building weight and 3D effects on the relative stiffness method are presented by Franzius et al. (2004) based on the work by Franzius (2004). Design charts and amended definitions of the modification factors based on parametric studies are presented, however it is concluded that the impact of these additional factors on the original Potts and Addenbrooke (1997) method are minimal and that, as the original method is conservative, it can be used with confidence. The 3D studies, however, only consider a building represented by an elastic slab lying symmetrically above a tunnel, not at an oblique angle to the route alignment.

Son and Cording (2005) investigate the influence of relative shear stiffness of masonry facades in relation to soil stiffness and recommend that this be considered when using strain damage criteria for predicting building damage. The relative stiffness (RS) relationship is
given between the building shear stiffness and the soil as

\[ RS = \frac{E_sL^2}{G_{\text{build}}Hb} \]  

where \( E_s \) is the soil stiffness, \( L \) the length of the building, \( H \) the building height, \( G_{\text{build}} \) the elastic shear stiffness of the building and \( b \) the wall thickness. Relationships between this relative stiffness and the ratio of angular distortion of a building (\( \beta \)) to the change in ground slope of a greenfield profile (\( \Delta GS \)) are given for a range of numerical and model tests. Using these charts a modified (normalised) angular distortion is determined and used in conjunction with the tensile strain to determine the modified building damage parameter. Interaction with the Potts and Addenbrooke (1997) method using relative axial and bending stiffnesses is not explored nor is the method referenced.

The influence of a building on surface settlements in 3D has also been under investigation at Oxford University using finite element methods. Work presented in theses by Lui (1997), Augarde (1997), Bloodworth (2002) and Wisser (2002) detailing the development and use of 3D finite element models of masonry structures and tunnels is discussed in section 2.5.4. The results of their analyses show the effect of a masonry structure on surface settlements and include the effects of building weight and tunnelling obliquely beneath a building. A plot of surface settlements is given in figure 2.11 comparing a greenfield profile (a) and the profile including a building after tunnel construction underneath (b). This research confirms the importance of the influence of soil-structure interaction including both the building stiffness and weight and that this problem is an inherently three-dimensional one.

The prediction of damage to buildings due to tunnel induced ground movements in 2D is generally based on the methods of Burland and Wroth (1974) and Boscardin and Cord- ing (1989). The influence of the building stiffness in two dimensions can be included by using the relative stiffness method of Potts and Addenbrooke (1997). For assessments incorporating full 3D soil-structure interaction effects though, conventional methods cannot handle the problem (Potts, 2003) and numerical methods must be used.
2.5.4 Numerical methods

Numerical methods are now being used increasingly frequently for the prediction of damage to buildings due to tunnelling. Inadequate computing hardware and software resources have previously been a significant barrier to the use of numerical methods for modelling tunnel-induced building damage however, as the number of recent papers discussed below shows, the use, particularly of finite element methods, is now relatively common.

Two-dimensional modelling

Frischmann et al. (1994) present results of the prediction of settlements and damage of the Mansion House in London due to tunnelling using finite element methods. A description of the project and the results of the modelling and the field measurement of displacements is given in section 2.5.3 of this thesis. The soil was modelled using a linear elastic model, justified on the grounds that the area of interest was outside the zone of possible non-linear behaviour. This is a simplification as the non-linear effects of the typical response of London clay under small strains have been shown to be significant as discussed in section 2.3.2. The ground loss of 2.0-2.5% was modelled by applying radial displacements to the excavated tunnel. The masonry facade was modelled as simple beam elements with uniform elastic modulus of 1GPa and Poisson’s ratio of 0.2. It is interesting that the predictions only gave a reasonable fit to the field data when the vertical support of orthogonal walls was
modelled using vertical springs at the ends of the wall under investigation and the stiffness of the springs optimised. This indicates that the 3D geometry of the building needed to be taken into account. Reduced stiffness expected in the hogging mode of deformation of walls due to existing cracks was considered. Settlements predicted using this method were then applied to detailed models of the walls to predict crack damage.

The work of Potts and Addenbrooke (1997) is discussed in section 2.5.3 of this thesis in relation to the influence of building stiffness on settlement profiles due to tunnelling. Further aspects of the finite element models used in their parametric study are discussed here. Modelling was undertaken assuming plane strain conditions with soil modelled as a non-linear elastic-plastic material. Non-linear pre-yield behaviour was modelled and a Mohr-Coulomb yield surface and plastic potential used to model the plastic behaviour. The soil was given properties representative of London Clay. A linear elastic surface beam was used to represent the building and its interface with the soil was assumed to be rough. Stiffness properties for the beam were a variable in the analyses and independent bending and axial stiffnesses were prescribed. In addition, analyses were undertaken where the beam was given the stiffness of a one, three or five story structure with concrete slab floors each with a value of $E_c = 23 \times 10^6 \text{kN/m}^2$, area $A = 0.15 \text{m}^2$/m and $I_{slab} = 2.8125 \times 10^{-4} \text{m}^4$/m. The analysis did not include the building weight or any three-dimensional effects.

Calabresi et al. (1999) use a plane strain finite element model to prediction damage to the Castel S. Angelo in Rome due to tunnelling. Ground conditions consisted of silty alluvial sands overlying stiff clay at a depth of 30m. The tunnel depth ranged between 23 and 26m and was 11.8m in diameter. Soil was modelled using an elastic-plastic model with pre-failure non-linearity implemented as the Hard Soil model in the software PLAXIS. The structure is a massive masonry cylinder 64m in diameter and 35m high, founded on a 6m thick slab. The justification for the use of a plane strain idealisation of the structures was that its out of plane dimension is large in comparison to the tunnel diameter. This method is not appropriate for modelling a building over a long tunnel in a soil-structure interaction problem. The foundation slab and masonry cylinder were modelled as linearly elastic-perfectly plastic materials with compressive strength of 1.6MPa and modulus of 1.5GPa.
for the slab and a reduced modulus to account for internal voids in the masonry cylinder. Volume losses ranging from 0.5-1.5% were prescribed by imposing radial contractions on the tunnel outline. Finite element results were compared to empirical predictions including the building stiffness modification factors of Potts and Addenbrooke (1997). The finite element analysis gave good agreement in terms of damage prediction parameters. No case study data was presented to verify the predictions.

Throughout the literature, both for analytical studies and for the plane strain analyses discussed above, beams representing buildings are generally given simple linear isotropic elastic properties based on the material of the wall or building. The methods used to derive properties for the beam are empirical and are not consistently undertaken. Recognition that beams may need to be more complicated than simply linear elastic isotropic is found initially in Burland and Wroth (1974) who consider that the properties of beams representing facades might be affected by the physical geometry of the facade. They note that the ratio $E/G$ may vary from isotropic elastic theory ($E/G=2(1+\nu)$) due to the number of openings affecting the relative bending and shear stiffness. Examples are given for the relationship between $L/H$ ratio and critical tensile strain for values of $E/G$ of 0.5 (very stiff in shear), 2.6 (isotropic) and 12.5 (very flexible) which exhibit differing responses. No indication is given as to the appropriate choice of a value of $E/G$ to use for any particular facade geometry or independent consideration of $E$ or $G$ values.

In addition to these elastic considerations, cracking is generally dealt with by reducing the modulus of the representative beam by arbitrary amounts or the introduction of measures such as hinges where cracks were observed to develop. No rigorous analyses of the effects on beam properties of such factors as material, cracks, windows or the building dimensions, especially for short buildings is used to assign properties to the simple equivalent beams.

An evaluation of damage to masonry buildings using 2D numerical analysis, including the modelling of a non-linear masonry wall (rather than using simplified elastic beams) was undertaken by Miliziano et al. (2002). For a tunnel driven symmetrically under a series of regularly spaced (in the longitudinal direction) orthogonal masonry walls, each structure was represented by an equivalent masonry wall made up of a number of discrete linear
elastic elements (representing the masonry bricks) separated by horizontal and vertical interfaces of elastic-perfectly plastic material with a tensile limit strength (representing the mortar). An elasto-plastic Mohr Coulomb model was used for the soil with Young’s modulus increasing linearly with depth. The main conclusion is to confirm the importance of the inclusion of the surface structures in tunnelling-induced damage prediction as neglecting the soil-structure interaction lead to an overprediction of damage. Further analyses of masonry walls are presented by Boonpichetvong and Rots (2004) who present 2D tunnelling analyses including a full masonry facade modelled using a fracture mechanics approach. Despite the soil being modelled as simply linear elastic, the impact of soil-structure interaction is still clear.

**Three-dimensional modelling**

The ground movements around the advancing heading of a tunnel develop as a 3D process. The geometric considerations when tunnels pass obliquely under a building, the transitory nature of soil movements during the passage of the tunnel and the thickness of the building walls are also 3D effects. Finite element analysis in 3D is therefore needed to capture fully the appropriate geometry and thus predict damage to structures more realistically. Barriers to the wide use of three-dimensional finite element modelling include the modelling of tunnel installation procedure including the tunnel lining, the choice of a suitable constitutive model for the soil and the computer hardware required to run large complex models. These barriers are progressively being tackled due to research into 3D modelling and the use of improved commercial software capable of modelling in 3D.

Research at Oxford University has been described in section 2.3.2 in relation to the modelling of greenfield sites. The focus of the work has been the development of a three-dimensional model that provides realistic simulation of the interaction between a structure and the ground during tunnelling. The work by Chow (1994) and Houlsby (1999) on the development of constitutive soil models and Augarde (1997) on three-dimensional modelling of tunnel excavation and tunnel lining has already been described. Work by Lui (1997) as part of the same project involved the development of a numerical model for a
masonry building and a scheme to connect the building to the ground surface. The masonry structure was modelled as a series of facades constructed from 2D plane stress elements connected using specially developed tie elements. The material model for the masonry was an elastic no tension model with a small residual tensile strength. Elastic lintels were introduced above door and window openings in the facade.

Finite element analyses including the building were undertaken as described by Lui (1997), Augarde (1997) and presented by Burd et al. (2000). The building was shown to influence the greenfield settlements as discussed in section 2.5.3. The damage predicted on the facades was also significantly different when the building was included in the analysis with the tunnel, compared to the case when the greenfield settlements were applied to the building. The result of the research to date is the development of a detailed 3D model. Verification of the model by comparison with case studies was reported by Bloodworth and Houlsby (1999) and Bloodworth (2002). Wisser (2002) described the extension of the model to include the ability to model compensation grouting. There exist opportunities for further verification using new case study information described in section 2.6.

Computer resources used in the simulation of tunnelling using the model developed at Oxford are significant. The non-linear soil model combined with the no tension masonry model and the large number of solution steps required to fully capture the details of the interaction lead to large memory requirements and long run times. Bloodworth et al. (1999) note that for a typical large analysis, run times were of the order of two weeks on the fastest serial platform available to the research group at the time, a Sun Ultra 2 having 300MHz and 512Mb of RAM. The use of the (now superseded) Oxford Supercomputing Centre machine, OSCAR, significantly reduced run times from around 20 minutes per calculation step to four minutes. This time is still a significant barrier, however, to the widespread use of such complex models, as design consultants are unlikely to have access to high powered supercomputers and will want analyses that can be completed in matters of hours. Possibilities for reducing run times include the development of more efficient solvers, more advanced hardware (such as the newer, more powerful supercomputers) or more efficient methods of numerical modelling.
Examples of 3D finite element modelling used on commercial projects include the Netzel and Kaalberg (2000) description of damage assessment of masonry structures in Amsterdam for the construction of the North/South Metroline. Construction in soft sandy soil, generally 21 to 32m below ground level, of 6.5m twin tunnels was assessed for potential damage to the adjacent masonry structures. A long building comprising ten 16m high, 18m long and 7m wide connected apartments was modelled. The foundation of the building consists of 14m long timber end bearing piles. A 3D finite element analysis was undertaken using a no-tension model for the masonry using plane stress elements with openings for windows and doors. Piles were modelled using truss elements and non-linear springs represented the toe resistance and skin friction. In this study, however, the tunnel was not modelled. Theoretical greenfield subsurface settlements at pile toe level were imposed on the piles sequentially at depth to model the passage of the tunnel. By imposing greenfield settlements rather than modelling the tunnel construction, the building-soil interaction was ignored. Nevertheless, the results of the study are interesting as they indicate that the worst damage was due to the transient effects of the longitudinal settlements. No field data is given as a comparison with the numerical model.

An investigation of the influence on tunnel induced settlements of a multistorey framed structure is described by Dias and Kastner (2002). Tunnel construction on Line 2 of the Cairo Metro through soft alluvial soils 25m under a six-storey structure is simulated. The soil was modelled as elastic-perfectly plastic material and the building frame as elastic beams representing columns supporting floors comprised of shell elements. The building was found to influence the transverse settlement profile by acting in a rigid manner causing the profile to become a straight slope and maximum settlement to increase. Moments and shear forces in the building members were found to be affected by the transient passage of the tunnel, but no damage assessments were undertaken, nor were the results compared to field measurements. Further work on the interaction of framed concrete structures in tunnelling situations is presented by Mroueh and Shahrour (2002 and 2003) who simulated tunnel construction in an elastic-perfectly plastic soil based on the Mohr-Coulomb criterion under a linear elastic framed building. Two analyses were performed, one including the
building self weight and one with a weightless building. For both cases a fully coupled analysis with the building included produced less relative displacements at foundation level and lower induced forces in the building frame than an uncoupled analysis with the greenfield displacements simply applied to the building. The authors also concluded that to ignore the building weight in the assessment leads to underestimation of the tunnelling induced forces in the building. A similar analysis of tunnelling under a full model of a concrete frame building is presented by Jenck and Dias (2004). The soil was modelled as elastic-perfectly plastic and the building frame as elastic concrete beams representing columns supporting floors comprised of shell elements. The influence of the building weight and stiffness are shown, with the soil settlement profile and the member forces in the frame building changing depending on the stiffness and weight of the building.

All of the analyses described above include a full finite element model of the building. This uses significant computer hardware and means that run times are quite long. Modelling of the full building in three dimensions is often thought to be inappropriate for design in practice as design programmes can be too long if they include such analyses as noted by Fricker and Alder (2001). The author is unaware of any current research into the simplified modelling of buildings in three dimensions using surface beams that aims to make analysis easier and faster. In 2D, building facades and load bearing walls are represented by surface beams with properties derived in a number of empirical ways as described in this review. The methods discussed for the formulation of such simplified beams are not consistent, however, and differ greatly between models. There is no equivalent of the building as a beam in two dimensions in use in 3D analysis. The development of such a simplified model would facilitate analysis without the need to model the complete structure to determine its influence on ground settlements. This is one of the primary aims of this project and will be discussed in Chapter 3.

2.5.5 Current damage assessment methodology

Currently the assessment of structural damage is undertaken in a three-stage process. A preliminary Stage 1 assessment is carried out based on predicted greenfield settlements
(ignoring any soil-structure interaction). Buildings at locations where maximum settlement is greater than 10mm and slope greater than 1 in 500 are deemed to be at risk and are thus subjected to a second stage assessment. In Stage 2, hand methods are used to predict the limiting tensile strains in the structure and classify the risk of damage according to the categories of Boscardin and Cording (1989). In this stage current practice accounts for the contribution of the structure’s stiffness in 2D using the relative stiffness method of Potts and Addenbrooke (1997). These stage one and two methods are known to be conservative and are widely used. For buildings classified to be at moderate risk (or worse) of damage, a third stage of assessment is carried out. It is only in Stage 3 that engineers might utilise finite element models to fully assess individual structures in 3D.

2.6 Case studies of building damage prediction

Many of the papers discussed above in relation to numerical modelling contain case study data which is compared to the numerical predictions. In general, the case studies in the literature all provide evidence that buildings modify the greenfield settlement profile. A large proportion of these are commercial projects where the case study data were not intended to be used for further research, merely presented as a description of the project. Case study data that were obtained specifically for the purpose of research is generally of higher quality and more useful.

Recently published data from the collaborative research project undertaken by Imperial College and sponsored by London Underground Limited, the Department of the Environment Transport and the Regions (DETR) and the Engineering and Physical Sciences Research Council (EPSRC) and other industry partners contained in Burland et al. (2001) includes numerous case studies where data were collected during the construction of the Jubilee Line Extension in London. The volumes include case studies of 21 buildings, 14 of which are load bearing masonry with five reinforced concrete and two steel and masonry framed structures. The majority are on pad or strip footings, with five on piles and five on rafts. The ground conditions existing at the sites are mostly London Clay or the Lambeth
Group. All of the structures lie above the twin tunnels of the Jubilee Line extension.

A large number of the structures were protected from damage by compensation grouting. This process, while interesting in itself, has been the subject of other research at Oxford University (Wisser, 2002) so case studies where this technique is employed are not considered here. The most appropriate case studies for the research on soil-structure interaction described in this thesis are those where no protective measures have been implemented. These include the site at the Murdoch, Clegg and Neptune House complex at Moodkee Street, Rotherhithe. The buildings on the site are load bearing masonry. The full case studies are presented by Withers (2001b) but a brief description of the case is given below.

Murdoch, Clegg and Neptune Houses in Rotherhithe are three storey masonry buildings containing flats founded on strip footings (Withers, 2001b). The tunnels underneath were driven by two EPB TBM s in the lower sandy strata of the Lambeth Group. Monitoring took place during early 1996 prior to and during construction and continued until August 2000 and included precise levelling of points on the exterior walls of all buildings and horizontal measurements with tape extensometer along two external walls. The predicted and measured settlement response of the buildings were compared (Withers, 2001b) and some unexpected results relating to the stiffness effect of the buildings are evident. All three buildings were assumed to behave in a rigid manner during the settlement predictions. From the measured data, only Clegg House behaved as rigidly as expected, whereas the other two buildings responded more flexibly than predicted and displayed settlement profiles closer to the greenfield profile, particularly Murdoch House. Differences were attributed to assumptions about volume loss and trough parameter.

A significant pointer to the expected uses of this data is given by the editors, stating that they have “opened up access to a ‘gold mine’ of data. Now that gold has to be mined. This further sifting, assimilation, processing, analysis and interpretation of data is a vital...activity”.
Chapter 3

Aims and Outline

3.1 Conclusions from literature review and research opportunities

This chapter summarises current approaches to analysing building response to tunnelling from the literature, identifying opportunities for current research. The aims and outline of the project are then described.

The assessment of ground movements and building damage due to soft ground tunnelling has historically been undertaken in practice using empirical methods. These methods, as described in section 2.2, did not include the building stiffness, weight or three-dimensional effects. The building stiffness can be included in two-dimensions by using the relative stiffness method of Potts and Addenbrooke (1997). This method however, does not include the building weight and only considers two dimensions. The recent extensions to the model described by Franzius (2004) and Franzius et al. (2004) allow for the building weight and consider a three-dimensional situation with a tunnel passing symmetrically under a building. The need to include full three-dimensional effects is apparent not only due to the potential for damage caused by the transient longitudinal settlement trough but also the possibility of asymmetric and eccentric geometric situations and the twisting of buildings. Equally important, as noted by Franzius (2004), is the ability to include a model for the
building capable of modelling the behaviour of masonry (not just an elastic model) and to utilise further comparisons with the Jubilee Line Extension (JLE) case studies in the development of fully 3D capable methods.

This echoes the conclusions presented by Burland in summarising the lessons learned from the research on the JLE project (Burland et al., 2002). These lessons included that load bearing walls in hogging can be significantly more flexible than those in sagging and that this behaviour should be considered in any relative stiffness analysis; that transitory movements during the passage of a tunnel can be more severe than the final movements after the tunnel has passed (as evidenced by the measurements made at the Treasury building and at Keeton’s Estate where transitory deflection ratios were more severe than final values); that the relative stiffness approach of Potts and Addenbrooke (1997) was used to good effect in improving the previous methods of building damage assessment; and finally that, despite these improvements, damage to some buildings was less than predicted by the assessment methods used which may have resulted in too much protective compensation grouting work being undertaken. It was noted that methods for including buildings in settlement predictions “urgently need extending to three dimensions” (Burland et al., 2002). To include three-dimensional effects fully in predictions relating to complex soil-structure interactions, numerical models must be used as conventional methods cannot handle this problem as noted by Potts (2003).

The body of numerical analysis discussed in sections 2.3.2 and 2.5.4 includes analyses in which the modelling of tunnels and buildings is undertaken in two dimensions. In the majority of these analyses the building is represented by a simple linear elastic beam (for example Frischmann et al. (1994) and Potts and Addenbrooke (1997)). In general, no rigorous analysis of the methods for determining properties for such beams has been undertaken to ensure that the beam responds to ground movements in the same way that the building does. Franzius (2004) also notes the importance of the need to develop beam models capable of including masonry behaviour and accounting for twist in 3D.

Numerical models developed at Oxford University include three-dimensional models with a non-linear soil model, masonry structure, staged tunnel excavation and lining installation.
These models capture 3D ground effects associated with tunnelling, the impact of the building on ground movements and damage caused to a masonry structure. These 3D models, however, are quite complex, and their use too time consuming to be applied routinely on all design projects or for parametric investigations. Another result of this complexity is the fact that only a single building is usually modelled. In urban areas the ability to model multiple buildings simultaneously to model their combined interaction with tunnel induced ground movements would be beneficial. This could involve the use of multiple buildings comprised of simplified surface beams or the surrounding of a full model of a building of interest with simplified beam models of adjacent buildings.

The conclusions above drawn from the review of literature indicate opportunities for current research. The aims and outline of the project are given below.

### 3.2 Project aims

The aim of this project is the development and testing of a new three-dimensional approach to the numerical modelling of masonry buildings using surface beams. It involves extending the surface beam method of modelling masonry buildings to three dimensions; the development of a procedure for determining properties for surface beams representing masonry buildings; the inclusion in the new surface beam models of the ability to capture the different behaviour of masonry in different modes of deformation; and the use of the new beam models in 3D numerical models of building response to tunnelling.

Attention is focused here on the opportunity to simplify the modelling of the building, rather than the soil or tunnel. It has been discussed above how complex features of numerical simulation such as the sequential excavation of the tunnels, soil model that accounts for small strain non-linearity, the inclusion of tunnel lining and surface structures are required to ensure that the simulation predicts realistic surface settlements. It is thought, however, that the full building structure is not required to be modelled. For the purpose of assessing the building response in three dimensions, the objective is to develop a surface beam model of a building that responds to ground movements as the full building does. This could be
considered as the opposite of the approach taken by structural engineers when modelling full buildings but only including a simplified model for the ground.

The development of new numerical simulations of tunnelling necessitates the verification of such models against case study data. There is currently available recent case study data specifically related to building response to tunnelling from the JLE project which has to date not been extensively utilised. The opportunity therefore exists to use this data for the verification of any new methods developed and comparison with current numerical models.

### 3.3 Project outline

The soil-structure interaction problem outlined above is addressed in this research project in three phases. In Phase 1 of the project, described in Chapters 4 and 5, finite element analyses of masonry buildings are undertaken and the results used to develop models for two new surface beams (an elastic and a non-linear model) with appropriate properties determined so that the beams react to ground movements like a full building facade. Timoshenko (shear capable) beams are chosen to model the facades. These beams are implemented into the OXFEM finite element program as described in Chapter 6, as 3D elements allowing the full modelling of 3D movement.

Phase 2 involves finite element modelling of full masonry structures as a series of connected surface Timoshenko beams in 3D with the beams modelled using the procedure developed in Phase 1. Tunnel construction is then simulated underneath the buildings using the numerical methods detailed in Chapter 7 and the simplified building response compared to that of a full building model. Example analyses include symmetric and oblique alignments of the building with respect to the tunnel and are described in Chapter 8.

Phase 3 involves comparison between assessments of ground movement and building response made using the new simplified beam method and case study data. The buildings used for the case studies detailed in Chapter 9 are Murdoch, Clegg and Neptune Houses under which the London Underground Jubilee Line Extension was constructed.
Chapter 4

Building Facade Analysis - Elastic

4.1 Introduction

Chapters 4 and 5 of this thesis describe a study undertaken to investigate the response of building facades to ground displacements using finite element analysis. The results of the study are used to develop a new constitutive model for surface beams for the modelling of building facades, such that the beam response closely approximates the behaviour of full facades in soil-structure interaction situations.

As discussed in Chapters 2 and 3, it is common practice in soil-structure interaction problems studied using 2D analysis, to approximate a building as a beam on the ground surface. Current methods of assigning properties to surface beams to model masonry structures are generally not based on any rigorous analysis of the response of buildings to ground movements. Usually, simple linear elastic isotropic properties are assigned to the beam. Burland and Wroth (1974), however, noted that openings in a facade may impact its response to settlements, but provided no rigorous design guidance.

The work described here involves an investigation into the appropriateness of assigning simple elastic properties; finds that for certain building facades current methods could be improved; and goes on to establish a new method for modelling a two-dimensional masonry facade as a surface beam for use in three-dimensional finite element analyses.
Factors which influence the interaction between a facade and ground movements considered in this study include: the facade material, overall dimensions, amount and layout of openings, and whether the ground movements cause hogging or sagging of the facade. The influence of these factors are determined by the modelling of both linear elastic and masonry facades which have a range of dimensions and differing amounts of openings. A typical facade with examples of ground displacement is shown in figure 4.1.

A description of the finite element method is not given in this thesis as it is reported extensively in the literature. References on finite element analysis methods used by the author throughout this project include Astley (1992) and Cook et al. (1989).

The study described in Chapters 4 and 5 comprises two series of different finite element
analyses; one series for facades with linear elastic material properties (described in Chapter 4) and one for facades made from a non-linear no-tension ‘masonry’ material (described in Chapter 5). Table 4.1 gives an overview of the finite element analyses performed for this study and the overall range of key parameters considered; a detailed description of all analyses is included in sections 4.2 and 5.3. In each of Chapters 4 and 5, a description of each series of analyses is followed by a critical evaluation of the results and a comparison with current methods of modelling these facades as beams. A new procedure for determining elastic properties to attribute to beams called the equivalent elastic beam method is developed based on the results of the analyses with the linear elastic facades in Chapter 4. A new non-linear equivalent masonry beam model is developed based on the masonry facade analyses described in Chapter 5.

4.2 Finite element analysis of elastic facades

4.2.1 Introduction

This section describes finite element analyses of building facades where a linear elastic facade material is assumed. An outline of the analyses performed is followed by presentation and discussion of the results.

4.2.2 Software

All finite element analyses in this project were undertaken using the Oxford University Civil Engineering Research Group in-house finite element software OXFEM. This software is written in FORTRAN 90 and was originally developed by Burd (1986) and Houlsby but has since been supplemented by numerous research students including Augarde (1997), Liu (1997), Bloodworth (2002) and Wisser (2002). A good description of its origin and development is given by Bloodworth (2002). A description of the numerical techniques used in the solution of finite element problems in OXFEM is given in section 7.5. Mesh generation and post-processing were undertaken using external programs as described below.
4.2.3 Facade meshes

A description of the full range of elastic facades analysed is given in table 4.2 and the facade meshes are shown in Appendix A. The commercial software package I-DEAS, produced by the Structural Dynamics Research Corporation was used for mesh generation. The simulation application within the I-DEAS software was found to be acceptable by Liu (1997) and Augarde (1997) for generating meshes for geotechnical problems involving buildings.

Groups of facades with similar amounts of openings representing windows are grouped into families in table 4.2. Families E5 and E7 have some facades with slightly different percentages of openings due to geometrical constraints.

Once the geometry of each facade was defined, a modular form of mesh generation was used rather than using the mapped or free meshing facilities in I-DEAS as these can both lead to inconsistencies in meshes repeatedly generated for similar facades of differing sizes. This method facilitated mesh generation for the large number of analyses with different geometries and ensured consistency such that mesh differences were not a cause of error.

The modular method involved the design of a mesh block which could be repeated to form full facade meshes. Two mesh blocks were designed, one with a window and one without. A crossed triangle form of mesh with six noded plane triangle elements was chosen to maintain symmetry. Both blocks were constructed manually in I-DEAS by specifying nodal coordinates. The blocks are shown in figures 4.2 (a) and (b). The block containing the window comprises 106 nodes and 56, six noded, linear strain triangle elements, while the no-window block contains 104 nodes and 64 elements. Full facades constructed by the repetition of the blocks are shown in Appendix A, which contains all the facades analysed which included window openings. Facades without windows are listed in table 4.2 and comprise the appropriate number of plain mesh blocks. The two examples shown in figures 4.3 and 4.4 are meshes for analyses E13 (no windows) and E33 (18.75% windows).

The meshed facades were exported from I-DEAS to ABAQUS (the commercial finite element software) format and then to a format readable as input files by OXFEM by using the CONVERT FACADE program written by Augarde (1997). These files contain node
Table 4.2: Schedule of facade meshes for elastic analyses

<table>
<thead>
<tr>
<th>Facade Family</th>
<th>Facade Number</th>
<th>Length (m)</th>
<th>Height (m)</th>
<th>L/H Ratio</th>
<th>Openings (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>E11</td>
<td>60</td>
<td>8</td>
<td>7.50</td>
<td>0</td>
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<tr>
<td>E1</td>
<td>E12</td>
<td>40</td>
<td>8</td>
<td>5.00</td>
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</tr>
<tr>
<td>E1</td>
<td>E13</td>
<td>20</td>
<td>8</td>
<td>2.50</td>
<td>0</td>
</tr>
<tr>
<td>E1</td>
<td>E14</td>
<td>40</td>
<td>12</td>
<td>3.33</td>
<td>0</td>
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<tr>
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<td>E15</td>
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<td>12</td>
<td>1.67</td>
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</tr>
<tr>
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<td>E16</td>
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<td>16</td>
<td>1.25</td>
<td>0</td>
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<tr>
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<tr>
<td>E3</td>
<td>E31</td>
<td>60</td>
<td>8</td>
<td>7.50</td>
<td>18.750</td>
</tr>
<tr>
<td>E3</td>
<td>E32</td>
<td>40</td>
<td>8</td>
<td>5.00</td>
<td>18.750</td>
</tr>
<tr>
<td>E3</td>
<td>E33</td>
<td>20</td>
<td>8</td>
<td>2.50</td>
<td>18.750</td>
</tr>
<tr>
<td>E3</td>
<td>E34</td>
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<td>12</td>
<td>3.33</td>
<td>18.750</td>
</tr>
<tr>
<td>E3</td>
<td>E35</td>
<td>20</td>
<td>12</td>
<td>1.67</td>
<td>18.750</td>
</tr>
<tr>
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<td>E36</td>
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<td>18.750</td>
</tr>
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<td>E37</td>
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<td>18.750</td>
</tr>
<tr>
<td>E5</td>
<td>E51</td>
<td>60</td>
<td>8</td>
<td>7.50</td>
<td>9.375</td>
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<tr>
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<td>8</td>
<td>2.50</td>
<td>9.375</td>
</tr>
<tr>
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<td>12</td>
<td>3.33</td>
<td>10.000</td>
</tr>
<tr>
<td>E5</td>
<td>E55</td>
<td>20</td>
<td>12</td>
<td>1.67</td>
<td>10.000</td>
</tr>
<tr>
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<td>E56</td>
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<td>9.375</td>
</tr>
<tr>
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<td>E57</td>
<td>12</td>
<td>16</td>
<td>0.75</td>
<td>9.375</td>
</tr>
<tr>
<td>E7</td>
<td>E71</td>
<td>60</td>
<td>8</td>
<td>7.50</td>
<td>5.625</td>
</tr>
<tr>
<td>E7</td>
<td>E72</td>
<td>40</td>
<td>8</td>
<td>5.00</td>
<td>5.625</td>
</tr>
<tr>
<td>E7</td>
<td>E73</td>
<td>20</td>
<td>8</td>
<td>2.50</td>
<td>5.625</td>
</tr>
<tr>
<td>E7</td>
<td>E74</td>
<td>40</td>
<td>12</td>
<td>3.33</td>
<td>6.250</td>
</tr>
<tr>
<td>E7</td>
<td>E75</td>
<td>20</td>
<td>12</td>
<td>1.67</td>
<td>6.250</td>
</tr>
<tr>
<td>E7</td>
<td>E76</td>
<td>20</td>
<td>16</td>
<td>1.25</td>
<td>5.625</td>
</tr>
<tr>
<td>E7</td>
<td>E77</td>
<td>12</td>
<td>16</td>
<td>0.75</td>
<td>6.250</td>
</tr>
</tbody>
</table>
Figure 4.2: Facade mesh blocks with and without window

Figure 4.3: Facade mesh E13, 20m x 8m, no windows

Figure 4.4: Facade mesh E33, 20m x 8m, 18.75% windows
and element information, to which is added information about the material, boundary conditions including any ground displacements and options for data output.

### 4.2.4 Description of elastic finite element analyses

The material properties used for the linear elastic isotropic facades were: Young’s modulus $E$, of 10,000MPa and Poisson’s ratio $v$, equal to 0.2. These properties are typical for masonry when making the assumption that the material is linear elastic (Liu, 1997).

Each of the facades described above was subjected to an imposed displacement at base level. The form of the displacements applied was chosen so that the resulting stresses at the base of the facade could be easily compared to the stresses that would theoretically occur at the base of a linear elastic beam subjected to the same displacements. To facilitate these comparisons, a parabolic load distribution at the base was chosen as the desired result (ignoring self weight) after applying displacements to the facade base as this involves parameters that are convenient for comparison. The form of the parabolic load distribution chosen is shown in figure 4.5. The expression for the parabolic load distribution shown can be written by inspection as,

$$q = \frac{w_o x (L - x)}{L^2} - w'$$  \hspace{1cm} (4.1)
where $w_o$, $w'$ and $L$ are defined in figure 4.5 and $x$ is the distance along the facade base. If the assumption is made that the area under the parabola sums to zero (i.e. the total load applied to the facade is zero), $w'$ can be found to be equal to $2w_o/3$. Utilising the result that $d^2M/dx^2 = q$ (Timoshenko, 1957) where $M$ is the bending moment we can integrate to find the expression for moment as,

$$M = \frac{w_ox^2}{3L^2} \left(2xL - x^2 - L^2\right)$$  \hspace{1cm} (4.2)

To derive an expression for the displacement to be applied to give this bending moment and thus the desired parabolic load, an appropriate beam theory must be chosen. The simplest beam theory is Bernoulli-Euler theory (as described by Timoshenko, 1957) where beam displacements are described by assuming that stresses present are those associated with bending only and shear stresses are ignored. The relationship between curvature and moment in this case is,

$$\frac{d^2y}{dx^2} = -\frac{M}{EI}$$  \hspace{1cm} (4.3)

where $y$ is the vertical displacement, $x$ is the distance along the beam, $d^2y/dx^2$ is the curvature, $E$ is the Young’s modulus and $I$ is the second moment of area.

This is acceptable for long slender beams, however Timoshenko (1957) notes that where a beam is relatively short or deep, shear effects can be significant. It is considered that shear effects will be significant in the analysis of building facades due to their potentially deep geometries and as a result, such effects should be considered in the derivation of the imposed displacements to be applied to the facades. Beam theory where the analytical solution for the deflection includes shear terms is known as deep or Timoshenko beam theory after Timoshenko (1957). The relationship between moment and curvature in this case is given by Timoshenko as,

$$\frac{d^2y}{dx^2} = -\frac{1}{EI} \left(M + \frac{EIq}{kAG}\right)$$  \hspace{1cm} (4.4)
where $E$ is the Young’s modulus, $I$ is the second moment of area, $k$ is a shape factor (used to correct for shear averaging through a cross-section; see section 6.2) equal to $2/3$ for a rectangular cross section, $A$ is the cross sectional area, $M$ is the bending moment from equation 4.2 and $q$ is the expression for distributed load from equation 4.1. Further discussion regarding Timoshenko beam theory is given in Chapter 6. Solving for displacement $y(x)$ (using the condition that there is no displacement at either end of the beam to solve for the constants of integration) gives,

$$y(x) = \frac{w_o x(x - L)}{180 E I L^2} \left( 2x^4 - 4x^3 L + x^2 L^2 + xL^3 + L^4 \right) - \frac{w_o x^2(x - L)^2}{2AGL^2} \quad (4.5)$$

Two different boundary conditions were analysed. The first, the rough case, uses the assumption that the soil will restrain the building facade at ground level leading to the lower boundary being fixed in the horizontal direction. The neutral axis for the facade is thus at the base. The rough case finds support in the literature from Burland and Wroth (1974) in their discussion of settlement related building damage. The second case, called the smooth case, assumes that no lateral restraint is provided by the soil at the base and horizontal slip can occur. For the smooth case, the neutral axis is at the mid-height of the facade. Burland et al. (1997) note that the smooth case may be more realistic due to evidence presented of case study buildings where slip occurs at ground level in soil-structure interaction situations. Both cases are analysed here for completeness.

The calculation of second moment of area, $I$, will differ under each boundary condition regime, leading to different applied displacements for smooth and rough based facades. The displacement regime for each facade comprises the values calculated at each node using equation 4.5. For the elastic analyses, the direction of the imposed displacements is chosen such that, for the smooth case, displacements cause sagging of the facade and for the rough case, applied displacements cause hogging of the facade. These boundary conditions mirror those chosen by Burland and Wroth (1974), who use these two conditions in their analytical investigations. The opposite cases need not be analysed as they will simply produce the negative results of the analysed cases.
Commonly a Gaussian displacement profile is used in studies of tunnelling induced ground movements. The sixth order displacement function described above is used instead in this section of work for two reasons. Firstly, to facilitate ease of comparison of loads arising at the facade base; a parabolic distribution there is convenient for this purpose, and secondly to allow Gaussian ground displacement profiles to be used in the testing and check analyses described in Chapter 6. By using two different displacement profiles in this way it is hoped that the beam models developed in this work will be more robust.

4.2.5 Output and post-processing

Results from OXFEM include the stress and strain results at all Gauss points and nodal values of force and displacement. The desired values for this investigation were, however, the stress values along the base of the facade. These stress values enable the response of the facade to the applied displacements in the finite element analysis to be compared to the analytical response described in equation 4.1. It was therefore necessary to derive expressions to determine the stresses at the base of the facade consistent with the nodal force output. This was done by approximating the stress distribution as a piecewise linear distribution as shown in figure 4.6. Expressions for the nodal forces due to a uniformly and linearly distributed load on an element as shown in figure 4.7 were combined to solve for the stress at each element. The general solution for a facade comprising \( N \) elements along its base was found to be,

\[
\begin{bmatrix}
\frac{5l^2}{36} & \frac{l^2}{9} & 0 & 0 & 0 & \ldots & 0 \\
\frac{l^2}{9} & \frac{l^2}{3} & \frac{l^2}{9} & 0 & 0 & \ldots & 0 \\
0 & \frac{l^2}{9} & \frac{l^2}{3} & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & \ldots & 0 & \frac{l^2}{9} & \frac{l^2}{3} & \frac{l^2}{9} \\
0 & \ldots & 0 & 0 & 0 & \frac{l^2}{9} & \frac{5l^2}{36}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\vdots \\
\sigma_N
\end{bmatrix} =
\begin{bmatrix}
\frac{Q_0l}{6} + \frac{P_1}{3} \\
(P_1 + Q_1 + P_2)l/3 \\
\vdots \\
(P_{N-1} + Q_{N-1} + P_N)l/3 \\
\frac{Q_Nl}{6} + \frac{P_N}{3}
\end{bmatrix}
\tag{4.6}
\]

where \( P \) and \( Q \) are nodal forces at the middle and end nodes of the elements respectively (shown in figure 4.6), \( l \) is the length of each element and \( \sigma_i \) is the average stress at the base.
Figure 4.6: Determination of load distribution at facade base from nodal forces

Figure 4.7: Nodal forces equivalent to uniform and triangular distributed loads of element $i$. This approach required the lengths, $l$, of all elements to be the same. This equation is solved for the stress at each element which can then be plotted and a parabolic line of best fit assumed (in the form of the distribution shown in figure 4.5).

4.2.6 Results

Finite element stiffness

The results of the finite element analyses of the facades are the stress distributions at the base of the facade. These can be characterised by the intercept value, $w'$, and the difference between the highest and lowest points on the stress curve, $w_o$ defined in figure 4.5.
CHAPTER 4. BUILDING FACADE ANALYSIS - ELASTIC

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Figure 4.8: Example stress distributions at facade bases - Effect of increasing % openings

stiffness, $K$ of the facade as $w_o$ divided by the displacement at the midpoint gives

$$K = \frac{w_o}{y(L/2)}$$  \hspace{1cm} (4.7)

where $y(L/2)$ is the vertical applied displacement at the midpoint of a facade of length $L$.

For each facade, define $K_{fe}$ as the finite element stiffness which is found using $w_o$ from the finite element analysis results and the applied central displacement, $y(L/2)$ as

$$K_{fe} = \frac{(w_o)_{fe}}{y(L/2)}$$  \hspace{1cm} (4.8)

Examples of the stress distributions from which the $K_{fe}$ values are derived are shown in figure 4.8 for 60m long, 8m high facades with differing amounts of openings (facades E11, E31, E51 and E71 in table 4.2).

Theoretical stiffness value for a beam

For a theoretical deep beam of length $L$ we can define the stiffness in a similar manner to equation 4.7, as the stress divided by the displacement at the midpoint. The applied
displacement, \( y(x) \) is given by equation 4.5. Solving the expression for displacement at the midpoint where \( x = L/2 \) gives the following (where \( C_1 = 11/5760 \) and \( C_2 = 1/32 \))

\[
y(L/2) = \frac{w_o L^4}{EI} (C_1) + \frac{w_o L^2}{AG} (C_2)
\]

Using the definition of stiffness \( K \) from equation 4.7 and the result from equation 4.9, we define \( K_{beam} \) as the theoretical stiffness of a beam as

\[
K_{beam} = \frac{w_0}{\frac{w_o L^4}{EI} (C_1) + \frac{w_o L^2}{AG} (C_2)}
\]

which simplifies to

\[
K_{beam} = \frac{1}{\frac{L^4C_1}{EI} + \frac{L^2C_2}{AG}}
\]

Presentation of finite element analysis results

For the presentation of results for this phase, the stiffness \( K_{fe} \) for each facade is normalised by dividing by the appropriate value of theoretical beam stiffness \( K_{beam} \) to obtain a normalised stiffness ratio of \( K_{fe}/K_{beam} \). This provides an interesting result as the response of the facade in comparison to the response of a theoretical deep beam (as is commonly used to model such facades described in Chapter 2) can be easily established.

Shown below in figures 4.9 and 4.10 are plots of normalised stiffness ratio against length to height (\( L/H \)) ratio for smooth based models in sagging and rough based models in hogging. Each chart contains four plots, one for each family of facades (E1, E3, E5 and E7) with a similar percentage of the face area removed to simulate windows or doors. Percentages of openings over the facade area are not identical within families due to geometrical constraints on the arrangement of the facade mesh blocks used to build the full facades. For this reason, the lines joining the data points in each family are not significant except to indicate trends for facades with similar amounts of openings and for ease of viewing.

A normalised stiffness ratio of unity indicates that the facade is responding to the applied
Figure 4.9: 
Smooth based facades in sagging - Normalised stiffness ratio

Figure 4.10: 
Rough based facades in hogging - Normalised stiffness ratio
displacement in the same fashion as a theoretical deep beam. Considering facades with no windows initially, family E1 with no windows can be seen to respond in a similar fashion to a deep beam for length to height ratios in excess of about 3.0. As $L/H$ reduces, however, the behaviour deviates from that predicted by beam theory and the normalised stiffness ratio falls, indicating that the finite element model is less stiff than a theoretical deep beam of the same dimensions. The normalised stiffness ratio continues to fall as $L/H$ reduces.

The second factor to consider is the amount of openings in the facade. The effect of increasing the amount of openings in the facades on their response to the applied displacements is evident from the results of families E7, E5 and E3. For facades with identical $L/H$, as the percentage of windows in the facade increases, a reduction in normalised stiffness ratio is observed. While the results for family E1 with no windows for high values of $L/H$ agree well with the theoretical beam stiffness, facades with openings exhibit decreasing stiffness (the family of curves shifts downwards) as the amount of openings increases.

The charts of normalised stiffness ratio against length to height ratio ($L/H$) thus appear to exhibit two distinct sections. For high values of $L/H$, the facade dimensions are such that the only factor causing the response to deviate from that predicted by beam theory is an increasing amount of windows. However for values of $L/H$ less than about 3.0, the overall dimensions of the facade ($L$ and $H$) have a greater effect on the response than the proportion of windows. This can be seen in the charts by the narrowing in the gap between the response of the windows families, and a simultaneous drop towards the origin in the normalised stiffness plots. From inspection of equation 4.11, $K_{beam}$ can be seen to approach infinity as $L/H$ approaches zero. This causes the normalised stiffness ratio ($K_{fe}/K_{beam}$) to plot towards zero as $L/H$ falls, assuming that $K_{fe}$ does not tend to infinity in the same way. The possibility that a different mechanism is acting in the finite element analysis at low values of $L/H$ is discussed further in section 4.3.2. The boundary between the section where the percentage of windows is the dominant factor (right hand side) and where the overall dimensions of the facade dominate (left hand side) is denoted the critical length to height ratio, $(L/H)_{crit}$. This is observed to occur at a value of approximately $L/H = 3.0$.

The concept of the critical length to height ratio is illustrated in figure 4.11. Each side of
the dividing line of \((L/H)_{\text{crit}}\) is approximated as linear, giving a bi-linear response for the facade over all ranges of \(L/H\). This concept is later utilised in the development of the new equivalent beam models described in section 4.3.

Figure 4.11: Critical length to height ratio, \((L/H)_{\text{crit}}\)

From these results it is evident that the commonly used approximation of a building facade (or building) as a beam with isotropic elastic properties is only appropriate for long, low buildings facades with no openings. For shorter facades or those with openings (or both) the commonly used deep beam with linear elastic properties attributed based on a simple plain wall does not respond to ground displacements in the same manner as the facade. A new method involving a procedure to assign appropriate properties to surface beams therefore needs to be developed such that elastic beams can more accurately represent building facades for all ranges of \(L/H\) and for differing amounts of openings.

4.3 Development of equivalent elastic beam method

This section describes the development of a new procedure to assign appropriate properties to elastic surface beams such that they respond to ground movements in a similar manner to elastic building facades of all sizes and with varying amounts of openings. This procedure is called the equivalent elastic beam method and it assigns properties to elastic Timoshenko beams as described below. As discussed in section 2.5.2, buildings are subject to both bending and shear effects due to soil settlements and Timoshenko beam theory allows for
the inclusion of both (as opposed to Bernoulli-Euler beams with only bending effects). Let the stiffness of such a beam be $K_{\text{equiv}}$ and be given by

$$K_{\text{equiv}} = \frac{1}{L^4C_1 + \frac{L^2C_2}{A^*G}}$$

where $C_1$ and $C_2$ are constants given above for equation 4.9 and $E$ and $G$ are the Young’s and shear moduli of the material respectively. The values of $A^*$ and $I^*$ are the effective cross sectional area and effective second moment of area respectively of a building facade with geometric properties $A$ and $I$. These values, once derived below, will be used as inputs for the shear and bending stiffness of the elastic beam.

The approach developed below, with appropriate bending and shear stiffnesses found for the beam by investigation of geometric properties $A$ and $I$, differs from the approach of Burland and Wroth (1974) who note that differing amounts of openings may be allowed for by manipulation of the ratio $E/G$ directly. Weaknesses of this $E/G$ ratio approach include: that no guidance is given as to appropriate values of the moduli to use for differing amounts of openings (arbitrary high and low values are chosen for extreme cases only); no modification is given for effects of the overall geometry ($L/H$) of the buildings, which is required as discovered in the analysis in the previous section; and values of $E$ and $G$ presented are not based on any robust analysis of the soil-structure interaction problem in hand. The new approach attempts to address these issues.

The procedures described below are thus concerned with the determination of appropriate effective values, $A^*$ and $I^*$, from the geometry of any given facade. The aim is to develop a method that gives values such that when the derived $A^*$ and $I^*$ are used in the calculation of $K_{\text{equiv}}$ it approximates the stiffness value $K_{fe}$ observed from the results of the finite element analyses. For the presentation of results for this phase, the comparison of the theory and finite element results is again undertaken after normalising stiffness values by dividing by the appropriate value of $K_{\text{beam}}$ for the facade dimensions.

Determination of $A^*$ and $I^*$ consists of two stages. Stage 1 consists of a procedure based on the facade geometries to determine value of effective shear area, $A^*$ from consideration
of shear (denoted part (a)) and the effective second moment of area, $I^*$ from consideration of bending (denoted part (b)) for deep beams. After this stage, the results are shown to give good agreement between $K_{fe}$ and $K_{equiv}$ for the region where $L/H > (L/H)_{crit}$ (where the amount of openings is the dominant variable) but poor agreement for facades with $L/H < (L/H)_{crit}$. For the region of $L/H < (L/H)_{crit}$ a second stage of analysis is thus added. For Stage 2, two alternative methods have been developed to augment the stage one geometric procedure. The first is denoted the \textit{Linear Depression} method and the second is the \textit{Limiting Height} method. As for the Stage 1 procedure, each alternative Stage 2 model has two parts, part (a) to determine $A^*$ and part (b) to determine $I^*$. The derivation and detail of these new procedures is described below.

### 4.3.1 Stage 1 - Geometry based procedure

**Part (a) - Shear**

Consider a short cantilever beam as shown in figure 4.12(a). For the plain beam, if we make the simplifying assumption that the shear strain $\gamma$ is uniform throughout the cross section then $\gamma$ is given by

$$
\gamma = \frac{V_s}{L} = \frac{\tau}{G} = \frac{F}{AG}
$$

(4.13)

where $V_s$ is the vertical displacement due to shear, $L$ is the length, $G$ is the shear modulus, $F$ is the applied force, $A$ is the cross sectional area and $\tau$ is the mean shear stress. Thus

$$
V_s = \frac{FL}{AG}
$$

(4.14)

For the beam with openings in figure 4.12(b) composed of $n$ vertical strips (here $n = 3$) where the cross sectional area and length of strip $i$ are $A_i$ and $L_i$ respectively, $V_s$ can be given by

$$
V_s = \frac{F}{G} \left( \frac{L_1}{A_1} + \frac{L_2}{A_2} + \cdots + \frac{L_n}{A_n} \right)
$$

(4.15)
CHAPTER 4. BUILDING FACADE ANALYSIS - ELASTIC

Figure 4.12: Beams in shear

where \( n \) is the total number of vertical strips and each \( A_i \) is net of any openings. For an equivalent deep beam let

\[
V_s = \frac{FL}{A^*G}
\]

such that

\[
\frac{FL}{A^*G} = \frac{F}{G} \left( \frac{L_1}{A_1} + \frac{L_2}{A_2} + \cdots + \frac{L_n}{A_n} \right)
\]

and finally

\[
A^* = \frac{L}{\sum_{i=1}^{n} \left( \frac{L_i}{A_i} \right)}
\]

where \( A^* \) is the effective cross-sectional area, \( L_i \) is the length of a vertical strip \( i \) of facade, \( A_i \) is the cross-sectional area of a vertical strip and \( n \) is the number of vertical strips.

Part (b) - Bending

Consider a plain beam as shown in figure 4.13(a). The second moment of area \( I \) is

\[
I = \frac{tH^3}{12}
\]
where $t$ is the thickness. To calculate the second moment of area for the beam with openings, take $n$ horizontal strips as shown in figure 4.13(b) where strip $j$ has height $h_j$, face area $A_{fj}$ and thickness $t$. Take $b_j$ as the distance from the midpoint of the strip to the neutral axis of the facade (shown at the mid-height of the facade, as for a smooth based analysis). To account for the openings, in each strip with windows the effective height $h'_j$ is calculated as $h'_j = A_{fj}/L$, with $A_{fj}$ being the face area of the strip net of windows. For a strip with no windows the effective height $h'_j$ is equal to the height $h_j$. For each strip the second moment of area $I_j$ is given using the parallel axis theorem as

$$I_j = \frac{th'_j^2}{12} + th'_jb_j^2$$

and the effective second moment of area for an equivalent deep beam is given by

$$I^* = \sum_{j=1}^{n} I_j$$

where $n$ is the number of horizontal strips of facade.

**Stage 1 Comparison of predicted and finite element results**

The values determined for $A^*$ and $I^*$ using the Stage 1 procedure outlined above are used to calculate the stiffness $K_{equiv}$ of a range of equivalent deep beams. These values
are compared to the stiffness of the facades $K_{fe}$ from the finite element analyses in the charts shown in figure 4.14. In the figures, normalised stiffness ratios of $K_{fe}/K_{beam}$ and $K_{equiv}/K_{beam}$ are plotted against the length to height ratio ($L/H$).

From figure 4.14 it can be observed that the geometric Stage 1 procedure gives good agreement with the observed finite element stiffness for high values of $L/H$. For facade families E1 and E7 (no windows and 5.625% windows respectively) the equivalent beam results closely match the observed finite element response in the high $L/H$ region. For families E5 and E7 with higher percentages of windows, the match is not as close, with approximately a 10% difference on average, but the trend is much better than a theoretical deep beam (normalised stiffness ratio of unity).

For values of $L/H$ below the previously defined critical value of $L/H = 3.0$ however, the equivalent stiffness determined by the procedure above remains far too high in comparison with the finite element response. It is apparent that the geometric Stage 1 method for assigning properties $A^*$ and $I^*$ to equivalent deep beams copes well with the region where the amount of openings dominates the response ($L/H > (L/H)_{crit}$) but not for smaller values of $L/H$. A second stage of analysis is therefore proposed below to determine more appropriate equivalent beam properties for the region $L/H < (L/H)_{crit}$.

### 4.3.2 Stage 2 - Methods for facades with $L/H < (L/H)_{crit}$

To determine more appropriate values of $A^*$ and $I^*$ such that the stiffness response of facades with values of $L/H$ less than $(L/H)_{crit}$ is replicated by the equivalent deep beams, two alternative methods are considered.

**Method 1 - Linear Depression**

In this method the effective values $A^*$ and $I^*$ for facades in the region where $L/H < (L/H)_{crit}$ are calculated as described in stage one above initially but are then multiplied by the ratio of $L/H$ for the model to $(L/H)_{crit}$. This approach makes use of the assumption described in figure 4.11 of a bi-linear normalised stiffness relationship. If the normalised
Figure 4.14: Comparison of finite element stiffness and predicted stiffness from Stage 1 (smooth based)
stiffness ratio is denoted the NSR, the section from the origin in the normalised stiffness plot to the stiffness value at \((L/H)_{\text{crit}}\) under consideration yields the relationship

\[
\frac{NSR_{L/H}}{L/H} = \frac{NSR_{(L/H)_{\text{crit}}}}{(L/H)_{\text{crit}}}
\]  

(4.22)

This same relationship will hold between values of \(K_{\text{equiv}}\) for different values of \(L/H\) less than \((L/H)_{\text{crit}}\) and thus values for \(A^*\) and \(I^*\) can be calculated as

\[
A^* = \frac{L}{\sum_{i=1}^{n} \left( \frac{L_i}{A_i} \right)} \times \left[ \frac{L/H}{(L/H)_{\text{crit}}} \right]
\]  

(4.23)

\[
I^* = \left( \sum_{j=1}^{n} I_j \right) \times \left[ \frac{L/H}{(L/H)_{\text{crit}}} \right]
\]  

(4.24)

Method 2 - Limiting Height

In this alternative method, instead of assuming a straight line relationship to calculate \(I^*\) and \(A^*\) if \(L/H < (L/H)_{\text{crit}}\), these quantities are fully recalculated using only a limiting height, \(H_{\text{lim}}\) for the facade equal to \(L/3\) above which the facade material is ignored. \(L/3\) is chosen as the value for \(H_{\text{lim}}\) as \((L/H)_{\text{crit}}\) is equal to 3.0. The physical interpretation of this approach is the assumption that material is only mobilised in the lower section of the structure when tall structures respond to ground movements. Saint-Venant’s principle (Love, 1944) could be said to apply in this case which states that strains produced in a body, due to the application of a system of forces statically equivalent to zero force and zero couple, are of negligible magnitude at distances which are large when compared with the linear dimension. It is as though in this instance the facade starts acting like a long vertical bar, with stresses confined to the base region. In contrast to the Linear Depression method, which is a purely empirical method, the Limiting Height method is thus based on an approximation related to the assumed mechanics of the situation.
These assumptions lead to $I^*$ and $A^*$ being calculated for $L/H < (L/H)_{crit}$ as

$$A^* = \frac{L}{\sum_{i=1}^{n} \left( \frac{L_i}{A_{i,lim}} \right)}$$

$$I^* = \sum_{j=1}^{n} I_{j,lim}$$

where $A_{i,lim}$ is the cross sectional area of a vertical strip calculated using $H_{i,lim}$. $I_{j,lim}$ is the second moment of area of a horizontal strip calculated using $H_{j,lim}$. For the calculation of $I_{j,lim}$ the neutral axis about which the moment is taken is assumed to remain unchanged (at the mid-height of the full facade for smooth based facades and at the base for rough facades). This concept is described in figure 4.15.

### 4.3.3 Comparison of predicted and finite element results

The predicted stiffness using the values of $A^*$ and $I^*$ produced by both stage two methods described above as well finite element stiffness values are given in table 4.3 for the smooth based facades in sagging. Results are given under headings of the two Stage 2 methods, but it should be noted that the values of $A^*$, $I^*$ and stiffness $K_{equiv}$ for facades with $L/H > 3.0$ are the same under each method as it is only below $(L/H)_{crit}$ that the additional (and differing) Stage 2 procedures apply. Stiffnesses predicted by each method are normalised
(by dividing the stiffness by $K_{beam}$) and plotted in figures 4.16 and 4.17 along with the observed stiffness from the finite element analyses for each family of facades.

On inspection of figures 4.16 and 4.17 for the region $L/H < (L/H)_{crit}$, both the Limiting Height and the Linear Depression method can be seen to give reasonable agreement with the observed stiffness values from the finite element analyses. Both methods, however, give values slightly higher than the finite element stiffness, for low values of $L/H$. On comparing the two methods, the Linear Depression method, can be seen from figures 4.16 and 4.17 to give stiffness values slightly closer to those observed. This method is also the simpler of the two to calculate and as such is chosen as the most appropriate method.

A series of analyses with rough bases and a hogging displacement were undertaken, but tables and charts of the results of the predicted and finite element stiffness values of these have not been presented. The results, however, again indicated that new procedure produces appropriate beam properties that give stiffness values in reasonable agreement with the observed finite element results.

### 4.4 Conclusion

A new approach for assigning material properties to elastic beams to more accurately model building facades of varying dimensions and amounts of openings has been developed. The effective properties $A^*$ and $I^*$ are determined using the new process which is based on geometric criteria and are used in conjunction with the material properties to give the stiffness $EI^*$ in bending, $GA^*$ in shear and $EA^*$ axially. This approach is based on effective geometric properties of the beams rather than material properties (as suggested by Burland and Wroth (1974)) and is thus felt to be more robust. This process is shown to be equally valid for both a smooth or a rough base boundary condition assumption. Assessments of the ability of elastic beams with these properties to model facades in finite element analyses of tunnelling are given in Chapter 8 and Chapter 9.
Table 4.3: Predicted and observed stiffness - Smooth facades in sagging

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<th>Facade Details</th>
<th>Predicted Method 1</th>
<th>Predicted Method 2</th>
<th>Obs. F.E.A.</th>
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<td>Limiting height</td>
<td></td>
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Figure 4.16: Predicted and observed normalised stiffness ratio - Method 1 (smooth based)
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Figure 4.17: Predicted and observed normalised stiffness ratio - Method 2 (smooth based)
Chapter 5

Building Facade Analysis - Masonry

5.1 Introduction

This Chapter describes a series of finite element analyses of masonry facades. These analyses are similar to those in Chapter 4 with elastic facades, except that in addition to the impact of facade dimensions, openings and boundary conditions, here a non-linear constitutive model is used to represent masonry brickwork. The full range of facade analyses is summarised in Table 4.1 with the masonry analysis schedule in Table 5.1. Before the finite element analyses are described, an introduction to the performance of masonry walls and facades subject to ground displacements is given based on a literature review.

5.2 Review of literature

Masonry is a popular building material having historically been extensively used throughout the United Kingdom in the construction of houses, flats and public buildings. The behaviour of masonry as a structural material is more complex, however, than the linear isotropic elastic behaviour that is often assumed in simple analyses. While the assumption of a linear elastic model for masonry may be convenient for simplicity, it is clear that masonry is an inhomogenous material comprising either brick or stone and mortar joints.
Liu (1997) describes typical masonry characteristics including its low tensile strength and propensity to crack under tensile load; anisotropy due to the mortar joints; varying stiffness between the mortar and the blockwork; and the differing behaviour of cracked and uncracked masonry walls. Masonry should thus be modelled using a non-linear approach. Much effort has been devoted to the development of a constitutive model for masonry which captures these characteristics, a number of which are described by Liu (1997).

As discussed in Chapter 2, when surface beams are used to represent a masonry building or facade they are often given linear elastic properties (e.g. Frischmann et al., 1994). It is evident, however, that the approximation of masonry walls and facades by an elastic surface beam may not fully capture the necessary masonry characteristics. A key characteristic overlooked in a linear elastic beam approximation is cracking causing loss of stiffness of the masonry. An attempt to deal with this was introduced in section 2.5.1 in relation to building damage. Burland and Wroth (1974) proposed an approach where a linear elastic response is assumed up to a limiting tensile strain, beyond which the response is not considered. This limiting tensile strain is, however, related to the point at which cracking becomes visible and as Green et al. (1976) note, masonry actually loses its tensile strength at a much lower tensile strain. Green et al. (1976) state that this lower tensile strain is best determined from physical testing.

Burland and Wroth (1974) further note that an important consideration is the mode of deformation of masonry structures; in particular the difference between hogging and sagging. They give examples of walls subjected to angular strains in both hogging and sagging and show that for walls in sagging, where the foundations offer restraint, reasonably uniform distortions occur without significant cracking. For walls in hogging (and thus no restraint at the top tensile face) with the same amount of angular strain, damage is much more severe with significant cracking. Cracking of masonry walls in hogging is shown to be more likely to occur at low deflection ratios ($\Delta/L$) with cracking propagating more rapidly than for walls in sagging, which for the same values of $\Delta/L$ sustain little or no damage.

Examples of cracking of masonry model walls in both hogging and sagging are given by Cox (1980) who modelled masonry walls using a friction table. A friction table consists
of a table across which a horizontal rough surface slides, much like a treadmill. Walls or building frames can be constructed flat on the rough surface and the movement of the surface from top to bottom of the building simulates gravity. Ground movements at the base of the building are simulated using a beam on which the building rests which can be straight or curved to simulate heave or subsidence. Cox found that in the sagging case, tension cracks were partly suppressed by the restraining action of the ‘soil’ at the base of the wall and the brickwork ‘arching’ over the settlement profile and thus supporting itself. In the hogging case, however, little arching was seen as the top of the building is unrestrained and tension cracks were much more severe. Examples of facades subject to ground displacements on the friction table are shown in figure 5.1 in which the greater severity of cracks due to hogging is visible.

Real masonry walls are found to exhibit similar behaviour. In a study of a three-storey, 40m long, 11m wide masonry building subjected to ground movements due to the removal of nearby trees, Cheney and Burford (1974) show sections of external masonry facade subjected to heaving ground (and thus hogging deformation) exhibited wider and progressively worse cracks than other sections of the same building with different deformation modes. Similar observations of the response of a separate four-storey, 75m long, 10m wide brick masonry structure subjected to differential settlement were made by Green et al. (1976)
and these also show that facade sections in hogging were more severely damaged than sections in sagging. The most serious damage occurred in the area of the facade with the worst hogging deformation, where cracks of the order of 10mm wide were observed, whereas in the sagging regions the maximum crack width was 4mm.

More recent ground movements were observed during the tunnelling of the London Underground Jubilee Line Extension (Burland et al., 2001). While these case studies are described in detail in Chapter 9, it is useful to examine briefly the comparative hogging and sagging behaviour of some of the masonry buildings here. The complex including Murdoch, Clegg and Neptune Houses at Moodkee Street, Rotherhithe (described in section 2.6 of this thesis) exhibited some interesting features which differed in hogging and sagging. For Murdoch House, the 40m long northern and southern facades were both in hogging and exhibited flexible responses with the observed base displacements closely following a Gaussian greenfield ground profile. Neptune House, particularly the western facade, exhibited a stiff response over the sagging section with the facade rigidly ‘spanning’ the expected greenfield trough. The base of the facade, however, followed the greenfield profile (i.e. acted flexibly) in the hogging region (Withers, 2001b). This case study is discussed in detail in Chapter 9 of this thesis.

Finite element analyses also display evidence of masonry facades interacting differently with sagging and hogging ground displacements. During the development and testing of the masonry constitutive model described in section 5.3, Liu (1997) analysed masonry facades subjected to imposed ground displacements. It was observed that once cracked, the masonry walls lost bending stiffness leading to additional strain and thus damage occurring more rapidly than in the uncracked state. A parametric study was also conducted by Liu (1997) using two-dimensional finite element analyses of a masonry building over a tunnel. The stiffness and weight of the masonry facade and its eccentricity over the tunnel were varied. In sagging regions the masonry was observed to form arches. The stress redistribution due to the arching resulted in reduced differential settlement: due to the arches, the pressure on the ground redistributed, with contact pressure in the region of each arch base increasing and the ground pressure remote from an arch base reducing.
This caused soil under the arch bases to settle more than other areas, causing flattening of the settlement trough and the indication of a ‘stiff’ facade response as shown in figure 5.2. This arching was not observed in facade regions in hogging due to the lack of restraint at the top of the facade and the tensile nature of the horizontal strains in the ground in hogging zones, preventing the formation of stress arches. Liu proposed that the response of a masonry facade could be considered as a beam with supports located at the soil profile inflection points, between which, in the sagging region, the soil profile would be a straight line and outside, in the hogging region, the profile would depend on the facade stiffness.

Boonpichetvong and Rots (2004) present 2D finite element analyses including a masonry facade modelled using a fracture mechanics approach. The facade was placed in the hogging region of tunnelling surface displacements as this was considered the most critical area. The response of the facade indicated that up to a critical level of angular distortion (of 1/800) the facade suffered minimal cracking damage, but beyond this critical level the facade softened and cracks propagated more rapidly. Son and Cording (2005) used a distinct element approach with different properties for masonry bricks and mortar joints in investigating masonry facades adjacent to excavations. They found that the lower the stiffness of the facade, the closer it followed a greenfield surface profile. Crack development was found to further decrease the stiffness of the building and cause the building to more closely follow the greenfield profile.

A number of 3D finite element analyses of tunnels under masonry structures including
full soil-structure interaction in 3D, are described by Liu (1997). Augarde (1997) and Burd et al. (2000) also discuss the same analyses (the details of which were described in section 2.5.4). Here, among other observations, it was found that in analyses where the tunnel alignment was oblique under the building, the front facade, which was mostly in sagging, exhibited stress arches leading to flattening of the soil response profile. The front facade thus responded with an almost linear settlement variation from end to end with only minor damage. The rear facade of the structure, however, was predominantly in hogging. No stress arches formed in the rear facade, leaving it with less resistance to the ground displacements and it thus exhibited a much more flexible response. The ground profile under the rear facade followed a profile similar to the greenfield profile with large differential settlements and curvatures. Significant cracking damage was evident at the top of this facade caused by the tensile strains induced by the hogging displacement.

From this review of published literature regarding the performance of masonry walls and facades subject to ground movement it is clear that there is a significant difference in the performance of facades that are subjected to sagging and those that are subjected to hogging deformations. Facades in sagging tend to exhibit arching effects and thus appear to act in a stiff, rigid manner spanning any ground movements, whereas facades in hogging quickly lose stiffness in bending due to the propagation of cracks caused by tensile strains and tend to act in a more flexible manner with the facade base following any ground movements without affecting them. Surface beams designed to represent a masonry facade should therefore also reflect this difference in hogging and sagging response.

5.3 Description of masonry finite element analyses

5.3.1 Introduction

This section describes finite element analyses of masonry facades undertaken using a similar approach to the analyses of linear elastic facades described in section 4.2.4 of this thesis. Two-dimensional facades are subjected to an applied ground displacement and their re-
sponse is investigated with a view to developing a simple non-linear surface beam model to represent masonry facades that responds appropriately in hogging and sagging for use in finite element analyses of tunnelling.

5.3.2 Meshes

The meshes used in the analyses are identical to those used for the elastic analyses described in Chapter 4. The full range of masonry facades analysed is given in table 5.1 with the key variables of facade dimensions and proportion of openings. Masonry facades are identified using a similar scheme to the elastic facades in table 4.2 with the reference ‘M’ used to indicate that the constitutive model is a non-linear masonry model. The facade meshes are shown in Appendix A.

5.3.3 Masonry material model

The constitutive model used to represent the masonry is an elastic no tension model developed by Liu (1997) in which material under compression behaves elastically, while material in tension will crack where tensile stress develops, after which stiffness in a direction perpendicular to the crack reduces to a small value. The approach taken is a macroscopic one to modelling masonry where the material is treated as a continuum and the properties of the bricks and the mortar are averaged rather than accounted for separately as they would be in a microscopic approach (which is computationally more expensive). Additionally, a smeared cracking approach is adopted where the cracked masonry is assumed to remain as a continuum as opposed to cracks being considered individually. Liu (1997) includes three states in the constitutive model: elastic intact, single cracked and double cracked; each of which has its own stiffness matrix as described below. The model is implemented in the OXFEM finite element program where the modified Euler solution scheme is used to deal with the non-linearity of the material (Burd, 1986). Further details about the solution techniques in OXFEM can be found in section 7.5.
### Table 5.1: Schedule of facade meshes for masonry analyses

<table>
<thead>
<tr>
<th>Facade Family</th>
<th>Facade Number</th>
<th>Length (m)</th>
<th>Height (m)</th>
<th>L/H Ratio</th>
<th>Openings (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>M11</td>
<td>60</td>
<td>8</td>
<td>7.50</td>
<td>0</td>
</tr>
<tr>
<td>M1</td>
<td>M12</td>
<td>40</td>
<td>8</td>
<td>5.00</td>
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<tr>
<td>M1</td>
<td>M13</td>
<td>20</td>
<td>8</td>
<td>2.50</td>
<td>0</td>
</tr>
<tr>
<td>M1</td>
<td>M14</td>
<td>40</td>
<td>12</td>
<td>3.33</td>
<td>0</td>
</tr>
<tr>
<td>M1</td>
<td>M15</td>
<td>20</td>
<td>12</td>
<td>1.67</td>
<td>0</td>
</tr>
<tr>
<td>M1</td>
<td>M16</td>
<td>20</td>
<td>16</td>
<td>1.25</td>
<td>0</td>
</tr>
<tr>
<td>M1</td>
<td>M17</td>
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<td>16</td>
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<td>0</td>
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<tr>
<td>M3</td>
<td>M31</td>
<td>60</td>
<td>8</td>
<td>7.50</td>
<td>18.750</td>
</tr>
<tr>
<td>M3</td>
<td>M32</td>
<td>40</td>
<td>8</td>
<td>5.00</td>
<td>18.750</td>
</tr>
<tr>
<td>M3</td>
<td>M33</td>
<td>20</td>
<td>8</td>
<td>2.50</td>
<td>18.750</td>
</tr>
<tr>
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<td>3.33</td>
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<tr>
<td>M3</td>
<td>M35</td>
<td>20</td>
<td>12</td>
<td>1.67</td>
<td>18.750</td>
</tr>
<tr>
<td>M3</td>
<td>M36</td>
<td>20</td>
<td>16</td>
<td>1.25</td>
<td>18.750</td>
</tr>
<tr>
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<td>M37</td>
<td>12</td>
<td>16</td>
<td>0.75</td>
<td>18.750</td>
</tr>
<tr>
<td>M5</td>
<td>M51</td>
<td>60</td>
<td>8</td>
<td>7.50</td>
<td>9.375</td>
</tr>
<tr>
<td>M5</td>
<td>M52</td>
<td>40</td>
<td>8</td>
<td>5.00</td>
<td>9.375</td>
</tr>
<tr>
<td>M5</td>
<td>M53</td>
<td>20</td>
<td>8</td>
<td>2.50</td>
<td>9.375</td>
</tr>
<tr>
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<td>M54</td>
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<td>12</td>
<td>3.33</td>
<td>10.000</td>
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<td>1.67</td>
<td>10.000</td>
</tr>
<tr>
<td>M5</td>
<td>M56</td>
<td>20</td>
<td>16</td>
<td>1.25</td>
<td>9.375</td>
</tr>
<tr>
<td>M5</td>
<td>M57</td>
<td>12</td>
<td>16</td>
<td>0.75</td>
<td>9.375</td>
</tr>
<tr>
<td>M7</td>
<td>M71</td>
<td>60</td>
<td>8</td>
<td>7.50</td>
<td>5.625</td>
</tr>
<tr>
<td>M7</td>
<td>M72</td>
<td>40</td>
<td>8</td>
<td>5.00</td>
<td>5.625</td>
</tr>
<tr>
<td>M7</td>
<td>M73</td>
<td>20</td>
<td>8</td>
<td>2.50</td>
<td>5.625</td>
</tr>
<tr>
<td>M7</td>
<td>M74</td>
<td>40</td>
<td>12</td>
<td>3.33</td>
<td>6.250</td>
</tr>
<tr>
<td>M7</td>
<td>M75</td>
<td>20</td>
<td>12</td>
<td>1.67</td>
<td>6.250</td>
</tr>
<tr>
<td>M7</td>
<td>M76</td>
<td>20</td>
<td>16</td>
<td>1.25</td>
<td>5.625</td>
</tr>
<tr>
<td>M7</td>
<td>M77</td>
<td>12</td>
<td>16</td>
<td>0.75</td>
<td>6.250</td>
</tr>
</tbody>
</table>
Figure 5.3: Elastic no-tension model principal stresses (after Liu, 1997)

The stresses on a masonry element are shown in figure 5.3. For these stresses $\sigma_x$, $\sigma_y$ and $\tau_{xy}$, the incremental stress and strain are related by the tangent stiffness matrix $D$ by

$$d\sigma = D d\epsilon$$  \hspace{1cm} (5.1)

If both principal stresses, $\sigma_1$ and $\sigma_2$, are compressive then the material is elastic and the incremental stiffness is given by

$$D = \frac{E}{1 - \nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix}$$  \hspace{1cm} (5.2)

If major principal stress $\sigma_1$ is tensile while $\sigma_2$ is compressive, the material is singly cracked in a direction perpendicular to the major principal stress giving an incremental stiffness in the principal stress space of

$$D' = \begin{pmatrix} 0 & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & 0 \end{pmatrix}$$  \hspace{1cm} (5.3)

which can be multiplied by a transformation matrix using $\theta$ to give $D$ in $x$-$y$ coordinate space where $\theta$ is the inclination of the principal strain direction, taking compression as
positive. If both $\sigma_1$ and $\sigma_2$ are tensile, the material is doubly cracked and the incremental stiffness is set to a small residual value, $E_r$.

The material in the model is given an infinite compressive strength and a low tensile strength as shown in figure 5.4. The material is considered cracked when the tensile stress reaches a critical value, $c$, as shown. A residual stiffness for the material is required for numerical stability and to avoid a multitude of unrealistic minuscule cracks appearing (Liu, 1997). The residual stiffness, $E_r$, is calculated using a residual stiffness factor $f$ as

$$ E_r = f \times E $$

In addition, the stiffness is reduced gradually as tensile stress develops to avoid problems associated with crack re-closure as described by Liu (1997). A stiffness reduction rate parameter $\epsilon^r$ controls the speed at which the stiffness decreases. The notion of cracking strain, $\epsilon^r$ was introduced by Liu to monitor the cracking process and any reclosing during stress and state updating. This was necessary as stress normal to a cracking direction is ideally zero after cracking.

Elastic lintels above the windows in the mesh were used to avoid failure of the masonry above the openings. This not only avoids complications associated with material failure but replicates the common situation found in real masonry buildings, where lintels are used. Over-integration with 16 Gauss points is used during solution to smooth the integration
Table 5.2: **Material properties for no-tension masonry model**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$\nu$</td>
<td>$\gamma$</td>
<td>$f$</td>
<td>$\epsilon_{cr}$</td>
</tr>
<tr>
<td>$1.0 \times 10^7$ kPa</td>
<td>0.2</td>
<td>20 kPa/m</td>
<td>0.01</td>
<td>10 kPa</td>
</tr>
</tbody>
</table>

Table 5.3: **Material properties for elastic lintels**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$\nu$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>$1.0 \times 10^7$ kPa</td>
<td>0.2</td>
<td>20 kPa/m</td>
</tr>
</tbody>
</table>

due to the non-linear nature of the material.

The properties of the non-linear masonry and elastic lintel materials are given in table 5.2 and table 5.3 where: $E$ is Young’s Modulus in compression, $\nu$ is Poisson’s ratio, $\gamma$ is self weight, $f$ is the residual stiffness factor, $c$ is the residual tensile strength, and $\epsilon_{cr}$ is the stiffness reduction ratio.

### 5.3.4 Displacement and boundary conditions

For each facade analysis, displacements are imposed at base level and the response is analysed. These imposed displacements are the same form as the those applied for the elastic facade analyses of section 4.2.4 and given by equation 4.5. The magnitude of the displacements applied for the masonry analyses, however, is different to those for the elastic analyses. Appropriate magnitudes of displacement for each facade were found by using the damage category chart of Boscardin and Cording (1989) and the interaction charts for deflection ratio ($\Delta/L$) based on the one presented by Burland (1997), as described below.

For the case of a facade with $L/H = 1.0$, Burland used the limiting strain values ($\epsilon_{lim}$) proposed by Boscardin and Cording (1989) (and given as table 2.2 in this thesis), to produce the interaction chart in figure 2.8; using the equations presented as 2.17 to 2.21 in this thesis to do so. From this chart, making the assumption that horizontal strain at the ground surface ($\epsilon_h$) is zero (i.e. there is only bending and shear strains present, not additional ground induced horizontal strains), values of $\Delta/L$ can be read off for a chosen damage category. From the midpoint displacement ($\Delta$) so found, equation 4.5 is used to
Table 5.4: Maximum central displacement for masonry facades

<table>
<thead>
<tr>
<th>Facade Number</th>
<th>MX1</th>
<th>MX2</th>
<th>MX3</th>
<th>MX4</th>
<th>MX5</th>
<th>MX6</th>
<th>MX7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Boundary</td>
<td>r s</td>
<td>r s</td>
<td>r s</td>
<td>r s</td>
<td>r s</td>
<td>r s</td>
<td>r s</td>
</tr>
<tr>
<td>Displacement (mm)</td>
<td>72</td>
<td>121</td>
<td>41</td>
<td>58</td>
<td>20</td>
<td>23</td>
<td>44</td>
</tr>
<tr>
<td></td>
<td>41</td>
<td>20</td>
<td>28</td>
<td>31</td>
<td>22</td>
<td>18</td>
<td>18</td>
</tr>
</tbody>
</table>

calculate the corresponding value of \( w_o \) which is then used to determine the displacement to apply at all nodes along the base of the facade, again using equation 4.5.

For this series of analyses the appropriate final level of damage for the masonry facades was chosen as category two (slight) damage. To determine the deflection ratio \( \Delta/L \) to apply to each facade to achieve this damage, interaction charts for each value of \( L/H \) for the masonry facades given in table 5.1 were produced (using the same approach as Burland (1997)) and examples are given in figure 5.5. From these charts, values of \( \Delta/L \) were read off for \( \epsilon_h = 0 \) at the upper boundary of damage category two. The maximum resulting displacement at the centre of each of the various facade shapes is given in table 5.4 for hogging (‘MX1’ represents the value for facades M11, M31, M51 and M71); sagging displacements are similar but negative.

As with the elastic facades, two boundary conditions were used in the analyses, a smooth (s) base where the base of the facade is free to move horizontally and a rough (r) base where horizontal movement is restrained. Displacements causing both hogging (h) and sagging (s) were imposed on each facade with each boundary condition.

### 5.3.5 Calculation procedure

Each analysis is conducted in two stages. In the first stage, the facade self weight is applied. Displacements and strains are then reset to zero and in the second stage, the imposed ground displacements are applied. Facade strains and crack damage is thus solely a result of the applied ground displacements. To consider the response of the facade as the displacements develop, the total displacement \( y_t \) at each node is divided by the number of steps in the stage, \( n \), and a value of \( y_t/n \) is applied at each step. Thirty steps are used for these facade analyses with results output at steps one, three, five, 10, 20 and 30.
Figure 5.5: Interaction charts for deflection ratio and horizontal strain showing damage categories (assuming a rough base boundary condition)
5.3.6 Results

Presentation of finite element results

Results for the elastic analyses presented in section 4.2.6 of this thesis are based on the stress at the base of the facades. For the masonry facade analyses in this section, however, this approach to presenting the results is not appropriate. It is not possible to infer the reaction of the full facade from the reaction at the base as it is in an elastic facade, due to the non-linear nature of the masonry constitutive material used and its ability to crack. The stress at a facade base is still a useful result to investigate but in order to gain a full understanding of the reaction of the facades to the imposed displacements, contour plots of stress and strain, crack patterns and damage over the whole facade are required.

Cracking strain, $\epsilon^{cr}$, is used to describe the damage to the masonry structures due to the ground displacements, following the approach of Liu (1997). Total strain can be considered as comprising two parts, the elastic strain, $\epsilon^e$, and the cracking strain, $\epsilon^{cr}$ as follows

$$\epsilon^{total} = \epsilon^e + \epsilon^{cr}$$

(5.5)

Liu (1997) concluded that is is reasonable to use the cracking strain (excluding the elastic part of the deformation) rather than the total strain to describe the severity of damage to masonry facades. Cracking strain can be resolved into three components: $\epsilon_1^{cr}$ representing the component normal to the initial crack; $\epsilon_2^{cr}$ representing the component normal to any second crack; and $\gamma_{12}^{cr}$ representing the shear deformation across the crack (Liu, 1997). This approach differs from that of Boscardin and Cording (1989) who use total tensile strain in their correlated damage grades. The cracking strain approach was, however, found by Liu (1997) and Burd et al. (2000) to be a reasonable approach for masonry structures and as such, damage grades based on cracking strain are presented here.

Crack patterns can also be predicted from the finite element results and are presented by drawing short lines on the facade in the direction of the expected crack for points at which the crack strain exceeds $500 \mu \epsilon$. The density of the lines indicates the severity of cracking;
a double thick line is drawn when strain exceeds $1000 \mu \epsilon$ and a triple line when it exceeds $1500 \mu \epsilon$.

**Effects of hogging and sagging boundary conditions**

Results for analysis M13 are presented for imposed hogging (h) and sagging (s) displacements for both rough (r) and smooth (s) boundary conditions. For each of these cases results are presented for normal cracking strain ($\epsilon_{cr}^1$), crack pattern, damage grade and stress trajectory at the final calculation step (30). Results are shown in figures 5.6 to 5.9.

Facades in hogging are seen to crack vertically for almost their full height in the centre, with cracking and damage concentrated at the top of the facade in the areas of high strain, as can be expected due to the unrestrained nature of movement at this boundary. Damage is also evident at the bottom corners of the hogging facades where shear strains also develop. The hogging facade with the rough boundary condition can be seen to sustain more serious
(a) Normal cracking strain, $\epsilon_{1}^{cr}$  

(b) Predicted crack pattern

(c) Damage category

(d) Stress trajectory

Figure 5.7: M13rs Rough/Sagging

(a) Normal cracking strain, $\epsilon_{1}^{cr}$  

(b) Predicted crack pattern

(c) Damage category

(d) Stress trajectory

Figure 5.8: M13sh Smooth/Hogging
damage than that with the smooth boundary condition. This is possibly due to the rough boundary forcing the neutral axis to lie at the ground surface of the rough based facade. The amount of moderate and severe damage overall is higher than expected given the choice of applied displacement magnitude discussed above. This is possibly due to the non-linear nature of the masonry material and the use of the cracking strain to determine the grades, as opposed to the elastic theory (and total strain values) upon which the damage grades were originally developed. Cracks in the hogging facades are mostly vertical indicating that they are mainly bending related rather than shear induced, although there are some diagonal shear cracks in areas of high shear strain near the supports.

Sagging facades can be seen to exhibit the predicted stress arching (most evident from the stress trajectory plots), with significant crack damage occurring under the arch, but very little above it due to the arch’s supporting action. This is more pronounced in the rough based facade which has a more clearly defined arch under which mainly diagonal and some horizontal cracking appears, indicating vertical downwards movement of the
facade material. Arching in the smooth facade is much less pronounced due to the lack of horizontal support for potential arch foundations at the base. Cracking here is mostly vertical indicating spreading of the facade horizontally at the unrestrained lower boundary. No significant arching is evident in the facades in hogging except for the lower corners of the rough based facade. Here small arches are evident due to the shape of the applied displacement profile at this position, where a small sagging zone exists.

The changing response of the facade as the imposed displacement increases is shown in figure 5.10. Here the maximum central displacement is plotted against average vertical force at each node along the facade base as the calculation progresses. Data are plotted at calculation steps one, three, five, 10, 20 and 30. For both hogging and sagging after a certain number of steps, the relationship becomes a straight horizontal line indicating the overall incremental stiffness of the facade is zero (i.e. no more force is generated as the displacements increase). The force lines appear to tend towards a value of 80N which again indicates that the facade weight is the only contributor to this contact force, as the nodes are at intervals every half metre (and the weight is 160N/m). The slight differences in the sagging forces are thought to be due to differing arch shapes in the smooth and rough case. The loss of stiffness appears to occur by calculation step 10 (smooth: $\Delta/L=0.038\%$, rough: $\Delta/L=0.033\%$).

Figure 5.11 shows the development of cracks as the displacement increases. The hogging facade can be seen to be cracked almost to its full height by step 10 ($\Delta/L=0.033\%$) and the arch in the sagging case can also be observed due to the cracking occurring underneath. These observations confirm the response indicated in figure 5.10.

For facades in hogging, ground heave thus appears able to occur unrestrained once cracking of the facade reduces the effective flexural rigidity to zero, due to the lack of restraint at the top of the facade meaning that arching can not occur. Facades in sagging, although cracking also occurs (possibly to a greater extent), exhibit an action where the facade is supported by the arch. This arching action allows the masonry facade to ‘span’ sagging displacements as if it was a stiff beam. These observations hold for both boundary conditions, rough and smooth and concur with the observations in the literature.
To investigate the impact of the other factors found to be critical in the elastic facade analysis, the overall facade dimensions and the amount of openings in the facade, further results are discussed below.

It is apparent from the results presented above that similar effects are evident for facades with both rough and smooth base conditions (although those facades with rough bases appear to suffer more damage) and thus similar conclusions can be drawn about facade response from analyses with either base condition. Contour plots for facades with rough bases only are therefore presented in the remaining sections.

**Effect of increasing amounts of openings**

Results are presented in this section for facades M33 (18.750% windows), M53 (9.375% windows) and M73 (5.625% windows). These facades have the same overall dimensions (20m x 8m) as facade M13 discussed above. The various plots given in figures 5.12 and 5.13 are for the final analysis step (step 30, $\Delta/L=0.100\%$)

Facades with openings exhibit similar key characteristics to those without openings: facades in sagging can again be seen to exhibit arching, whereas those in hogging do not. An effect
Figure 5.11: Development of cracks in masonry facades with rough bases
Figure 5.12: Effect of openings - Rough/Hogging
(a) M33 Crack pattern  
(b) M33 Damage category  
(c) M33 Stress trajectory  

(d) M53 Crack pattern  
(e) M53 Damage category  
(f) M53 Stress trajectory  

(g) M73 Crack pattern  
(h) M73 Damage category  
(i) M73 Stress trajectory  

Figure 5.13: Effect of openings - Rough/Sagging
of adding openings to the facades can be clearly seen in the concentrations of crack damage at the corners of the windows which lead to additional damage. Damage can also be seen to be spread over greater areas of the facade, the higher the number of openings. The stress arching in the sagging facades also differs slightly due to openings in the facades, as the stress arches must find their way around the windows and doors. Two stress arches, rather than one, develop in facades M53 and M33 in sagging (figure 5.13) due to the pattern of windows. The additional damage due to this occurrence is particularly evident in facade M33 where significant damage can be seen around windows close to the lower corners of the facade as the stress arches attempt to reach the base.

To determine if the addition of windows has any effect on the rate at which the facades crack sufficiently to lose their overall stiffness, figure 5.14 showing the average nodal force along the base as the displacement is applied (similar to figure 5.10). For all facades, irrespective of the amount of openings, the direction of imposed displacement (or indeed the boundary condition), the cracked state, evidenced by horizontal force plots, is reached by step 10 (smooth: $\Delta/L=0.038\%$, rough: $\Delta/L=0.033\%$). Hogging facades can be seen to all reach a force representing the self weight of the facade consistently for both rough and smooth boundary cases (the differing self weights being due to the differing amounts of openings). Sagging facades, however, exhibit different force values near the weight of the facade, but not all agreeing exactly. This is thought to be due to the amount of the facade weight supported by the different arch effects within individual facades.

Increasing the amount of openings appears to have little effect on the general nature of the facade response or the maximum damage grade, although crack patterns and damage grade differ due to window layouts and there exists localised damage around window corners. Increasing the amounts of windows does not appear to affect when the facade loses its stiffness. Sagging facades again experience arching which is absent in hogging.
CHAPTER 5. BUILDING FACADE ANALYSIS - MASONRY

Figure 5.14: Force displacement relationship at facade base - Differing % windows

Effect of differing overall geometry

The response of facades with differing overall dimensions ($L/H$) were analysed for the M1 family with no windows (facades M11, M12, M14, M15, M16 and M17) for hogging and sagging. Figure 5.15 shows plots for facade M16 for the final calculation step (step 30). Other facades were analysed but are not shown here.

As would be expected, despite the differing overall dimensions of the facades, those in

![Figure 5.15: Cracking plots for facade M16](a) M16 Crack pattern: Rough/Hogging (b) M16 Crack pattern: Rough/Sagging

Figure 5.15: Cracking plots for facade M16
sagging exhibited arching, whereas those in hogging did not. One notable feature of the facades with low values of $L/H$ is that the damage is confined to the lower portion of the facade in the sagging case. This echoes comments made earlier in section 4.3.2 about facades potentially having a ‘limiting height’ above which they are not affected by ground movements. This can be seen in figure 5.15 only to be true in sagging and is due to the arching effect whereby material above the arch is supported and not damaged by the ground movements.

Figure 5.16 shows the average nodal force (for the rough hogging case only for clarity) along the base as the displacement is applied to this family of windowless facades. The cracked state (again, shown by the response flattening to a horizontal line) is reached by all facades by step 10 (M11: $\Delta/L=0.040\%$, M12: $\Delta/L=0.034\%$, M14: $\Delta/L=0.036\%$, M15: $\Delta/L=0.046\%$, M16: $\Delta/L=0.052\%$) with the exception of facade M17 (step five, $\Delta/L=0.025\%$). With all facades reaching the cracked state at similar calculation steps, this suggests that there is no relationship between the overall geometry of the facade and the curvature at which it cracks sufficiently for it to effectively lose its overall stiffness.
Results summary

The results of the finite element analyses of masonry facades described here confirm the assertions discussed in the literature review (section 5.2) that facades subjected to hogging deformations act in a flexible manner once the deformations reach a certain level, due to cracking reducing their stiffness. Facades subjected to sagging deformations also crack as deformations develop, but exhibit arching which allows the facade span over sagging displacements. The finite element analyses thus confirmed that the response of facades in hogging and sagging is different. Cracking in the hogging facades is mainly vertical indicating the predominant influence of bending effects. The amount of base relative displacement at which the facades reach the cracked state is not influenced significantly by differing amounts of facade openings or the overall dimensions of the facade but is generally consistently in the range of in the range of $\Delta/L = 0.03-0.05\%$. These conclusions are used below to develop a new constitutive model for surface beams which represents the behaviour of masonry facades.

5.4 Development of equivalent masonry beam model

5.4.1 Introduction

This section describes the development of a new constitutive model for surface beams which includes characteristics such that it can better model the particular actions of masonry facades when responding to ground movements. The model is called the *equivalent masonry beam* (EMB) model and is based on the equivalent elastic beam model described in section 4.3. Two approaches are considered: the main approach where equivalent masonry beams in hogging lose their bending stiffness; and an alternative approach where alternative equivalent masonry beams are given a reduced initial stiffness which increases if they go into sagging.
5.4.2 Conclusions from masonry facade analyses

The finite element analyses in section 5.3.6 above demonstrated the particular responses of masonry facades subjected to ground movements in a variety of situations. The main conclusions relevant to the development of a constitutive model for beams to represent these masonry features are that:

- Cracks develop in masonry facades as the magnitude of the strains caused by the development of curvature of the facade increases;
- Once cracked, masonry facades lose their stiffness; and
- Facades in sagging, as they crack, develop stress arches within the facade allowing them to span sagging displacements, but facades in hogging exhibit predominantly vertical cracks and do not form arches leaving any heave relatively unopposed by actions within the facade.

5.4.3 Development of EMB model

The constitutive model described here for equivalent masonry beams is based on the conclusions above. This main approach involves reducing the bending stiffness of beams going into hogging. The model has the following key components.

The equivalent masonry beam model is shown in figure 5.17. The beam has initial elastic properties based on the equivalent elastic beam method, derived from the geometry of the facade and the amount of openings as described in section 4.3. These are the flexural rigidity $EI^*$, the shear stiffness $GA^*$ and the axial stiffness $EA^*$.

If ground displacements develop hogging curvature in the beam, once a critical curvature, $\kappa_{\text{crit}}$, is reached, the flexural rigidity is reduced to a small residual value to represent the loss of stiffness caused by cracking of the facade. It is not reduced completely to zero as this would cause numerical complications. The flexural rigidity is thus reduced to $f_b EI^*$ where $f_b$ is a residual stiffness factor.
Figure 5.17: Equivalent masonry beams in bending

Flexural rigidity in the equivalent masonry beam model is not reduced for facades in sagging but remains set at $EI^*$. There is thus a non-linear elastic relationship in bending as shown in figure 5.17. This closely resembles the masonry facade constitutive model described by Liu (1997).

Another similarity to the Liu (1997) masonry facade model included here in the equivalent masonry beam model is the gradual reduction of stiffness for the beam in hogging. Gradual reduction of stiffness is included to provide a smooth transition from the ‘uncracked’ to the ‘cracked’ state (from flexural rigidity $EI^*$ to $f_b EI^*$) for the masonry beams. This represents the physical situation where, as individual cracks develop due to increasing hogging curvature, the overall stiffness of the facade reduces. The inclusion of a smooth curve rather than a bi-linear approach also aids numerical stability during calculation.

The equations governing the gradual reduction of stiffness are similar to those used by Liu (1997) and comprise the moment-curvature relationship and its derivative

$$M = EI \kappa \left( \frac{\kappa_{crit} + f_b \kappa}{\kappa_{crit} + \kappa} \right)$$  \hspace{1cm} (5.6)

$$\frac{dM}{d\kappa} = EI \left[ \frac{1 - f_b}{\left( 1 + \frac{\kappa}{\kappa_{crit}} \right)^2} + f_b \right]$$  \hspace{1cm} (5.7)
In tunnelling simulations with staged tunnel construction, curvature of surface beams representing a building may reverse during the calculation. This might occur when transient curvatures caused by the passage of tunnel construction underneath a building are hogging but the final curvature is sagging (or vice versa). In this case, the stress path retraces the non-linear curve shown in figure 5.17.

Shear stiffness is not reduced in hogging or sagging, but takes the value as determined by the equivalent elastic beam method. This is based on the assumption that the beam’s bending stiffness is lost once cracked as there is no possibility of tensile stress transfer across cracks, but that shear stiffness is retained despite the cracking due to interlock between ragged vertically cracked surfaces. Axial stiffness is also assumed to be unaffected by hogging or sagging displacements. This approach could be considered simplistic, but the equivalent masonry beam model has been developed in this way firstly to capture the principal mechanisms (cracking due to bending in hogging causing loss of bending stiffness, and arching in sagging) and secondly, to avoid over complication by including other relatively minor details (such as potential changes to shear or axial stiffness due to cracking).

5.4.4 Development of Alternative EMB model

This section describes an alternative constitutive model developed for the equivalent masonry beams. The alternative approach takes the opposite starting point to that in section 5.4.3: initially the flexural rigidity of the beam is a low value (as if the facade has already cracked). This value, $EI_B^*$, represents the stiffness value determined by the equivalent elastic beam approach reduced by the residual stiffness factor, $f_b$, as follows

$$EI_B^* = f_b EI^*$$  \hspace{1cm} (5.8)

If the facade goes in to hogging, this reduced rigidity is maintained. In sagging however, the bending stiffness increases gradually. Under this approach the value that the beam
stiffness approaches in sagging, is

\[
\left( \frac{1}{f_b} \right) EI_B^* = EI^*
\]

which is the value of bending stiffness, under by the equivalent elastic beam approach.

The alternative approach is less representative of the physical situation evident for masonry facades than the main approach which is more closely based on the conclusions from the literature and facade analysis given in section 5.4.2. That is: stiffness reduces as hogging curvature increases, replicating the loss of stiffness in the finite element analyses discussed above as \( \Delta/L \) increases. The alternative approach, however, is less physically meaningful as, once the formation of arches in masonry facades in sagging had occurred, they would not gain additional stiffness as \( \Delta/L \) increases. The use of the alternative approach is, however, thought to have possible advantages for numerical analysis as it may be computationally more efficient and is thus worth investigating.

The relative merits of both approaches are discussed in detail in Chapter 8 along with a discussion of appropriate values for the critical curvature \( \kappa_{crit} \) and residual bending stiffness \( f_b \) parameters.
5.4.5 Comparison with finite element results

There is no simple stiffness relationship (as for the elastic beams) to enable a comparison between an analytical assessment of the equivalent masonry beam model and the finite element results in the manner presented in section 4.3.3 for the elastic facades. An assessment of the equivalent masonry beams is performed following their numerical implementation in the OXFEM finite element program and is described in Chapters 7 and 8.

5.5 Conclusion

A new approach for using a non-linear elastic constitutive model to allow surface beams to model masonry facades of varying dimensions and amounts of openings more accurately has been introduced. This equivalent masonry beam model has a flexural rigidity which reduces to a low residual value in hogging to simulate the relatively flexible response of a cracked facade in that state. Bending stiffness is retained in sagging to simulate a rigid arching response. Axial and shear properties are linearly elastic. This model is shown to be equally valid for both a smooth or a rough base boundary condition assumption.

This approach is based on the conclusions of the discussion of masonry facades in the literature review in this Chapter as well as the results of the finite element analyses of masonry walls and facades undertaken for this study.

It is important to note that the developments described in Chapters 4 and 5 have been developed based on analyses of the response of elastic and masonry walls and facades subjected to imposed ground displacements. The results of these analyses thus do not include soil-structure interaction effects, being isolated analyses with the building alone subject to imposed displacements.

Following the implementation of new beam elements and the equivalent beam models into the finite element program OXFEM, described in Chapter 6, analyses including soil-structure interaction are undertaken in Chapter 8 where the new beams are used to represent masonry facades in three-dimensional analyses of building response to tunnelling.
Chapter 6

Beam Elements

6.1 Introduction

The primary aim of this project is to develop an improved assessment procedure for building response to tunnelling by advancing the use of beam elements to represent masonry building facades in numerical models. In this chapter, the beam theories introduced briefly in Chapter 4 are revisited and discussed in detail through a review of the theories and their finite elements in the literature. Timoshenko beam finite elements are chosen for use in this study to represent building facades. A description of their formulation, implementation in to OXFEM (including the new equivalent elastic and masonry beam models) and testing is given in this Chapter. Finally, beam elements representing building facades are subjected to imposed ground displacements their response is compared to the response of the full facades from Chapter 4.

6.2 Literature review

The simplest beam theory is the classical theory known as Bernoulli-Euler theory. Under this theory, for a beam with its centroidal axis along the $x$-axis, of cross sectional area, $A$, second moment of area (about the $y$-axis), $I$, and Young’s modulus, $E$, under the action
CHAPTER 6. BEAM ELEMENTS

Figure 6.1: Bernoulli-Euler beam theory (after Astley, 1992)

of bending moment, \( M \), shear force, \( Q \), and axial force, \( P \), the resulting displacements are \( u(x) \) and \( w(x) \) in the \( x \) and \( z \) directions respectively (Astley, 1992). The key assumption for displacements under this theory is that plane sections initially perpendicular to the centroidal axis, remain plane and perpendicular to the axis after deformation. Figure 6.1 shows a beam element based on this theory of length, \( L \), with transverse end displacements, \( w_1 \) and \( w_2 \), rotation of the end planes \( \theta_1 \) and \( \theta_2 \) and rotation of the neutral axis, \( \mu_1 \) and \( \mu_2 \) (axial displacements \( u_1 \) and \( u_2 \) are not shown). The requirement that cross-sections remain perpendicular to the neutral axis means that \( \theta_1 \) and \( \theta_2 \) equal \( \mu_1 \) and \( \mu_2 \) respectively.

The deflection equation for such a beam can be derived using the fact that \( \theta = \mu \) and \( \mu = dw/dx \) as described by Timoshenko (1957) and given as

\[
\frac{d^2w}{dx^2} = -\frac{M}{EI} \tag{6.1}
\]

The strain energy (SE) per unit length (ignoring axial effects) is thus

\[
SE/\text{length} = \frac{1}{2}EI \left(\frac{d^2w}{dx^2}\right)^2 \tag{6.2}
\]

The assumption that plane sections remain perpendicular to the centroidal axis necessarily implies that shear strain, \( \gamma_{xz} \), is zero. This in turn implies that shear stress and shear force are zero. The only loading case resulting in zero shear force is a constant bending moment,
and thus Bernoulli-Euler theory strictly holds only for this case. As Astley (1992) notes, this formulation ignoring stresses due to shear can also be used for other load cases, but is acceptable only for long slender beams as errors incurred in displacements by ignoring shear effects are of the order of $H/L^2$, where $H$ is the depth of a beam and $L$ is the length. Where a beam is relatively short or deep, shear effects can, however, be significant (Timoshenko, 1957). As discussed in Chapter 4, it is evident that shear effects are indeed significant in the analysis of building facades and such effects were considered in the derivation of the imposed displacements applied to the facades for this thesis. Shear effects are thus also required in any surface beams used to represent building facades.

An analytical beam theory with shear effects included is the theory known as deep or Timoshenko beam theory. The critical difference in Timoshenko theory is that the assumption that plane sections remain plane and perpendicular to the neutral axis is relaxed to allow plane sections to undergo a shear strain $\gamma$. The plane section still remains plane but rotates by an amount, $\theta$, equal to the rotation of the neutral axis, $\mu$, minus the shear strain $\gamma$ as shown in figure 6.2. The rotation of the cross section is thus

$$\theta = \mu - \gamma$$  \hspace{1cm} (6.3)

which by using $\mu = dw/dx$ leads to

$$\frac{d\theta}{dx} = \frac{d^2 w}{dx^2} - \frac{d\gamma}{dx}$$  \hspace{1cm} (6.4)

and the beam deflection equation (which replaces equation 6.1) becomes

$$\frac{d\theta}{dx} = -\frac{M}{EI}$$  \hspace{1cm} (6.5)

The expression for strain energy in the beam now consists of a bending and a shear term. Timoshenko theory assumes that shear strain is constant over the cross-section. In reality, the shear stress and strain are not uniform over the cross section so a shear coefficient, $k$ is introduced as a correction factor to allow the non-uniform shear strain to be expressed as a
Figure 6.2: A Timoshenko beam element (after Astley, 1992)

constant. The shear coefficient approximates the correct integrated value of strain energy due to shear \( (1/2\tau\gamma) \) as an assumed constant average or centreline value (Astley, 1992). The value of the coefficient depends on the shape of the cross section and was originally introduced by Timoshenko (1921); details regarding its derivation and appropriate values are presented below. Using the shear coefficient, average shear strain is taken as \( \gamma = Q/kAG \) and the strain energy per unit length due to bending only thus becomes

\[
SE/\text{length} = \frac{1}{2} EI \left( \frac{d\theta}{dx} \right)^2 + \frac{1}{2} kAG\gamma^2
\]  

(6.6)

The Timoshenko theory presented above can be used to develop beam elements which include shear displacements for use in finite element analyses. Numerous different finite element formulations of Timoshenko beams exist and many early types are described by Thomas et al. (1973) including the generally acknowledged starting point for Timoshenko beam development: the McCalley (1963) beams as developed by Archer (1965) with four degrees of freedom comprising \( w \) and \( \theta \) (as defined above and shown in figure 6.2) at each of two nodes. Detailed derivations for the stiffness matrix of the Archer beams are given by Przemieniecki (1968) (who notably neglects to include the shear coefficient) by utilising a displacement formulation. Davis et al. (1972) present a formulation (including the shear coefficient) for the same four degree of freedom element, based on exact static solutions.
for equilibrium of a beam with forces and moments applied only at the nodes. This leads to shear force being constant along the beam. The lateral deflection is a cubic polynomial function of \( x \) and the cross sectional rotation is a dependent quadratic function. Additional discussion of both this element and the use of various interpolation functions is given below.

A large number of other two node Timoshenko beam formulations exist in the literature, but as noted by Narayanaswami and Adelman (1974), Dawe (1978) and others, some confusion exists regarding the appropriate rotational degree of freedom to be used at the nodes. In some cases, the shear strain has been used (Thomas et al., 1973), in others the value of \( \frac{dw}{dx} \) (Severn, 1970 and Nickel and Secor, 1972) as well as various combinations of cross section rotation \( \theta \), shear strain and \( \frac{dw}{dx} \). As Dawe (1978) and Davis et al. (1972) point out: the correct rotational degree of freedom is the cross sectional rotation, \( \theta \); the use of \( \frac{dw}{dx} \) does not enable the clamped end boundary condition to be represented correctly.

Higher order Timoshenko beams, having more than four degrees of freedom, also exist in the literature. Kapur (1966) develops a beam element by considering shear and bending displacements separately, with a cubic displacement function assumed for both the bending and the shear deformation. A translational and a rotational degree of freedom is introduced at each node for both bending and shear leading to a two node, eight degree of freedom element. Davis et al. (1972) criticise the Kapur beam for its inability to couple forces and displacements correctly when adjacent elements are not colinear, due to the separation of bending and shear. In their beam discussed above, shear force is the first derivative of bending moment thus coupling bending and shear and eliminating the need for the additional degrees of freedom of Kapur (1966). Nickel and Secor (1972) propose an element with seven degrees of freedom which include \( w, \frac{dw}{dx} \) and \( \theta \) at each of two end nodes and \( \theta \) at an additional mid-element node with a cubic function for \( w \) and a quartic function for \( \theta \). Dawe (1978) presents an element with three nodes, each of which has degrees of freedom of \( w \) and \( \theta \) leading to a coupled situation with a quintic variation of \( w \) and a quartic variation of \( \theta \). Dawe concludes that the approach coupling the correct primary variables (\( w \) and \( \theta \)), is superior to an approach where independent functions are assumed as it leads to considerably fewer degrees of freedom being required for a given order of
interpolation.

Higher order elements have also been developed by Levinson (1981) and Heyliger and Reddy (1988) that account for the true shear stress distribution throughout the beam cross section and removes the need to use Timoshenko’s shear coefficient. The higher order shear stress distribution means that the Timoshenko assumption of cross sections remaining plane is relaxed to allow warping into a non-planar surface.

Timoshenko beam elements with low order interpolation functions can exhibit a very stiff response as the beams become very thin. This phenomenon is known as shear locking and is a result of inconsistencies in interpolation functions used for \( w \) and \( \theta \). Beam elements with linear interpolation for both these variables are particularly susceptible (Reddy, 1997). To overcome locking a number of techniques are proposed including reduced order integration (Prathap and Bhashyam, 1982). Reddy (1997)(three-nodes) and Kosmatka (1994)(two-nodes) give a number of examples of formulations with different order interpolations in his development of locking-free Timoshenko beam elements. An approach to choosing appropriate interpolation functions for beam elements using Hermitian and Lagrange polynomials is presented by Augarde (1997) and is discussed in section 6.4.1.

The shear coefficient, \( k \), was introduced by Timoshenko (1921 and 1922) as a correction factor (originally denoted \( \lambda \), where \( \lambda = 1/k \)) to be applied to the average shear stress and shear strain to obtain centreline values to be used in the beam theory. In his original introduction of the theory, Timoshenko gave the shear correction factor the value of 2/3 for rectangular beams, with the value depending on the shape of the cross section. As Levinson (1981) notes, from its inception, the calculation of the ‘best’ shear coefficient has become in itself a ‘small research industry’ with much debate over the appropriate definition and derivation of the value. The most generally well regarded and widely used values are due to Cowper (1966) who clarifies the original definition and derives, using three-dimensional elasticity theory, a new formula to calculate \( k \) values for any cross section. Cowper (1966) tabulates standard formulae for a variety of cross sections and for a standard rectangular
cross section gives the formula including $\nu$, the Poisson’s ratio for the material as

$$k = \frac{10(1 + \nu)}{12 + 11\nu}$$

(6.7)

Throughout the early development of Timoshenko beams, only straight elements were considered. Curved beams have, however, also been developed and are useful in analysing curved structural elements such as tunnel linings. Curved beams suffered from *membrane locking*, a phenomenon similar to the shear locking described above for straight beams. Attempts at formulating locking-free curved beam elements include those of Ramesh Babu and Prathap (1986) with two nodes and linear interpolation for all variables, and Prathap and Ramesh Babu (1986) with three nodes and quadratic interpolation. Both these formulations suffer from locking and errors due to lack of consistency between shear and membrane fields caused by poor choice of nodal degrees of freedom. Day and Potts (1990) attack this problem by utilising Mindlin plate theory (for the inclusion of shear deformation in plate elements) to develop shear capable curved beams for modelling structural components in plane strain finite element analyses. Here the plane strain versions of the Mindlin three-dimensional plate equations (as described by Zienkiewicz (1997) and Astley (1992) among others) are used by Day and Potts to formulate locking and error free curved plain strain (and axi-symmetric) beam elements. In particular, they note and clarify the confusion in the literature regarding the appropriate nodal degrees of freedom to use. Potts and Zdravkovic (1999 and 2001) give a good description of the Mindlin beam formulation and its use for two-dimensional analyses. Such beams are also used by Potts and Addenbrooke (1997) to represent buildings in plane strain in their work on the relative stiffness method for soil-structure interaction in tunnelling described in Chapter 2.

Another approach to including shear effects is the use of *hybrid* beams as described by Bakker (2000). Here, the necessity for shear flexibility is provided, but rotational degrees of freedom are condensed out of the beam elements by replacing the requirement for continuity of rotations with continuity of bending moments between adjacent beam elements. Another beam developed without rotational degrees of freedom is the overlapping formulation of Phaal and Calladine (1992) used by Augarde (1997) for tunnel lining elements. In this
approach each node acts as the centre node of one element and the end node of two other elements with each element overlapping two others. This overlapping provides continuity of slope without the need for rotational degrees of freedom.

6.3 Choice of appropriate beam element

The beam elements chosen for use in this study are based on the straight two node beam elements first described by Davis et al. (1972); the similar and more recent description of the formulation of these beams by Astley (1992), however, will be followed here.

The primary task for the beams in this research is the representation of building facades which are generally considered to be straight or able to be represented by a combination of straight sections. This removes a need for curved beam elements. It is also considered that higher order beams with more than two nodes are not required. The use of higher order beams would introduce greater incompatibility of displacements between beam and soil elements away from the nodes, as is discussed below. Higher order beams accounting for the true stress and strain distribution through a cross-section are also not required, with the Timoshenko approach to including shear considered adequate.

The aim of this thesis is to expand beam element representations of buildings into three dimensions. This means that plane strain beam formulations such as the Mindlin beams described above, which are excellent for use representing long structures such as tunnel linings or retaining walls in plane strain but cannot be used for a discreet structure above a tunnel in three dimensions, will not be adequate for this study. Fully three-dimensional beam elements (often known as frame elements) are thus required; the capabilities for bending deformation in two planes, axial deformation and torsion about their own axis are all necessary. Such beams need to be able to be located anywhere in three-dimensional space. The Timoshenko theory described above for a simple two node plane beam with no axial effects is therefore expanded for use in formulating such three-dimensional elements.

This research aims to represent facades as equivalent beams on the surface. A recent
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approach to modelling masonry facades as equivalent frame systems, comprised of beams arranged in a grid was recently described by Roca et al. (2005). This is an intermediate approach between the full modelling of masonry facades and the use of surface beams.

From the analysis in Chapter 4, the necessity to independently specify shear \((GA)\) and bending \((EI)\) rigidities for beam elements representing a building facades was made clear. The beam element formulation for this study is chosen to enable such specification. Notably, this has not historically been possible in some popular commercial finite element codes. The version of ABAQUS available at the start of this research (version 5.8, released 1998), contained Timoshenko beam elements that could be used in plane or 3D analyses, but did not allow separate specification of \(EI\) and \(GA\); the shear rigidity was simply assumed to be half the bending rigidity. This would be unacceptable for use on this project, given the newly developed methods of determining beam properties with un-linked rigidity values. By late 2002, after this research was underway, a new version of ABAQUS (version 6.3, released September 2002) included the facility for the user to specify independent values for shear and bending rigidity, indicating that the software developers had also decided that independent specification of these properties was a useful attribute.

6.4 Formulation of Timoshenko beam elements

The beam element implemented is shown in figure 6.3. It is straight with six degrees of freedom at each node, three translations and three rotations. The formulation of the element stiffness matrix for this beam comprises contributions from four separate actions: one axial, one torsional and two bending, one about each of two orthogonal local axes. The contributions from axial and torsional effects to the element stiffness are formulated in the conventional manner and are detailed in Appendix B. The bending contributions to the stiffness matrix assume Timoshenko beam theory as described above. The contribution of bending in each orthogonal direction can be considered independently as the principal axes of the beam are chosen to coincide with the local \(x-y\) and \(x-z\) planes (Przemieniecki, 1968). These are denoted the \(out-of-plane\) and \(in-plane\) directions respectively.
6.4.1 Interpolation functions

In this section the bending shape functions in the $x$-$z$ plane are formulated. The derivation, based on Astley (1992), follows the common convention (used by Augarde (1997) in the original formulation of Bernoulli-Euler beams in OXFEM) of formulating the shape functions in terms of the non-dimensional variable $\bar{x}$ where

$$\bar{x} = \frac{x}{L}$$  \hspace{1cm} (6.8)

with element length, $L$, such that the two-node beam has nodes at $\bar{x} = 0$ and 1. As for a two-node Bernoulli-Euler beam element, the lateral displacement, $w(\bar{x})$, is expressed as

$$w(\bar{x}) = N_1(\bar{x})w_1 + N_2(\bar{x})\mu_1 + N_3(\bar{x})w_2 + N_4(\bar{x})\mu_2$$  \hspace{1cm} (6.9)

where $N_i(\bar{x})$ are shape functions associated with the displacements ($w$) and rotations ($\mu$) at the nodes. For a Timoshenko beam, however, we must also include variation for shear, $\gamma$. The simplest approach, from Astley (1992), is to assume that $\gamma$ is a constant equal to $\gamma_0$. This zeroth order interpolation therefore assumes a constant shear stress and shear force within each element. As $\gamma_0$ is an independent variable, the element thus has a fifth degree of freedom in bending. Rewriting the expression for lateral displacement using equation 5.3 to replace $\mu$ in terms of $\theta$ and $\gamma_0$ thus gives

$$w(\bar{x}) = N_1(\bar{x})w_1 + N_2(\bar{x})(\theta_1 + \gamma_0) + N_3(\bar{x})w_2 + N_4(\bar{x})(\theta_2 + \gamma_0)$$  \hspace{1cm} (6.10)
which upon rearranging leads to

$$\begin{align*}
  w(\bar{x}) &= N_1(\bar{x})w_1 + N_2(\bar{x})\theta_1 + N_3(\bar{x})w_2 + N_4(\bar{x})\theta_2 + N_5(\bar{x})\gamma_0 \\
  \text{(6.11)}
\end{align*}$$

where $N_5(\bar{x}) = N_2(\bar{x}) + N_4(\bar{x})$. In standard matrix form this reduces to

$$\begin{align*}
  w(\bar{x}) &= \mathbf{N}_b^e \mathbf{d}_{b\gamma}^e \\
  \text{(6.12)}
\end{align*}$$

where the bending shape functions and degrees of freedom for an element are respectively

$$\begin{align*}
  \mathbf{N}_b^e &= \{N_1 \ N_2 \ N_3 \ N_4 \ N_5\} \\
  \mathbf{d}_{b\gamma}^e &= \{w_1 \ \theta_1 \ w_2 \ \theta_2 \ \gamma_0\}^T \\
  \text{(6.13) \ (6.14)}
\end{align*}$$

The shape functions $N_i$ are generated by the method described by Augarde (1997) using Hermitian interpolating polynomials. Hermitian interpolation involves the use of both an expression for ordinate information at known points and its derivative to determine approximate values between known locations (Cook et al., 1989). Augarde notes that for beam elements, the requirements for shape functions satisfy the definition of Hermite polynomials as the rotations can be considered as the first derivatives of the lateral displacements under the assumption of small displacement theory. These Hermite polynomials can be derived from Lagrange polynomials. This is particularly useful for developing finite element programs, as Lagrange polynomials are used for the shape functions for continuum elements and the axial effects in beam elements, and are thus already in the program code.

For beam elements, one-dimensional interpolation is required along the element centreline necessitating the use of one-dimensional Hermite polynomials. These can be obtained directly from Langrange polynomials using

$$\begin{align*}
  H_{0i}^1 &= [1 - 2(x - x_i)L_i'(\bar{x})][L_i(\bar{x})]^2 \\
  H_{1i}^1 &= (x - x_i)[L_i(\bar{x})]^2 \\
  \text{(6.15) \ (6.16)}
\end{align*}$$
where \( H_{ji} \) is a Hermite polynomial of level \( r \) and derivative order \( j \) at node \( i \). The level of the Hermite polynomial indicates the highest order derivative used in the interpolation. \( L_i(\bar{x}) \) is the one dimensional Lagrange interpolating polynomial of degree \((n - 1)\)

\[
L_i(\bar{x}) = \prod_{j=1,j\neq i}^{n} \frac{x - x_i}{x_i - x_j}
\tag{6.17}
\]

and \( L'_i(\bar{x}) \) is the first derivative. The derivations of the particular interpolating polynomials for the shape functions \( N_1(\bar{x}) \) to \( N_4(\bar{x}) \) for the Timoshenko beam being implemented here are given in Appendix B.

### 6.4.2 Formation of element stiffness matrix

Assembly of the bending contribution to the element local stiffness matrix including the shear degree of freedom follows the conventional method. As is usual for beam elements, the strain-displacement matrix \( B \), is used such that the product \( Bd \) gives beam curvatures. The strain-displacement matrix relationship is thus \( B = d^2N/dx^2 \), the use of which gives

\[
B^e_b = \frac{1}{L^2} \begin{bmatrix} N''_1(\bar{x}) & N''_2(\bar{x}) & N''_3(\bar{x}) & N''_4(\bar{x}) & N''_5(\bar{x}) \end{bmatrix}
\tag{6.18}
\]

where \( B^e_b \) is the strain-displacement matrix for an element in bending and \( N''_i(\bar{x}) \) are the second derivatives with respect to \( \bar{x} \) of the bending shape functions (from Appendix B).

The local element stiffness matrix including the shear degree of freedom, \( K^e_{br} \), is thus formed in the usual manner using

\[
K^e_{br} = L \int_0^1 B^e_b D B^e_b d\bar{x}
\tag{6.19}
\]

where \( D \) is the material property matrix, including \( EI \), with Young’s modulus \( E \) and second moment of area \( I \) and \( kGA \) with shear coefficient \( k \), shear modulus \( G \) and cross-
sectional area $A$. This leads to

$$K_{b\gamma}^e = \begin{bmatrix}
12a & 6aL & -12a & 6aL & 12aL \\
6aL & 4aL^2 & -6aL & -2aL^2 & 6aL^2 \\
-12a & -6aL & 12a & -6aL & -12aL \\
6aL & 2aL^2 & -6aL & 4aL^2 & 6aL^2 \\
12aL & 6aL^2 & -12aL & 6aL^2 & (12aL^2 + b)
\end{bmatrix}$$

(6.20)

where $a = EI/L^3$ and $b = kGAL$.

The orthogonal plane in bending gives an identical stiffness contribution but in the second plane, assuming, as described above, that the out-of-plane and in-plane directions coincide with the principal directions of bending. Summing of the contributions of the two bending planes, the axial and the torsional effects leads to a $14 \times 14$ local element stiffness matrix, $K_{b\gamma}^e$, including the shear terms. A Timoshenko beam element formulated in this manner with 14 degrees of freedom, could not be used in place of a Bernoulli-Euler theory based frame element which would have only 12 degrees of freedom, without renumbering and modifying the surrounding mesh to accommodate the extra degrees of freedom (Astley, 1992). It is thus common practice to condense out the extra two shear degrees of freedom.

For each of the individual $5 \times 5$ bending stiffness matrices including shear, the condensation process involves extracting out the shear row of the matrix and solving for the constant shear strain $\gamma_0$. This is a simple process as $\gamma_0$ is unique to each element, and is only connected to the four other members of the array of bending degrees of freedom $d_{b\gamma}^e$. If we assume that loads are applied only at the nodal points, there would be no forces appearing in the force-displacement equation formed using the $\gamma_0$ row of the stiffness matrix. Solving for the constant shear strain under this assumption thus gives

$$\gamma_0 = \frac{1}{12aL^2 + b}[-(12aL)w_1 - (6aL^2)\theta_1 + (12aL)w_2 - (6aL^2)\theta_2]$$

(6.21)

If we let $c = 6aL/(12aL^2 + b)$, the relationship between the degrees of freedom can be
expressed as

\[
\begin{bmatrix}
w_1 \\
\theta_1 \\
w_2 \\
\theta_2 \\
\gamma_0
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-2c & -cL & 2c & -cL
\end{bmatrix}
\begin{bmatrix}
w_1 \\
\theta_1 \\
w_2 \\
\theta_2
\end{bmatrix}
\]

(6.22)

or in matrix notation

\[d_e^{\gamma} = T_b d_b^e\]

(6.23)

The condensed stiffness matrix for bending in one plane is thus

\[K^e_b = T_b^T K^{\gamma}_b T_b\]

(6.24)

In the formulation implemented into OXFEM, this condensation operation is performed on the full 14 × 14 matrix making use of an expanded condensation matrix, \(T\), yielding a 12 × 12 element local stiffness matrix. Transforming the local element stiffness matrix for the beams oriented in three dimensional space to a global element stiffness matrix for assembly and solution in the global domain is by the conventional method of pre and post-multiplication by a transformation matrix comprising direction cosines derived from the global coordinates of the element nodes.

The determination of the stiffness matrix in OXFEM is by one-dimensional Gaussian quadrature. With the shape functions including terms of order three and below and the \(B\) matrix containing terms of order one or lower, the solution for the stiffness matrix is given exactly by two-point Gauss quadrature.
6.4.3 Compatibility of elements

The Timoshenko beams implemented as described above are for use in conjunction with ten-noded tetrahedral continuum elements to model the ground. The two-noded surface beams will therefore lie two to an edge of a tetrahedron at the ground surface. Displacements interpolated between the nodes of the beam elements will differ from those along the edge of the continuum element. The different element types therefore exhibit incompatible displacements at locations other than nodal points, but compatible displacements at each node. This incompatibility results from the transverse displacement of the beam elements being described by a cubic interpolation function (the beam nodes having four contributing degrees of freedom: two rotations and two displacements) while the displacement along an edge of a soil element is described by a quadratic function (as there are three nodes each with one displacement degree of freedom).

The amount of lack of conformity (as measured by the difference in strain energy due to transverse displacements) introduced by the arrangement of having two two-noded Timoshenko beams meeting a 10-noded tetrahedral soil element side is lower than if Timoshenko beams with three nodes (two end nodes and a mid-side node) were to be used. It is, however, higher than if the two-noded Timoshenko beams were used in conjunction with 20-noded tetrahedral soil elements. The use of these soil elements, however, would lead to unnecessarily long run times for analyses due to the additional degrees of freedom. Ten-noded tetrahedra are thus used for the soil. The incompatibility of the chosen soil and beam elements away from the nodal points is not considered to introduce significant detrimental effects as long as element lengths are kept sensibly short at locations where the two element types meet.

6.4.4 Constitutive models and stress updating

The two constitutive models introduced in Chapter 4 are for use with the Timoshenko beam elements. The equivalent elastic beam model is linearly elastic with beam properties described in section 4.3 and is formulated in OXFEM in the conventional manner. The
The equivalent masonry beam model, however, is non-linear as described in section 5.4. The in-plane bending rigidity is dependent on the direction (hogging or sagging) and magnitude of the curvature of the beam. When using this constitutive model, the assignment of beam properties and stress resultant (moment) updating following solution for the displacements becomes more complex.

The solution techniques used in the OXFEM finite element code are fully described in section 7.5, however it is useful to briefly describe the approach adopted for the Timoshenko beams here. For non-linear material, a Modified Euler solution scheme is used in OXFEM where loading is applied incrementally. For each incremental step, the stiffness matrix for a Timoshenko beam element is dependent on the strain after the previous increment. The current bending strains are thus used to calculate the in-plane bending rigidity at each step according to equation 5.7 prior to the solution of displacements.

Following solution for displacements at each step, updating of strains and stresses takes place. Strains, $\epsilon$, at a beam node are updated incrementally according to

$$\epsilon^i = \epsilon^{(i-1)} + B \Delta d$$

(6.25)

where $i$ is the current step number and $d$ are the nodal displacements.

Stress updating for the Timoshenko beams is done on a total stress basis. Instead of the incremental approach adopted for other non-linear materials in OXFEM where

$$\sigma^i = \sigma^{(i-1)} + D \Delta \epsilon$$

(6.26)

the approach adopted for the Timoshenko beams makes use of the explicit total stress-strain relationship. For the non-linear relationship for in-plane bending, equation 5.6 is used to give the bending stress (moment) using the already updated current strain.
6.5 Implementation and testing of Timoshenko beams in OXFEM

The Timoshenko beams as described above were implemented into the OXFEM finite element program. This involved both the inclusion of new FORTRAN 90 code and the updating and amending of many existing subroutines within the program. Once implemented, testing and validation of the Timoshenko beam elements were undertaken. Validation of the Timoshenko beams included the following test problems, shown in figure 6.4, undertaken with beams with elastic properties except where noted:

1. Plane simply supported beam with mid-span point load;
2. Simply supported beam in three-dimensional space with multiple orthogonal mid-span point loads;
3. Plane cantilever beam with point load;
4. Plane simply supported beam with distributed load;
5. String of beams on an elastic soil;
6. Footing of beams on an elastic soil; and
7. Plane simply supported beam with equivalent masonry properties with distributed load.

For the first three test problems the finite element results for displacement under the point load exactly match the analytical answers (using Timoshenko beam theory) using only one element in problem 3 and two elements in each of problems 1 and 2. This is to be expected as the order of the interpolating polynomial for transverse displacement matches the order of the expression for the transverse displacement field for these problems, both being cubic functions. The input parameters and solutions to these tests are given in table 6.1.

For test problem 4, the finite element analysis does not provide an exact solution (as the order of the displacement function is higher than the order of the interpolating polynomial),
Figure 6.4: Test problems for validation of Timoshenko beam elements
### Table 6.1: Timoshenko beam test problems

#### Test Problem 1: Input Parameters

<table>
<thead>
<tr>
<th>A</th>
<th>L</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>7240mm²</td>
<td>3000mm</td>
<td>1.610 × 10⁸mm⁴</td>
<td>3.380 × 10⁸mm⁴</td>
</tr>
<tr>
<td>E</td>
<td>G</td>
<td>k</td>
<td>F</td>
</tr>
<tr>
<td>2.00 × 10⁵MPa</td>
<td>8.00 × 10⁴MPa</td>
<td>0.667</td>
<td>1.00 × 10⁶N</td>
</tr>
</tbody>
</table>

#### Test Problem 1: Results

<table>
<thead>
<tr>
<th>Analytical Expression</th>
<th>Analytical Result</th>
<th>FE Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_y = \frac{PL^3}{48EI} + \frac{PL}{4kAG} )</td>
<td>19.411mm</td>
<td>19.411mm</td>
</tr>
</tbody>
</table>

#### Test Problem 2: Input Parameters (both in-plane and out-of-plane)

<table>
<thead>
<tr>
<th>A</th>
<th>L</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>7240mm²</td>
<td>3000mm</td>
<td>1.610 × 10⁸mm⁴</td>
<td>3.380 × 10⁸mm⁴</td>
</tr>
<tr>
<td>E</td>
<td>G</td>
<td>k</td>
<td>F</td>
</tr>
<tr>
<td>2.00 × 10⁵MPa</td>
<td>8.00 × 10⁴MPa</td>
<td>0.667</td>
<td>1.00 × 10⁶N</td>
</tr>
</tbody>
</table>

#### Test Problem 2: Results

<table>
<thead>
<tr>
<th>Analytical Expression</th>
<th>Analytical Result ((v_y) and (w_z))</th>
<th>FE Result ((v_y) and (w_z))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_y = w_z = \frac{PL^3}{48EI} + \frac{PL}{4kAG} )</td>
<td>19.411mm</td>
<td>19.411mm</td>
</tr>
</tbody>
</table>

#### Test Problem 3: Input Parameters

<table>
<thead>
<tr>
<th>A</th>
<th>L</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>7240mm²</td>
<td>1000mm</td>
<td>1.610 × 10⁸mm⁴</td>
<td>3.380 × 10⁸mm⁴</td>
</tr>
<tr>
<td>E</td>
<td>G</td>
<td>k</td>
<td>F</td>
</tr>
<tr>
<td>2.00 × 10⁵MPa</td>
<td>8.00 × 10⁴MPa</td>
<td>0.667</td>
<td>1.00 × 10⁶N</td>
</tr>
</tbody>
</table>

#### Test Problem 3: Results

<table>
<thead>
<tr>
<th>Analytical Expression</th>
<th>Analytical Result</th>
<th>FE Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_y = \frac{PL^3}{3EI} + \frac{PL}{kAG} )</td>
<td>12.942mm</td>
<td>12.942mm</td>
</tr>
</tbody>
</table>

#### Test Problem 4: Input Parameters

<table>
<thead>
<tr>
<th>A</th>
<th>L</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>7240mm²</td>
<td>3600mm</td>
<td>1.610 × 10⁸mm⁴</td>
<td>3.380 × 10⁸mm⁴</td>
</tr>
<tr>
<td>E</td>
<td>G</td>
<td>k</td>
<td>q</td>
</tr>
<tr>
<td>2.00 × 10⁵MPa</td>
<td>8.00 × 10⁴MPa</td>
<td>0.667</td>
<td>3.00 × 10²N/mm</td>
</tr>
</tbody>
</table>

#### Test Problem 4: Results

<table>
<thead>
<tr>
<th>Analytical Expression</th>
<th>Analytical Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_y = \frac{qL^4}{384EI} + \frac{qL^2}{8kAG} )</td>
<td>21.634mm</td>
</tr>
</tbody>
</table>
Figure 6.5: **Convergence of finite element result and analytical solution for test problem 4**

but converges to the analytical solution with increasing mesh density. Table 6.1 gives the input parameters and analytical solution for this test while the convergence of the finite element results to the analytical solution with increasingly finer meshes is shown in figure 6.5. Convergence to within 2% of the analytical solution occurs for meshes with approximately 50 nodes and to within 1.4% for meshes with 70 nodes.

There appears to be no precise analytical solution to the problem in test 5 for a Timoshenko beam on elastic soil. A comparison was thus made between the beam displacement for this test using OXFEM and the displacement calculated by the program ABAQUS for the same test. The input parameters for the Timoshenko beams and the elastic soil as well as the results are given in table 6.2. Linear elastic soil and elastic Timoshenko beam elements were specified in each program. The Timoshenko beam formulations were similar, with independent specification of bending and shear properties. The displacements under the point load at the centre of the beam using the two programs were within 0.75%.

Problem 6 was also analysed by comparing the finite element results obtained using OXFEM and ABAQUS. The beam and soil properties as well as the load magnitudes are identical to those given for test problem 5. The sides of the square footing were all 3.0m long and
Table 6.2: Timoshenko beam test problem 5

<table>
<thead>
<tr>
<th>Beam Input Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A</strong></td>
<td>7240mm²</td>
</tr>
<tr>
<td><strong>L</strong></td>
<td>3000mm</td>
</tr>
<tr>
<td><strong>I</strong></td>
<td>1.610 × 10⁸mm⁴</td>
</tr>
<tr>
<td><strong>J</strong></td>
<td>3.380 × 10⁸mm⁴</td>
</tr>
<tr>
<td><strong>E</strong></td>
<td>2.000 × 10⁵MPa</td>
</tr>
<tr>
<td><strong>G</strong></td>
<td>8.000 × 10⁴MPa</td>
</tr>
<tr>
<td><strong>k</strong></td>
<td>0.85</td>
</tr>
<tr>
<td><strong>F</strong></td>
<td>1.000 × 10⁶N</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Soil Input Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>E</strong></td>
<td>1.000 × 10⁵kPa</td>
</tr>
<tr>
<td><strong>G</strong></td>
<td>3.356 × 10⁴kPa</td>
</tr>
<tr>
<td><strong>ν</strong></td>
<td>0.49</td>
</tr>
<tr>
<td><strong>Dimensions</strong></td>
<td>6.0 × 6.0 × 3.0m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Midpoint Displacement Results</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OXFEM</strong></td>
<td>5.585mm</td>
</tr>
<tr>
<td><strong>ABAQUS</strong></td>
<td>5.543mm</td>
</tr>
</tbody>
</table>

the footing was centred on the surface of the soil block which had the same dimensions as test 5. Midside vertical displacements under the point loads for OXFEM and ABAQUS were 5.43mm and 5.39mm respectively (a difference of 0.8%). The vertical displacements of the corner nodes were 2.40mm and 2.42mm respectively (also a difference of 0.8%).

The final test, problem 7, verifies the implementation of the non-linear constitutive model for the Timoshenko beams. It is a repeat of test 4, undertaken with 72 beam elements, but with equivalent masonry beams rather than elastic. Two tests were undertaken: test 7a where the equivalent masonry beam constitutive model described in section 5.4.3 is used; and test 7b using the alternative model as described in section 5.4.4.

The masonry properties for test 7a are, \( \kappa_{crit} = 1.0 \times 10^{-2} \), and \( f_b = 0.1 \) with other properties and loads for the test the same as in table 6.1 for test 4. Figure 6.6 shows the moment-curvature relationship for test 7a at the central node of the beam under a uniform load causing hogging applied over 10 incremental load steps. It also shows a theoretical bi-linear moment-curvature relationship, with no gradual reduction of stiffness for comparison. The flexural rigidity of the beam reduces as the hogging curvature develops as expected.

The masonry properties for test 7b are, \( \kappa_{crit} = -1.5 \times 10^{-2} \), \( f_b = 0.1 \), \( E_B = 2.0 \times 10^3 \), the load applied is 30N/mm and other properties are the same as in table 6.1 for test 4. Figure 6.7 shows the moment curvature relationship for test 7b, at the central node of
Figure 6.6: **Test 7a: Beam response with equivalent masonry beams**

the beam under a uniform load causing sagging that is applied over 100 incremental load steps. It also shows a bi-linear moment-curvature relationship, with no gradual increase of stiffness for comparison as well as the theoretical moment-curvature relationship if loading had continued beyond the 100 steps. The flexural rigidity of the beam increases as the sagging curvature develops as expected.

Following these problems it was concluded that the 12-degree of freedom Timoshenko beams were successfully implemented in OXFEM.

### 6.6 Finite element analyses with beam elements

As a more substantial check (before full 3D tunnelling analyses) of the performance of the beam elements and the robustness of the methods developed in Chapter 4 for assigning properties to the beams, a number of 2D analyses are undertaken. The applied displacement analyses of the elastic facades in Chapter 4 are repeated here with the facades replaced with beam elements and the results compared to the full facade analyses. In addition, new analyses with Gaussian displacements are described where the response of a facade and beams are compared. The beam element properties are determined using the equivalent beam methods developed in Chapter 4.
6.6.1 Beams elements with imposed displacement

The finite element analyses undertaken here involve applying the displacement described by equation 4.5 in sagging to a building with a smooth base. For these analyses, the building facade is represented by a string of elastic Timoshenko beam elements running between the nodes along the base of the facade, rather than the full facade. For these test problems only the in-plane, two-dimensional response of the beams is considered.

For each of the facades noted in table 4.2 and shown in Appendix A, the properties for the beams used here to represent them are determined by applying the equivalent elastic beam method described in section 4.3. This provides the effective cross sectional area $A^*$ and second moment of area $I^*$ given in table 4.3 for facades with a smooth base. The Young’s modulus, $E$, and shear modulus, $G$, for the beams are the same as for the elastic facades. The shear coefficient chosen is Timoshenko’s original value of $k = 2/3$ for consistency with the value used in the derivation of the displacement in equation 4.5.

As discussed in section 4.2.5, the output from OXFEM at nodal points is in terms of forces and displacements. To determine the equivalent stress along the base of the beams a similar approach to section 4.2.5 is adopted, however as the beams are two-noded, the nodal forces equivalent to a distributed load on the element differ from those shown in figure 4.7 for the
three-noded side of a facade element. For a two-noded beam the equivalent nodal forces for a uniformly and linearly distributed load are given in figure 6.8 (Cook et al., 1989), which are used to derive an expression similar to equation 4.6 for the beams.

The results of the finite element analyses are shown in figure 6.9. This figure compares the normalised stiffness ratio (NSR) for the finite element analyses of the beams \( (K_{febeam}/K_{beam}) \) with the previously detailed (in figure 4.16) NSR for the facade analyses \( (K_{fe}/K_{beam}) \) with the same applied displacement. The labels for the facade and beam families presented in each sub-figure refer to those given in table 4.2 and Appendix A.

It can be seen that the response of the full facades as characterised by the NSR is replicated reasonably well in the analyses with the Timoshenko beams. The agreement is particularly good for facades with no openings, for all values of \( L/H \), with the observed NSR of the beams closely matching the NSR of the facades and reducing for the tests with low \( L/H \) as expected. The trend evident in the facade analyses whereby the increasing amount of windows reduces the stiffness of the facades is also replicated in the beam analyses, with the NSR reducing as the amount of windows increases in the facade that the beam represents. The beams, however appear to react more stiffly than the facades for these analyses with a high percentage of openings. The NSRs for the beams in these analyses can be seen to be higher than those of the facades. The reason for this is that the equivalent properties applied to the beams result in them having a slightly higher stiffness than the facades. This is evident in figure 4.17 and is discussed in section 4.3.3.

Overall the results indicate that the beams with properties assigned using the equivalent elastic beam method replicate the response of the facades to an applied displacement...
Figure 6.9: Observed finite element Beam and Facade response to applied displacement (smooth based)
reasonably well. Such a conclusion, however, is to be expected as method to assign the beam properties was developed based on identical analyses to those described above. It is therefore desirable to test the beams under a different set of displacements.

6.6.2 Beams and facades with Gaussian displacement

To ensure that the Timoshenko beam formulation and equivalent elastic beam properties are robust, a series of finite element analyses is undertaken where Gaussian displacements are applied to the beams and the full facades. A Gaussian displacement profile is the classic profile assumed to describe the surface settlement trough arising from tunnelling at a greenfield site as described in section 2.2. The equation for vertical settlement is given as equation 2.1. The magnitude of the maximum vertical settlement chosen is $S_{\text{max}} = 0.01 \text{m}$.

For these analyses three different Gaussian displacement profiles are applied by varying the position and size of the building relative to the tunnel as shown in figure 6.10. These displacements are applied to the $20 \times 8 \text{m}$ facades denoted A13 (no openings) and A33 (18.75% openings) and beams representing these facades.

The results are as shown in figure 6.11. The results show the stress at the same nodal locations for the facades and the beams and indicate that the beams respond to the applied displacements in an acceptably similar manner. The jagged nature of the response of the facades with windows can be seen to be smoothed by the beams. Another difference is that while all the tests show good agreement between the beam and the facade results in the middle of the facade, the beams all exhibit larger stresses at the end points. This is
thought to be due to difference in dimensionality between the beams (1D line elements),
and the facades (2D planes) with stresses throughout the whole facade.

6.7 Conclusion

Timoshenko beams have been formulated and implemented successfully into the OXFEM
finite element program. Two constitutive models have been implemented for the beam ele-
ments: an elastic model and a non-linear no hogging model. The beam elements have been
tested and the response of the beams to simple applied displacements has been compared
favourably to the response of full facades. While useful for confirming the robustness of the
beam implementation and the new equivalent elastic and masonry beam models, the simple
applied displacement approach is merely a pre-cursor to the main focus of this research:
the use of beam elements to represent masonry buildings in three-dimensional numerical
models of tunnel construction. It is on this problem that the remaining chapters of this
thesis concentrate.
Figure 6.11: Facades E13 (no windows) and E33 (18.75% windows) and equivalent beam response to imposed Gaussian displacements (from figure 6.10)
Chapter 7

Composition of three-dimensional numerical model

7.1 Introduction

This chapter describes the composition of three-dimensional numerical models for analysis of building response to tunnelling. The procedure for mesh generation, the constitutive models used and the solution technique employed are outlined along with a description of the use of shared memory parallel computing. The numerical simulation of the tunnelling process is described. The computational task for all the three-dimensional analyses was performed using the Oxford in-house finite element software OXFEM.

7.2 Mesh generation and pre-processing

The pre-processing tasks necessary to prepare a three-dimensional numerical model for analysis in the OXFEM finite element program are similar for all the numerical models analysed in this thesis. They include geometric wire frame modelling, mesh generation, and production of the OXFEM input file.

Geometric modelling and mesh generation for all three-dimensional problems was under-
CHAPTER 7. COMPOSITION OF 3D NUMERICAL MODEL

Figure 7.1: **Wire frame model with tunnel stages and building footprint**

taken using the *modelling* module within the I-DEAS software described in Chapter 4 (in relation to the 2D work). The generation of a model, following the initial choice of geometric layout, commences with wire frame modelling of the soil and tunnel. The face of the soil block is drawn on a vertical plane and is then extruded in the direction of tunnelling to create a 3D block of soil. Building footprints are drawn on the soil surface. The cross section of the tunnel and its lining is then drawn on the soil face and extruded through the soil progressively to create partitioned volumes corresponding to the desired sequential tunnelling stages. Figure 7.1 illustrates this process, showing the wire frame generated for a tunnel with six construction stages under a building oblique to the tunnel alignment.

Once the wireframe model is complete, each region within the model is meshed using the I-DEAS *meshing* module. Element lengths are refined so that areas of local interest, such as the building footprint and the tunnel, have smaller elements than remote areas. Such an approach aims to strike a balance between an appropriate level of detail in areas of interest and the minimisation of the total number of degrees of freedom in the model.

Elements used for the soil and tunnel lining are 10-noded tetrahedral elements. These are chosen as it is considered that for modelling incompressible, undrained material behaviour in three dimensions using exact integration, the tetrahedron family of elements is superior to both the serendipity and Lagrangian cube families (Bell et al., 1991). 20-noded tetrahedral elements are considered to be the most suitable tetrahedral element type. Fewer
20-noded elements than 10-noded ones are required to model a problem with a similar level of accuracy. Problems with complex geometries, such as those for this thesis with tunnels and building footprints can require a large number of small elements. Modelled with 20-noded elements, such problems can have very large numbers of degrees of freedom, however, leading to computational inefficiency. For the analyses in this thesis, therefore, 10-noded tetrahedral elements are used for the soil as they are computationally more efficient while still being suitable for modelling incompressible material. The choice of elements for the tunnel lining is discussed in detail below.

Node positions known as *anchor nodes* in I-DEAS are defined around the building footprint allowing the building mesh, which is generated separately, to be added later and matched to the soil. Groups of elements such as the tunnel lining and the tunnel interior elements which are to be excavated at each stage are defined in I-DEAS for later use in the OXFEM input file. Groups of nodes such as all those on the surface or around the building footprint, are also defined to facilitate results output and post-processing.

The final task within I-DEAS is to check the mesh for any errors. Checks for multiple nodes or zero volume elements are simple to carry out and any duplicated or erroneous elements and nodes are removed. Checks can also be performed to ensure that elements do not have poor geometric conditioning which could lead to errors during calculation. Of particular concern are the elements comprising the tunnel lining, which, for the analyses in this thesis, consist of continuum elements as discussed below. A check for geometric conditioning successfully used by Wisser (2002) is the stretch, $\zeta$, of an element where

$$
\zeta = \sqrt{24} \times \frac{\text{radius of the largest circle that will fit inside the element}}{\text{longest edge of an element}}
$$

(7.1)

The minimum stretch suggested for acceptable geometric conditioning by I-DEAS is 0.05, however as Wisser notes, this value should be only considered as a guideline, as the appropriate value will depend on the problem analysed and the location of each element within the mesh. In addition, it is considered that any round-off errors associated with thin elements with poor geometric conditioning are insignificant as the calculations are undertaken
using double precision arithmetic.

The meshed model file is exported from I-DEAS into the ABAQUS commercial software format and is then converted into a format readable by OXFEM using a FORTRAN program written by Augarde (1997) as amended by Bloodworth (2002) called CONVERT5.

For analyses including a building, the meshes for the facades, consisting of six-noded plane stress triangular elements, are generated separately in I-DEAS and exported and converted to OXFEM format using the CONVERT FACADE program described in Chapter 4. The building information is then added to the soil input file. To connect the building facades, which are defined in local two-dimensional coordinate systems, to each other and to the soil block (defined in three dimensions), tie elements developed by Liu (1997) are used. Previously these tie elements were generated during an OXFEM run. This facility has been separated as part of this project as this pre-processing utility is not a core operation of the analysis program. A new program, TIE-GEN, now generates these tie elements.

The OXFEM input file is completed by adding material parameters, instructions for the staged tunnel construction and output requirements. Timoshenko beam elements for analyses where the building is represented by surface beams are also added at this stage.

### 7.3 Simulation of tunnelling

Numerical simulation of tunnel construction requires the modelling of three main activities: soil excavation, lining installation and volume loss generation. A range of techniques to simulate these three activities is discussed in section 2.3.2. In this thesis, the tunnel is constructed in a number of stages to simulate passage in the longitudinal direction under a building or buildings of interest. For each tunnel construction stage, modelling of soil excavation is achieved by the removal from the overall mesh of elements representing the interior of the tunnel and the imposition of nodal loads on the excavated boundary to remove surface tractions according to the method of Brown and Booker (1985).

Techniques used in this thesis for the modelling of tunnel lining and volume loss build on
Table 7.1: Material properties for concrete tunnel lining

<table>
<thead>
<tr>
<th></th>
<th>( G )</th>
<th>( \nu )</th>
<th>( \gamma )</th>
<th>( s_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( 1.25 \times 10^7 ) kPa</td>
<td>0.2</td>
<td>20.0 kPa/m</td>
<td>( 1.0 \times 10^6 ) kPa</td>
</tr>
</tbody>
</table>

previous work at the Oxford University. Augarde (1997) introduced the use of overlapping shell elements for modelling lining in three dimensions. Subsequently, Augarde and Burd (2001) and Wisser (2002) compared the use of these shell elements with the use of continuum lining elements. They concluded that despite the possibility of sub-optimal geometric conditioning of thin continuum elements in the lining (discussed above), surface settlement results due to a tunnel lined with continuum elements were smoother than with a shell element lining. Continuum lining elements are thus used for all tunnelling analyses in this thesis. Wisser (2002) also investigated the modelling of face support during tunnelling operations and concluded that face support is only necessary in analyses involving very soft ground; in stiffer soil the application of face support did not have a significant influence on the settlement profile and it is thus not applied during the analyses in this thesis.

The tunnel construction process is modelled as follows: at the start of an analysis, all elements in the soil block, including those representing the the interior of the tunnel and the lining, are soil elements. At each tunnel construction stage, the material properties of the lining elements are switched from soil to concrete. Linear elastic material properties are assumed for the concrete and the properties for the lining in all analyses in this thesis are given in table 7.1. The soil elements in the tunnel are then excavated.

Volume loss is modelled by the application of a uniform shrinkage to the tunnel lining. This is achieved by the application of nodal forces to the nodes in the lining elements (Wisser, 2002). The nodal load vector, \( f_e \), for each lining element is given by

\[
f_e = K_e d_e
\]  

(7.2)

where \( K_e \) is the element stiffness matrix and \( d_e \) is the element nodal displacement vector.
comprising the displacement vectors of each node such that

\[ d_e^T = [d_1 \ d_2 \ldots \ d_{10}] \] (7.3)

The displacement vector of node \( i \) is calculated by

\[ d_i = \delta_r \frac{n_i}{|n_i|} \] (7.4)

where \( n_i \) is the vector perpendicular to the tunnel axis pointing from node \( i \) to the tunnel axis and \( \delta_r \) is the required radial displacement.

The method of tunnel construction simulation outlined above can be considered as an ‘outcome driven’ method as the desired volume loss for the analysis is directly applied within the model. This differs from the alternative approach where the analyst attempts to model the exact tunnelling process that causes the volume loss. As described in Chapter 2, both these general approaches have been followed in numerical analyses reported in the literature. The outcome driven approach, as well as being simpler to implement, is more suitable to modelling situations where the volume loss is known in advance such as back-analyses or Class B or C predictions (i.e. during or after the event (Lambe, 1973)). In addition, the outcome driven approach is considered by some (Lee and Rowe, 1990a) to be more robust even for Class A predictions (before the event). This is due to the assertion that even if one is able to exactly model the tunnelling process, it is not possible to model the human factor in tunnelling operations. For example, during real construction the use of the same tunnelling machine may lead to different volume losses when operated by different crew; a well-trained, experienced tunnelling crew may operate the machine more accurately resulting in less volume loss than an inexperienced poorly trained crew.

### 7.4 Soil model

A discussion regarding the range of constitutive models used for soil in finite element analyses is given in section 2.3.2. This concluded with noting the importance of modelling
the small strain non-linearity of soil when assessing the effects of ground movements due to tunnelling. For the tunnelling analyses in this thesis the nested yield surface model by Houlsby (1999) is used which models the non-linearity of a clay soil at small strains. This model consists of a number of nested yield surfaces within an outer fixed von Mises surface. The model is a kinematic work hardening plasticity model in which the inner yield surfaces translate as plastic strain occurs. As a stress point moves in space and encounters a yield surface, the stiffness reduces and the yield surface moves with the stress point. The model thus simulates the non-linearity of response at small strains and the effect of recent stress history. Figure 7.2 illustrates the model in its initial state and for an example situation where with the yield surfaces have translated after a stress point moves from the origin to point A and back again.

Each nested yield surface is described by parameters defining its size and the amount of stiffness reduction as the yield surface is reached. The tangential shear stiffness, $G_i$, after yield surface $i$ has been reached is

$$ G_i = g'_\alpha \times G_0 \tag{7.5} $$

where $g'_\alpha$ is a parameter describing the decrease in shear stiffness at an inner yield surface and $G_0$ is the shear modulus. The triaxial yield strength, $c_i$, for yield surface $i$ is

$$ c_i = c'_\alpha \times c \tag{7.6} $$
Figure 7.3: Tangent shear stiffness variation with strain for nested yield surface model

where $c$ is the undrained shear strength of the material $c = 2s_u/\sqrt{3}$, $s_u$ is the undrained strength in triaxial compression and $c'_\alpha$ is a parameter governing the size of the particular inner yield surface. The outermost yield surface is fixed and defines the undrained shear strength of the material.

The stiffness response of soil under this model is illustrated in figure 7.3 which shows the change in tangent shear modulus of the soil with shear strain. Parameters used to plot this figure are those chosen as typical for London clay by Houlsby (1999).

The soil model allows for the undrained shear strength and shear modulus to increase with depth. The shear modulus at any depth, $z$, below the surface is given by

$$G_0 = G_0^s + \omega z$$  \hspace{1cm} (7.7)

where $G_0^s$ is the shear modulus at the surface and $\omega$ is the increase in shear modulus per metre depth. The undrained shear strength at depth $z$ is

$$s_u = s_{u0} + \mu z$$  \hspace{1cm} (7.8)

where $s_{u0}$ is the undrained shear strength at the surface and $\mu$ is the increase in strength with depth.
The parameters required to fully define the model are thus:

- initial shear modulus at the surface inside the first yield surface, \( G_0 \)
- undrained shear strength at surface, \( s_{u0} \)
- increase in \( s_u \) with depth, \( \mu \)
- increase in \( G_0 \) with depth, \( \omega \)
- unit weight, \( \gamma \)
- Poisson’s ratio, \( \nu \)
- \( n \) pairs of constants describing the \( n \) inner nested yield surfaces, \( c'_{\alpha}, g'_{\alpha} \)

The values used for each of the parameters above are given with the description of each analysis in Chapters 8 and 9.

### 7.5 Solution technique and calculation process

The solution of a finite element problem involves the solution of the set of equations

\[
K \, d = f
\]  
(7.9)

where \( d \) comprises the displacements at the nodal degrees of freedom, \( K \) is the stiffness matrix and \( f \) comprises the nodal forces. The method used for this task in OXFEM is a Frontal solution method based on Gaussian elimination. In a Frontal solver, the full global stiffness matrix is never fully assembled; the assembly process is interrupted at intervals when there are enough complete rows and columns in the stiffness matrix to perform the reduction by Gaussian elimination of complete columns (Astley, 1992). Entries in the stiffness matrix are complete when they will receive no more contributions from any other degrees of freedom. Entries may also be empty, having received none of their contributions or partially complete, having received some but not all. The empty and partially complete
CHAPTER 7. COMPOSITION OF 3D NUMERICAL MODEL

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The stiffness matrix for a non-linear material such as the no-tension masonry or the nested yield surface soil, depends on the current stress level. For the solution of such problems alternatives include an iterative approach such as the Newton-Raphson method or an incremental approach such as a modified Euler scheme (Cook et al., 1989). Both methods are available in OXFEM, with the Newton-Raphson solution scheme having been implemented as part of concurrent but unrelated research. For all analyses in this thesis, the modified Euler scheme is used with loading applied over a number of steps and the out-of-balance between the equilibrium force at the current stress level and the original applied force applied as a correction at the next step (Burd, 1986).

7.6 **Shared memory parallel computing**

Large three-dimensional finite element analyses can suffer from long run times, particularly when non-linear material models are used. The use of parallel computing significantly reduces the calculation time for large finite element analyses. Parallel computing involves the partitioning of time consuming calculations and the distribution of these to a number of different individual processors where independent calculations are computed in parallel, reducing the run time compared to a serial calculation (Graham, 1999). This reduction in run times is particularly useful for research where a large number of slightly different runs
of the same problem may need to be performed, for example in parametric studies.

There are two main types of parallel computing: distributed memory and shared memory systems. In a distributed memory system, each processor has its own local memory with processors linked together able to distribute information by message passing. A shared memory system consists of a number of processors all with access to a global memory store, meaning that each processor does not have its own memory (except for a small cache) but each has the same view of all memory globally. Communication between processors is by one processor writing data to the global memory and another reading those data; there is no need for direct communication between processors (McLatchie et al., 2003). Hybrid systems (also known as shared distributed systems) also exist, where each processor has its own memory as in a distributed system, but in addition, all processors can view each other’s memory. The main advantage of a shared memory system for programming and use is that there are no explicit communications between processors, simplifying programming and use. Two shared memory systems were used for this project.

The first shared memory system used for this project was the OSWELL system at the Oxford Supercomputing Centre (OSC). The OSWELL system consisted of 84 processors in four partitions comprising 24; 24; 12; and 24 processors with 48; 48; 24; and 48Gbytes of memory in each partition respectively, giving a total of 168Gbytes. A job ran within one partition such that a maximum of 24 processors could be used, although typically users request less than the full complement of processors in a partition for a job. Each of the 84 processors was a Sun Fire 6800 UltraSparc III 900MHz machine. User access to the system was through a head node comprising two Sun Fire V880 750MHz machines with 4Gbytes RAM and the use of a Sun GridEngine 5.3p2 job scheduling queue system (McLatchie et al., 2003).

The second shared memory system used for this project was the KONRAD system which replaced the OSWELL system at the OSC from August 2005. KONRAD consists of 24 processors; eight on each of three compute nodes with each node sharing 32Gbytes of memory. Each processor is a Dual-Core AMD Opteron 865 1.8GHz machine. The head node for the system is a Dual AMD Opteron 246 2.0GHz machine with 4Gbytes DDR, and
uses a Sun GridEngine scheduler.

For use in parallel computing the original FORTRAN90 code for OXFEM was parallelised. The method employed for this task was the use of the OpenMP standard for shared memory systems. Using OpenMP to parallelise existing serial code such as OXFEM involves inserting new OpenMP compiler directives into the code to indicate regions to be executed in parallel and give details about the parallelisation (Graham, 1999). Regions of the code parallelised generally include only the more time consuming operations. A significant advantage of the OpenMP standard is that the directives appear as comments to a serial compiler but operative code to a parallel compiler, allowing the same versions of the program to be maintained for both serial and parallel computing.

Although the OXFEM code was already parallelised prior to the start of this project, an older supercomputer (the OSC system known as OSCAR; a hybrid shared distributed system) was used previously. OSWELL, having been installed at the OSC in 2002, replaced OSCAR necessitating the the migration of the OXFEM code to the new system from the local civil engineering group network where the program is run in serial. This task involved the migration, parallel compilation and testing of the code on the new OSWELL system. This provided an opportunity for an upgrade of a number of sections of the code, facilitated by the more rigorous compilers on the supercomputer. This process of migrating the OXFEM code, compiling and testing was repeated when the KONRAD system replaced OSWELL.

The use of the supercomputer was beneficial in minimising run times for this project. Typical run times for analyses with approximately 13,000 degrees of freedom and 110 calculation steps used to take as long as 180 hours (approximately 8 days) to run in serial on the fastest civil engineering network machine at the time, a Sun 400MHz, 512Mbyte machine under UNIX. The use of the OSCAR supercomputer reduced such analyses to approximately five hours with eight parallel processors employed (Bloodworth, 2002). On OSWELL, during the current project, analyses with approximately 30,000 degrees of freedom and 115 calculation steps took around eight hours to run with eight parallel processors.
Chapter 8

Numerical modelling of building response to tunnelling:
Example analyses

8.1 Introduction

This Chapter describes examples of three-dimensional numerical analysis of building response to tunnelling. The analyses are used to evaluate the use of beam elements for modelling masonry building facades in 3D during finite element tunnelling simulations, using the developments described in the previous Chapters of this thesis. The analyses allow an evaluation of the 3D beam approach for modelling masonry buildings and the equivalent elastic and equivalent masonry beam models for assigning properties to the beams. This evaluation is by comparison with finite element analyses, based on previous work at Oxford University, containing full finite element models of the buildings. (Evaluation of the 3D beam approach with respect to tunnelling case histories is undertaken in Chapter 9). Two different problems are analysed: a symmetric problem where the tunnel passes square under the centre of a masonry building, and an oblique problem where the building is both eccentric to the tunnel centre line and skewed at an angle. Results are presented for
a number of finite element analyses of both the symmetric and oblique problems. Surface 
displacements and building damage predictions resulting from the finite element analyses 
using surface beam elements to represent the building, finite element analyses using full 
building facades and traditional hand assessments are compared.

8.2 Symmetric analysis

8.2.1 Description of analysis

The first example problem consists of a tunnel constructed with the centre line passing 
symmetrically beneath the middle of a rectangular building which is aligned with its front 
and rear facades perpendicular to the direction of tunnelling as shown in figure 8.1. The 
example problem is identical to that used by Augarde (1997), Liu (1997), Burd et al. (2000) 
and Wisser (2002) during their development of numerical methods for modelling tunnel 
construction, masonry facades and compensation grouting. Four different finite element 
analysis run types are undertaken with the symmetric problem as summarised in table 8.1: 
the first type (SGF) is a greenfield analysis with no building present; the second (SMF) 
includes a full masonry building on the surface made from 2D facades; thirdly, surface 
beams with properties determined using the equivalent elastic beam method are used to 
represent the building (SEB); and finally the building is represented by surface beams using 
the equivalent masonry beam model (SMB). The SGF and SMF analyses are repeats of 
the previous Oxford University work, but this is necessary so that comparisons with the 
surface beam analyses are not mesh dependent.

<table>
<thead>
<tr>
<th>FE Runs</th>
<th>Description of Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>SGF</td>
<td>Symmetric Greenfield</td>
</tr>
<tr>
<td>SMF</td>
<td>Symmetric with building made of 2D Masonry Facades</td>
</tr>
<tr>
<td>SEB</td>
<td>Symmetric with Equivalent Elastic Beams representing the building</td>
</tr>
<tr>
<td>SMB</td>
<td>Symmetric with Equivalent Masonry Beams representing the building</td>
</tr>
</tbody>
</table>
Figure 8.1: Symmetric example analysis layout

**Geometry and mesh**

The layout of the symmetric example problem is shown in figure 8.1. It consists of a tunnel of 5m inner diameter constructed at a centreline depth of 10m. On the surface is a building 20m long, 10m wide and 8m high. The building comprises 1m thick masonry walls and has windows and a door in the front and rear facades (which are identical) with dimensions as shown in figure 8.2. The end walls of the building are plain with no openings.

The tunnel lining is 0.25m thick concrete making the outer diameter of the tunnel 5.5m. Volume loss applied to the tunnel is 2%. Due to symmetry, only half the problem is analysed using a soil block with dimensions of 60m in all three directions.

The soil mesh for all the symmetric analyses is shown in figure 8.3 with the building mesh for run type SMF and the beam mesh for run types SEB and SMB shown in figure 8.4. The soil and tunnel mesh contains 9993 nodes and 6558 ten-noded tetrahedral elements. The building mesh for run type SMF has 1562 nodes and 679 six-noded triangular plane stress elements; and for run types SEB and SMB, the surface beams comprise 64 Timoshenko beam elements. The surface beam nodes are selected to match nodes on the building footprint of the soil mesh (which also correspond to nodes at the base of the facade mesh).
The boundary conditions applied to the soil are: fully fixed displacements (x, y and z displacements fixed) at the base; and horizontal displacements (x or y displacements as appropriate) fixed perpendicular to each vertical soil boundary. For run SMF the 2D plane stress elements in the building mesh have fixed horizontal (x displacement fixed) displacement at the plane of symmetry. For run types SEB and SMB the surface beams have fixed horizontal displacements (x displacement fixed) and fixed rotation about the y-axis at the plane of symmetry.

Soil and tunnel properties

The soil is modelled using the nested yield surface model described in section 7.4. The properties for the model are given in table 8.2 and are identical to those chosen to represent London Clay in previous analyses at Oxford University (for example Augarde, 1997 and Wisser, 2002). The soil is assumed to have a ratio of horizontal to vertical stress of 1.125. The elastic properties for the concrete tunnel lining are given in table 7.1.

Building and beam properties

For run type SMF, the building facades are assigned the no-tension masonry material model developed by Liu (1997) with the properties described in section 5.3 and given in
Figure 8.3: Symmetric example analysis soil mesh

Figure 8.4: Symmetric example analyses building meshes
Table 8.2: Material properties for nested yield surface soil model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_s^0$</td>
<td>$3.0 \times 10^3$ kPa</td>
</tr>
<tr>
<td>$s_{u0}$</td>
<td>60.0 kPa</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$3.0 \times 10^3$ kPa/m</td>
</tr>
<tr>
<td>$\mu$</td>
<td>6.0 kPa/m</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>20 kN/m$^3$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Surface | $\alpha$ | $\alpha'$ |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.020</td>
<td>0.900</td>
</tr>
<tr>
<td>2</td>
<td>0.040</td>
<td>0.750</td>
</tr>
<tr>
<td>3</td>
<td>0.060</td>
<td>0.500</td>
</tr>
<tr>
<td>4</td>
<td>0.100</td>
<td>0.300</td>
</tr>
<tr>
<td>5</td>
<td>0.150</td>
<td>0.200</td>
</tr>
<tr>
<td>6</td>
<td>0.200</td>
<td>0.150</td>
</tr>
<tr>
<td>7</td>
<td>0.300</td>
<td>0.100</td>
</tr>
<tr>
<td>8</td>
<td>0.500</td>
<td>0.050</td>
</tr>
<tr>
<td>9</td>
<td>0.700</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Table 5.2. Elastic lintels above the windows are also included (as shown in figure 8.2) with the properties given in table 5.3.

For runs SEB and SMB, the surface beams are assigned properties using the equivalent elastic and equivalent masonry beam methods as described in Chapters 4 and 5 respectively. Properties denoted as in-plane and out-of-plane refer to the planes of the 2D facade that the beams represent, where out-of-plane properties are those associated with displacements perpendicular to the plane of the facade. On application of those methods to the facades shown in figure 8.2, the effective cross sectional area, $A^*$, and the effective second moment of area in-plane, $I^*$, are calculated for both the front/rear facade and the end walls. For this building the base conditions are assumed to be smooth for the calculation of the beam properties. Appendix C contains an example of the full calculation of the properties for the beams to represent the front and rear facades, the results of which are given in table 8.3. Table 8.4 contains the properties for the beams representing the end wall. In tables 8.3 and 8.4: $E$ is Young’s modulus, $G$ is the shear modulus, $J$ is the torsional rigidity, $k$ is the shear coefficient and $\gamma$ is the self weight. All properties are the same for the equivalent elastic beams (EEB) and the equivalent masonry beams (EMB) with the exception of the critical curvature ($\kappa_{crit}$) and residual in plane bending factor ($f_b$) which are not applicable for the EEBs. Parametric studies of the effect of assigning different properties from those described in this section for the EEBs and EMBs are investigated in sections 8.2.4 and
Table 8.3: Material properties for surface beams: Front/rear building facade

<table>
<thead>
<tr>
<th>$E$</th>
<th>$G$</th>
<th>$A^*$</th>
<th>$I_{in}$</th>
<th>$I_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00$\times 10^7$ kPa</td>
<td>4.167$\times 10^6$ kPa</td>
<td>4.762 m$^2$</td>
<td>29.716 m$^3$</td>
<td>0.397 m$^4$</td>
</tr>
<tr>
<td>$J$</td>
<td>$k$</td>
<td>$\gamma$</td>
<td>$\kappa_{crit}$ (EMB only)</td>
<td>$f_b$ (EMB only)</td>
</tr>
<tr>
<td>1.353 m$^4$</td>
<td>0.85</td>
<td>20.0 kN/m$^3$</td>
<td>$1.0 \times 10^{-5}$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 8.4: Material properties for surface beams: End wall

<table>
<thead>
<tr>
<th>$E$</th>
<th>$G$</th>
<th>$A^*$</th>
<th>$I_{in}$</th>
<th>$I_{out}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00$\times 10^7$ kPa</td>
<td>4.167$\times 10^6$ kPa</td>
<td>3.333 m$^2$</td>
<td>17.778 m$^3$</td>
<td>0.278 m$^4$</td>
</tr>
<tr>
<td>$J$</td>
<td>$k$</td>
<td>$\gamma$</td>
<td>$\kappa_{crit}$ (EMB only)</td>
<td>$f_b$ (EMB only)</td>
</tr>
<tr>
<td>0.921 m$^4$</td>
<td>0.85</td>
<td>20.0 kN/m$^3$</td>
<td>$1.0 \times 10^{-5}$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

8.2.5 below.

The walls are assumed to have a self weight of 20 kN/m$^3$. Self weight is applied to the facades for run SMF while for the surface beams, loads are calculated using the facade dimensions described above and are applied at the nodal points on the surface beams. The loads differ from node to node due to the openings in the facades.

**Calculation procedure**

The finite element calculation proceeds in stages to simulate the stress history at the site and progressive construction of the tunnel as described in section 7.3. Table 8.5 shows the stages used for the symmetric analyses. In the first calculation stage, the building self weight and soil initial stresses are applied. In the second stage the building strains and displacements are reset to zero. This ensures that the strains, building damage and surface displacements recorded during subsequent tunnelling are due the construction of the tunnel alone. The tunnel is then constructed in six stages; the length of each stage in the $y$-direction varies and is given in table 8.5.

There are two exceptions to the general procedures. Firstly, for run type SMF with the building represented by masonry facades, the masonry material residual tensile strength, $c$ is initially set to 0.0 kPa while the building self weight is applied to the model (stages 1 and 2 from table 8.5). The residual tensile strength is then changed to 10 kPa at the
Table 8.5: Calculation stages for symmetric example analysis

<table>
<thead>
<tr>
<th>Stage</th>
<th>No. Steps</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>Application of building weight and initial soil stresses</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>Reset displacements and strains to zero</td>
</tr>
<tr>
<td>3</td>
<td>15 (30 for SMB)</td>
<td>Tunnel construction stage 1: 00.0 - 12.5m</td>
</tr>
<tr>
<td>4</td>
<td>15 (30 for SMB)</td>
<td>Tunnel construction stage 2: 12.5 - 25.0m</td>
</tr>
<tr>
<td>5</td>
<td>15 (30 for SMB)</td>
<td>Tunnel construction stage 3: 25.0 - 30.0m</td>
</tr>
<tr>
<td>6</td>
<td>15 (30 for SMB)</td>
<td>Tunnel construction stage 4: 30.0 - 35.0m</td>
</tr>
<tr>
<td>7</td>
<td>15 (30 for SMB)</td>
<td>Tunnel construction stage 5: 35.0 - 47.5m</td>
</tr>
<tr>
<td>8</td>
<td>15 (30 for SMB)</td>
<td>Tunnel construction stage 6: 47.5 - 60.0m</td>
</tr>
</tbody>
</table>

commencement of tunnelling (stage 3). This approach was used by Wisser (2002) to avoid any tensile stress accumulating in the facades during the self weight loading contributing to the cracking of the building in the later tunnelling stages. It is adopted here as this approach is thought to provide more realistic facade response predictions to the tunnelling.

Secondly, for the run type SMB, the accumulation of stress in the first two stages also needs to be avoided in order to allow for assessment of displacements simply due to tunnelling. For SMB runs, this is achieved by simply adding the masonry beam elements only after stage 2 is complete.

The number of calculation steps per stage is chosen as 15 steps for all analyses except the SMB analyses where the number of steps is chosen as 30 in order to reduce the out-of-balance forces generated during the calculation procedures to acceptable levels. The choice of the number of steps per stage is based on recommendations presented by Wisser (2002) who investigated differing numbers of steps and corresponding out-of-balance forces.

### 8.2.2 Greenfield analysis

This section details the results of the symmetric greenfield run SGF, with no building on the surface. The purpose of this run is to provide greenfield surface settlement results to act as a control for the later runs which include the building on the surface.

Figure 8.5 shows the surface contours after each tunnelling stage while figure 8.6 shows the surface profile development in 3D. The colours in the 3D plots match the colour key in the
relevant contour plots, but the surface grid shown is unrelated to the finite element mesh. This applies for all the contour and 3D plots presented in the remainder of this thesis.

Also throughout this thesis, the direction of tunnelling is referred to as the *longitudinal* direction while the direction perpendicular to the tunnel is called the *transverse* direction. Vertical surface displacements are shown in figure 8.7 along transverse lines at $y=25\text{m}$ (a) and $y=35\text{m}$ (b); along the longitudinal centre line of the tunnel (c); and along a longitudinal line under the end wall of the building $x=10\text{m}$ (d), for each tunnelling stage 1-6.

Charts 8.5 to 8.7 show a smooth greenfield surface profile developing as the tunnel progresses. The greenfield surface displacement results, however, show a transverse settlement trough that is wider and shallower than the semi-empirical Gaussian curve commonly attributed to greenfield settlements as discussed in section 2.2. The semi-empirical curve for this example problem is shown along with the transverse finite element settlements in figure 8.8. A greenfield trough that is shallower and wider than the Gaussian approximation is a common feature of finite element analyses as discussed in sections 2.3.2 and 2.4. This feature does not limit the usefulness of the SGF run for use as a control for the comparison with runs with buildings on the surface. The longitudinal trough displays steady state settlement approximately 15m behind the tunnel face, a distance of five tunnel diameters.

Model SGF thus provides a greenfield reference base against which the influence of including a building on the surface, by means of facades (SMF run), or surface beams (SEB and SMB runs), is compared in the remainder of the symmetric analyses described below.

### 8.2.3 Analysis with masonry building

Surface displacement results from run type SMF with the building comprising 2D masonry facades are presented in figures 8.9 to 8.11.

The influence of the building on the settlement profile is clearly observable as comprising two main effects. The first is the influence of the building weight adding to the magnitude of the displacement under all the building facades. This is particularly evident in the 3D
Figure 8.5: Greenfield run SGF: Surface displacement contours
(a) Tunnelling Stage 1
(b) Tunnelling Stage 2
(c) Tunnelling Stage 3
(d) Tunnelling Stage 4
(e) Tunnelling Stage 5
(f) Tunnelling Stage 6

Figure 8.6: **Greenfield run SGF: 3D Surface profile**
CHAPTER 8. 3D NUMERICAL MODELLING: EXAMPLE ANALYSES

Figure 8.7: Greenfield run SGF: Surface displacements
Secondly, the transverse surface profile is flattened by the presence of the building. In this analysis the building lies almost completely in a sagging zone of displacement. It is expected, therefore, that the masonry building will respond in a rigid manner with little sagging curvature; a response which is clearly evident in the SMF results. As described in section 5.2, this rigid response with a flat displacement profile, is due to arching effects within the masonry facades (Liu, 1997) acting to flatten the curvature of the surface settlements in sagging. These observations concur with those of Burd et al. (2000) in connection with an analysis of the same problem.

It is interesting to note that as the tunnelling progresses the front and rear facades settle in a rigid fashion with no tilt and minimal curvature. The end wall, while remaining in a very slight hogging state, could also to be considered to act rigidly as the tunnelling progresses. Following the final stage of tunnelling, it can be seen that the building remains tilted, with the front facade approximately 1.5mm lower than the rear. This is also evident from the end wall plot for tunnelling stage 6 (figure 8.11(d)).
Figure 8.9: Masonry building run SMF: Surface displacement contours
Figure 8.10: Masonry building run SMF: 3D Surface profile
Figure 8.11: Masonry building run SMF: Surface displacements
Table 8.6: Analyses with equivalent elastic surface beams

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEB</td>
<td>Properties given in tables 8.3 and 8.4</td>
</tr>
<tr>
<td>SEB2</td>
<td>As for SEB but with $EI_{in} \times 10$</td>
</tr>
<tr>
<td>SEB3</td>
<td>As for SEB but with $EI_{in} \times 1/5$</td>
</tr>
<tr>
<td>SEB4</td>
<td>As for SEB but with $EI_{in} \times 1/10$</td>
</tr>
<tr>
<td>SEB5</td>
<td>As for SEB but with $EI_{in} \times 1/100$</td>
</tr>
<tr>
<td>SEB6</td>
<td>As for SEB but with $EI_{in} \times 1/1000$</td>
</tr>
<tr>
<td>SEB7</td>
<td>As for SEB but with no self weight</td>
</tr>
</tbody>
</table>

8.2.4 Analysis with equivalent elastic surface beams

Results from run type SEB (symmetric run with equivalent elastic beams) are described in this section. Following the description of the initial run (SEB) which has elastic surface beams with properties as given in tables 8.3 and 8.4 representing the building, a range of analyses are used to investigate the impact of differing beam properties. These include the building weight and the in-plane bending stiffness assigned to the beams. The analyses described in this section are summarised in table 8.6.

Equivalent elastic surface beams

Results for run SEB are shown in figures 8.12 to 8.14. Building facades are represented by Timoshenko surface beams with equivalent elastic beam properties.

The results show the influence of the building weight adding to the magnitude of the surface displacements; the centre line vertical displacement of approximately 16mm being greater than the greenfield settlement of 14mm. In addition, the transverse settlement profile is again flattened compared to the greenfield profile. This is due to the relative stiffness of the elastic beams representing the building with respect to the soil. The transverse settlement under the beams (figures 8.14(a) and (b)) exhibits a flatter and smoother profile than the transverse settlement under the masonry facades (figures 8.11(a) and (b)). This is due to the fact that the beam has the same stiffness (and thus influence on the settlement profile) along its length leading to a smooth profile. The profile under the masonry facade, however, is influenced by the location of the windows and door openings and the arching...
Displ. (m)
-0.000
-0.001
-0.002
-0.003
-0.004
-0.005
-0.006
-0.007
-0.008
-0.009
-0.010
-0.011
-0.012
-0.013
-0.014
-0.015
-0.016
-0.017

(a) Tunnelling Stage 1
(b) Tunnelling Stage 2
(c) Tunnelling Stage 3
(d) Tunnelling Stage 4
(e) Tunnelling Stage 5
(f) Tunnelling Stage 6

Figure 8.12: Equivalent elastic surface beams run SEB: Surface displacement contours
Figure 8.13: Equivalent elastic surface beams run SEB: 3D Surface profile
Figure 8.14: Equivalent elastic surface beams run SEB: Surface displacements
effect. The use of surface beams with equivalent elastic model properties thus gives a smooth approximation of the masonry facade response. The resulting surface displacements closely match the magnitude of the displacements under the masonry facades (from run SMF) but exhibit less variation along the facade length. The use of 3D surface beams with properties attributed using the equivalent elastic beam model can be seen to give closely comparable surface settlements to those under the full masonry facade for this problem.

Influence of building stiffness

Analysis SEB was repeated with the in-plane bending stiffness for the beams varied by using the multiples as shown in table 8.6 (runs SEB2 to SEB6). These multiples were used for the in-plane bending stiffness properties for the beams given in tables 8.3 and 8.4. Figure 8.15 shows the surface displacements after the final tunnelling stage using surface beams with various in-plane bending stiffnesses. The effect of reducing the bending stiffness is clearly visible: it causes the profile to take a shape more like a greenfield profile, but with additional vertical displacement due to the self weight. The effects of reducing bending stiffness has been demonstrated previously by others (see section 2.5.4) including Potts and Addenbrooke (1997) and Jenck and Dias (2004). It is interesting to compare figures 8.15(a) and (b) with figure 2.10 which shows settlement profiles by Potts and Addenbrooke (1997) also tending towards the greenfield profile with reduction in bending stiffness. The only difference is that, no additional settlement due to the self weight is seen in figure 2.10 as this was not included in those results (self weight is included in the relative stiffness method subsequently by Franzius et al. (2004)). Jenck and Dias (2004) present similar results to those in figures 8.15(a) and (b) showing that increasing the stiffness of their building (by adding basement levels) causes the transverse settlement profile to be less like the greenfield.

The initial analysis, SEB, exhibits similar surface settlements to run SEB2 (with ten times the stiffness) and it is these runs that most closely match the surface displacements of run SMF with the full masonry facades. The importance of using an appropriate properties
for elastic surface beams is thus apparent: if the bending stiffness of the beam is too low, the settlement profile does not reflect the appropriate influence of the building; if the properties are too high, the surface profile will be too flat, overstating the influence of the building. It is acknowledged that this problem would be less sensitive to changes in the building stiffness than a problem where displacements are applied directly to the facade or building, due to the soil structure interaction. The properties assigned to run SEB using the methods developed in Chapter 4 of this thesis are thus considered to be appropriate as this run most closely matches the displacement profile of the masonry facades.

**Influence of building weight**

The influence of not including the building weight on the settlement profile due to tunnelling is investigated. Run SEB7 is identical to run SEB with the exception that the beams representing the building are assigned no self weight; all other properties of the beams are the same as those given in tables 8.3 and 8.4. Figures 8.16, 8.17 and 8.18 compare the surface contours, 3D surface profile and surface displacements for analysis SEB7 with no self weight and SEB with self weight.

The influence of the building weight is apparent from the magnitude of the maximum settlement under the building. The maximum settlement of the front facade in run SEB7 of 9.6mm, is less than the in run SEB of 16.4mm with the building weight included. The observed effect of the building stiffness without the self weight, particularly evident in figure 8.17 is to reduce the settlement under the beams, relative to the greenfield situation (maximum settlement of 14.1mm). The importance of including the building weight in these analyses is thus apparent in terms of settlement prediction. This concurs with the investigation into the effects of different building weights by Liu (1997). It is interesting that Franzius et al. (2004) note that although building weight slightly influences the Potts and Addenbrooke (1997) modification factors, the stiffness of the building is a much more significant factor for predicting accurate building damage.
Figure 8.15: Equivalent elastic surface beams (Tunnelling Stage 6): Influence of bending stiffness
Figure 8.16: Equivalent elastic surface beams run SEB7 with no self weight: Surface displacement contours compared to run SEB with self weight

Figure 8.17: Equivalent elastic surface beams run SEB7 with no self weight: 3D Surface profile compared to run SEB with self weight
8.2.5 Analysis with equivalent masonry surface beams

Results from run type SMB (symmetric run with equivalent masonry beams) are described in this section. Following the description of the central run (SMB22) which has masonry surface beams with properties as given in tables 8.3 and 8.4 representing the building, a range of analyses are used to investigate the impact of differing beam properties. These include the critical curvature, $\kappa_{\text{crit}}$ and residual bending stiffness factor, $f_b$ assigned to the beams. The analyses described in this section are summarised in table 8.7.

Equivalent masonry surface beams

Results from run SMB22 are described in this section. Results are shown in figures 8.19 to 8.21 for the run with the building facades represented by Timoshenko beams with equivalent masonry beam material with the properties given in tables 8.3 and 8.4. The parameters of critical curvature, $\kappa_{\text{crit}}$ and residual bending stiffness factor, $f_b$ for this central run (of the parametric study) were chosen as $1.0 \times 10^{-5}$ and 0.010 respectively. These base case values were chosen based on a range of pre-calculations and preliminary analyses. The effect of the choice of these values for $\kappa_{\text{crit}}$ and $f_b$ is further discussed below.

The results again show the influence of the beams flattening the settlement profile and
Figure 8.19: Equivalent masonry surface beams run SMB: Surface displacement contours
Figure 8.20: Equivalent masonry surface beams run SMB: 3D Surface profile
Figure 8.21: Equivalent masonry surface beams run SMB: Surface displacements
the additional vertical settlement due to the building weight. The shape of the transverse settlement profile is almost identical to that from the the SEB run with the equivalent elastic beams. The majority of the facade is in the sagging zone where the in-plane bending stiffness of the equivalent masonry beam model remains constant and thus it is unsurprising that the surface displacement results are the same as the SEB run. As with run SEB, the settlement profile closely resembles that under the masonry facade in run SMF but is smoother and flatter.

**Influence of $\kappa_{\text{crit}}$ and $f_b$**

The influence of the critical curvature, $\kappa_{\text{crit}}$ and residual bending stiffness factor, $f_b$ used in the equivalent masonry beam model is investigated in this section. Analysis type SMB was repeated using the range of different parameters shown in table 8.7. All other properties for the beams remained the same as those given in tables 8.3 and 8.4.

The choice of values for the parameters of $f_b$ and $\kappa_{\text{crit}}$ is influenced by a number of factors. Figure 8.22 shows theoretical moment-curvature diagrams for an equivalent masonry beam (EMB) in hogging. Moment-curvature plots including gradual stiffness reduction (as used in the EMB model and described in section 5.4.3) are shown along with bi-linear curves that would result from not using the gradual reduction of stiffness approach.

The choice of the residual bending stiffness ($f_b$) is driven by the aim of reducing the bending stiffness of the EMB in hogging to an appropriate level. It can be seen in figure

<table>
<thead>
<tr>
<th>Analysis</th>
<th>$\kappa_{\text{crit}}$</th>
<th>$f_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMB11</td>
<td>$1.0 \times 10^{-4}$</td>
<td>0.100</td>
</tr>
<tr>
<td>SMB12</td>
<td>$1.0 \times 10^{-4}$</td>
<td>0.010</td>
</tr>
<tr>
<td>SMB13</td>
<td>$1.0 \times 10^{-4}$</td>
<td>0.001</td>
</tr>
<tr>
<td>SMB21</td>
<td>$1.0 \times 10^{-5}$</td>
<td>0.100</td>
</tr>
<tr>
<td>SMB22</td>
<td>$1.0 \times 10^{-5}$</td>
<td>0.010</td>
</tr>
<tr>
<td>SMB23</td>
<td>$1.0 \times 10^{-5}$</td>
<td>0.001</td>
</tr>
<tr>
<td>SMB31</td>
<td>$1.0 \times 10^{-6}$</td>
<td>0.100</td>
</tr>
<tr>
<td>SMB32</td>
<td>$1.0 \times 10^{-6}$</td>
<td>0.010</td>
</tr>
<tr>
<td>SMB33</td>
<td>$1.0 \times 10^{-6}$</td>
<td>0.001</td>
</tr>
</tbody>
</table>
8.22(a) that the smaller the value of $f_b$, the greater the change in slope of the bi-linear target moment-curvature path and the tighter the gradually reducing stiffness curve. The incremental nature of the calculation procedure means that as lower values of $f_b$ are used, greater out-of-balance forces will develop in each incremental load step. To keep the out-of-balances forces within reasonable limits a greater number of steps per stage may be required for lower values of $f_b$. The number of steps per stage used in these analyses is 30 steps compared to the 15 steps for the other analysis types in this thesis.

The choice of the appropriate value of $f_b$ can be based on the equivalent elastic beam results for walls in hogging. These will be further discussed in the oblique example analysis section 8.3.4 as there is not a significant hogging section of wall in the symmetric analysis to draw any conclusions. For this analysis a value of $f_b=0.01$ is used (meaning that as hogging develops, the incremental stiffness in hogging trends towards 1% of the incremental stiffness in sagging).

The choice of the value of $\kappa_{crit}$ also influences the shape of the load path as shown in figure 8.22(b). The smaller the critical curvature, the closer the change in stiffness of the moment-curvature path is to the origin, the sharper the corner in the load path. This again has an impact on the development of out-of-balance forces during the incremental calculation with greater out-of-balance forces developing for smaller values of $\kappa_{crit}$. Thus, the smaller the critical curvature selected, the greater the number of calculation steps required in each stage.

The choice of the value of $\kappa_{crit}$ is based on the premise that there is a small hogging curvature that the masonry facade can sustain before cracking reduces its bending stiffness (as described in section 5.4.3). This point is the critical curvature, however, in the formulation of the EMBs a gradual reduction of stiffness approach is used to simulate the development of cracking leading to loss of stiffness, and to reduce the out of balance forces developed at this point. The range of values tested for $\kappa_{crit}$ (in table 8.7) is designed to be appropriate for this analysis, although it would be worthwhile future work to investigate smaller values of $\kappa_{crit}$ or the sudden reduction in stiffness approach.

Figure 8.23 shows the surface displacements after the final tunnelling stage with the various
Figure 8.22: Impact of differing residual bending stiffness \((f_b)\) and critical curvature \((\kappa_{crit})\) values

parameters from table 8.7 attributed to the beams. There is little observable difference between the transverse or centreline longitudinal settlements for all the runs.

There is an observable difference in the response of the end wall \((x=10m)\) however. This wall is in a slight hogging mode throughout the tunnel construction. It can be seen that for runs SMB32 and SMB33, the beams lose bending stiffness and the wall hogs significantly more than the masonry facade or the masonry beam analyses. This indicates that the critical curvature for these beams could be considered to be too small, and that they are losing their stiffness too rapidly.

The amount by which the beam stiffness reduces is another consideration. SMB33 hogs significantly more than SMB32. The remainder of the runs do not hog significantly, similar to the masonry facade, but are all tilted to a different degree. This analysis does not allow any firm conclusions to be made regarding the parameters \(f_b\) and \(\kappa_c\) apart from the indication that a critical curvature of \(\kappa_c=1.0 \times 10^{-6}\) may be too small as this causes the end wall to reduce its stiffness when the masonry facade does not. Further discussion of the choice of these parameters is given as part of the oblique analysis.
Figure 8.23: Equivalent masonry surface beams: Influence of $\kappa_{\text{crit}}$ and $f_b$
Alternative equivalent masonry beam approach

The settlement profile of surface beams using the alternative equivalent masonry beam model (outlined in section 5.4.4) is investigated in this section. This alternative approach involves the beams being assigned a reduced in-plane bending stiffness at the start of the analyses with which increases gradually if they go into a sagging mode of deformation. The settlement profile of equivalent masonry beams using both the standard and the alternative EMB approaches is compared to masonry facade, greenfield and equivalent elastic beam analyses in figure 8.24.

The alternative equivalent masonry beams can be seen to lead to a settlement profile that differs from the main EMBs. They are more flexible as a result of been assigned a low bending stiffness at the start of the analysis. This leads to their settlement profile differing from the masonry facade profile, in particular this can be seen in the end wall ($x=10$ m) plot.

The problem with the alternative approach is that it does not allow for the existence of a threshold level of hogging bending strain which the masonry walls can sustain prior to their loss of stiffness. This is accounted for with the main EMB model through the critical curvature ($\kappa_{\text{crit}}$). The greater $\kappa_{\text{crit}}$, the more gradual is the reduction in bending stiffness. It is recommended that the main EMB model be used and the use of the alternative EMB approach be discontinued.

8.2.6 Summary

There are a number of items of note arising from the symmetric analyses of building response to tunnelling. The surface profile for using equivalent elastic surface beams is very similar to the profile under a full masonry facade. This indicates that the Timoshenko beam elements with equivalent elastic beam properties based on the geometric method developed in this thesis, are successful for facades in sagging. Altering the in-plane bending stiffness of the elastic beams impacts the settlement profile in a similar manner to previously published investigations (Potts and Addenbrooke (1997), and Jenck and Dias (2004)).
(a) Transverse $y=25\text{m}$

(b) Transverse $y=35\text{m}$

(c) Longitudinal tunnel centre line $x=0\text{m}$

(d) Longitudinal end wall $x=10\text{m}$

Figure 8.24: Symmetric analyses: Comparison of numerical models
The symmetric analysis does not have a significant hogging section and as such does not allow the hogging capabilities of the equivalent masonry beams to be fully tested. The results of the SMB runs with masonry beams do not show a significant difference from the results of the SEB runs with the elastic beams. It can be seen, however, that the parameters $f_b$ and $\kappa_{\text{crit}}$ do have an impact on the response of the EMBs. In particular, a value of $\kappa_{\text{crit}}=1.0 \times 10^{-6}$ may be too small. The oblique analyses described in the following section will allow a fuller comparison of the response of the EMBs and the masonry facades in hogging.

### 8.3 Oblique analysis

#### 8.3.1 Description of analysis

The second example problem consists of a tunnel of the same diameter and depth as the symmetric problem under the same masonry building, but with the building aligned such that it is eccentric and oblique to the tunnel centre line. The layout of the problem is shown in figure 8.25. The geometry of this problem is similar to the oblique example used in previous work by Augarde (1997), Liu (1997) and Burd et al. (2000). Four different finite element analysis run types are undertaken with the oblique problem as summarised in table 8.8: the first (OGF) is a greenfield analysis with no building present; the second (OMF) includes a full masonry building on the surface made from 2D facades; thirdly, surface beams with properties determined using the equivalent elastic beam method are used to represent the building (OEB); and finally the building is represented by equivalent masonry surface beams (OMB).

<table>
<thead>
<tr>
<th>FE Run Name</th>
<th>Description of Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>OGF</td>
<td>Oblique Greenfield</td>
</tr>
<tr>
<td>OMF</td>
<td>Oblique with building made of 2D Masonry Facades</td>
</tr>
<tr>
<td>OEB</td>
<td>Oblique with Equivalent Elastic Beams representing the building</td>
</tr>
<tr>
<td>OMB</td>
<td>Oblique with Equivalent Masonry Beams representing the building</td>
</tr>
</tbody>
</table>
Figure 8.25: Oblique example analysis layout

Geometry, mesh and material properties

Figure 8.25 shows the layout of the oblique example problem. As for the symmetric analysis, it consists of a 5m inner diameter tunnel located 10m below ground level. On the surface is the same 20m long, 10m wide and 8m high masonry building with 1m thick walls shown in figure 8.2. The tunnel lining is again 0.25m thick and the applied volume loss is 2%. The soil block for the problem is 100m wide, 50m high and 60m long as shown. The building is located on the surface such that the facades are oriented at 45° to the tunnel alignment and the centre line of the tunnel passes under the front facade at a point 4.5m from the end wall as shown in figure 8.26.

The soil mesh for all the oblique analyses is shown in figure 8.27(a) with the building mesh for the full front and rear facades and end walls for run type OMF shown in figures 8.27(b) and 8.27(c). The soil and tunnel mesh contains 12682 nodes and 8612 ten-noded tetrahedral elements; for run type OMF, the building mesh has 1938 nodes and 836 six-noded triangular plane stress elements; and for run types OEB and OMB, the surface beams comprise 62 Timoshenko beam elements. The boundary conditions applied to the soil are: fully fixed displacements (x, y and z displacements fixed) at the base and horizontal displacements (x or y displacements as appropriate) fixed perpendicular to each vertical
soil boundary.

The material properties assigned to the soil, tunnel, building facades and surface beams are identical to those used for the symmetric analyses and are given in section 8.2.1 above.

**Calculation procedure**

The calculation proceeds in stages similar to the symmetric analyses except that the tunnel for the oblique analyses is constructed in eight stages. Table 8.9 shows the calculation stages and the construction lengths used for the oblique analyses.

As for the symmetric analyses there are two exceptions to the general procedures: the setting of the residual tensile strength for the masonry facades, $c$, to 0.0kPa while the building self weight is applied to the model (stages 1 and 2 in table 8.9), the residual tensile strength is then changed to 10kPa at the commencement of tunnelling (stage 3); and the addition of the masonry beam elements only after stage 2 is complete. Both are designed to avoid stress accumulating in the facades or beams during the self weight loading distorting the displacements due to tunnelling in the later stages.

The number of calculation steps per stage is chosen as 15 steps (Wisser, 2002) for all analyses except the OMB analyses where the number of steps chosen to reduce the out of
Figure 8.27: Oblique example analysis soil mesh
Table 8.9: Calculation stages for oblique example analysis

<table>
<thead>
<tr>
<th>Stage</th>
<th>No. Steps</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>Application of building weight and initial soil stresses</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>Reset displacements and strains to zero</td>
</tr>
<tr>
<td>3</td>
<td>15 (30 for OMB)</td>
<td>Tunnel construction stage 1: 0.0 - 10.0m</td>
</tr>
<tr>
<td>4</td>
<td>15 (30 for OMB)</td>
<td>Tunnel construction stage 2: 10.0 - 20.0m</td>
</tr>
<tr>
<td>5</td>
<td>15 (30 for OMB)</td>
<td>Tunnel construction stage 3: 20.0 - 25.0m</td>
</tr>
<tr>
<td>6</td>
<td>15 (30 for OMB)</td>
<td>Tunnel construction stage 4: 25.0 - 30.0m</td>
</tr>
<tr>
<td>7</td>
<td>15 (30 for OMB)</td>
<td>Tunnel construction stage 5: 30.0 - 35.0m</td>
</tr>
<tr>
<td>8</td>
<td>15 (30 for OMB)</td>
<td>Tunnel construction stage 6: 35.0 - 40.0m</td>
</tr>
<tr>
<td>9</td>
<td>15 (30 for OMB)</td>
<td>Tunnel construction stage 7: 40.0 - 50.0m</td>
</tr>
<tr>
<td>10</td>
<td>15 (30 for OMB)</td>
<td>Tunnel construction stage 8: 50.0 - 60.0m</td>
</tr>
</tbody>
</table>

balance forces is 30 steps.

8.3.2 Greenfield analysis

This section details the results of the greenfield run OGF with no building on the surface. Figure 8.28 shows the surface contours after each tunnelling stage, while figure 8.29 shows the surface profile development in 3D. Vertical surface displacements are shown in figure 8.30 along lines including: (a) the front facade (line FA-FB in figure 8.26); (b) rear facade (RA-RB); (c) north end wall (NA-NB); and (d) the south end wall (SA-SB), for each tunnelling stage.

The greenfield surface displacements for the oblique problem shown in figure 8.28 are not as smooth as the results for the symmetric problem from figure 8.5. Possible reasons for this are that the mesh for the oblique problem is coarser than for the symmetric problem, particularly at the surface over the tunnel centre line; and that with more tunnelling stages, in particular underneath the building footprint, the influence of the stage boundaries (Wisser, 2002) may be enhanced.

The greenfield finite element settlement trough is again shallower and wider than the Gaussian approximation with both curves being shown in figure 8.31. As with the symmetric problem, this feature does not limit the usefulness of the OGF run for use as a reference against which the influence of including a building on the surface, by means of facades
Figure 8.28: Greenfield run OGF: Surface displacement contours
Figure 8.29: Greenfield run OGF: 3D Surface profile
Figure 8.30: Greenfield run OGF: Surface displacements
8.3.3 Analysis with masonry building

Surface displacement results from run type OMF with the building comprising 2D masonry facades are presented in figures 8.32 to 8.34.

The influence of the building due to its self weight is clearly observable as the maximum settlement (under the corner of the front and north end walls) is 28.2mm (figure 8.34) compared to 16.9mm, the maximum settlement in the greenfield analysis (figure 8.30). The magnitude of the displacements along all walls are greater than those for the greenfield run.

The interaction of the building with the ground under each wall differs depending on the shape of the ground displacements under each individual wall. The front facade is mostly in a sagging region (with some slight hogging at the southern end). The response of the facade is essentially rigid for the first five tunnelling stages (figure 8.34(a)) with some slight hogging curvature developing in the final three stages (maximum $\Delta/L=0.004\%$).

The rear facade is mostly in a hogging zone and displays significant hogging curvature as tunnelling develops (maximum $\Delta/L=0.027\%$ in the final stage). The facade is much less
Figure 8.32: Masonry building run OMF: Surface displacement contours
Figure 8.33: Masonry building run OMF: 3D Surface profile
Figure 8.34: Masonry building run OMF: Surface displacements
Table 8.10: Oblique analyses with equivalent elastic surface beams

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>OEB</td>
<td>Properties given in tables 8.3 and 8.4</td>
</tr>
<tr>
<td>OEB2</td>
<td>As for OEB but with $EI_{in} \times 10$</td>
</tr>
<tr>
<td>OEB3</td>
<td>As for OEB but with $EI_{in} \times 1/5$</td>
</tr>
<tr>
<td>OEB4</td>
<td>As for OEB but with $EI_{in} \times 1/10$</td>
</tr>
<tr>
<td>OEB5</td>
<td>As for OEB but with $EI_{in} \times 1/100$</td>
</tr>
<tr>
<td>OEB6</td>
<td>As for OEB but with $EI_{in} \times 1/1000$</td>
</tr>
</tbody>
</table>

rigid than the front. This observation confirms those of Chapters 5 and 6 where masonry walls in hogging were observed to crack and lose their stiffness, thus suffering greater bending curvatures in soil-structure interaction situations than walls in sagging, which through arching, tend to suffer less curvature. This also concurs with the conclusions drawn by Liu (1997), Augarde (1997) and Burd et al. (2000) for analysis of the same problem.

The north end wall is entirely in the sagging region and, as expected, does not exhibit any significant curvature, although it tilts significantly in a rigid fashion (figure 8.34(c)). The south end wall is mostly outside the region of significant influence of the tunnelling (although there is a slight hogging section, see figure 8.34(d)). It does, however, tilt in a rigid fashion and settle approximately 6mm more than the greenfield profile.

8.3.4 Analysis with equivalent elastic surface beams

Following description of the initial run in this section (OEB) which has elastic surface beams representing the building with properties as given in tables 8.3 and 8.4, a range of analyses are used to investigate the effect of differing in-plane bending stiffness assigned to the beams. The analyses described in this section are summarised in table 8.10.

Equivalent elastic surface beams

Surface displacement results from run type OEB with the building represented by elastic beams are presented in figures 8.35 to 8.37.
Figure 8.35: Equivalent elastic surface beams OEB: Surface displacement contours
Figure 8.36: Equivalent elastic surface beams OEB: 3D Surface profile
Figure 8.37: Equivalent elastic surface beams OEB: Surface displacements
The results again show the influence of the building weight adding to the magnitude of the surface displacements relative to the greenfield runs with the maximum displacement of 27.2mm (figures 8.37(a) and (c)) much greater than the maximum greenfield displacement of 16.9mm. The maximum settlement is very similar to the OMF run.

The settlement profile under the equivalent elastic beams is also flattened compared to the greenfield run OGF, due to the relative stiffness of the elastic beams with respect to the soil. For all stages, the elastic beams can been seen to tilt in response to the tunnelling, but do not exhibit any significant curvature.

The settlement profile can also be compared to that under the masonry facades in run OMF. The settlement under the beams for the front and rear walls (figures 8.37(a) and (b)) exhibits a flatter profile with less curvature than the settlement under the masonry facades (figures 8.34(a) and (b)). This is a similar observation to the symmetric analyses in section 8.2.4. In the oblique analyses, however, the difference between the masonry facades and the equivalent elastic beams in hogging is more clear with the elastic beams displaying negligible curvature in hogging (rear facade maximum $\Delta/L=0.0017\%$) compared to the masonry facades (rear facade maximum $\Delta/L=0.0269\%$) but exhibiting similar magnitude and settlement profiles in sagging.

The use of surface beams with equivalent elastic model properties gives very similar surface settlements to those under the full masonry facade for areas in sagging. The resulting surface displacements of the elastic beams in hogging sections however, despite closely matching magnitudes, exhibit negligible curvature under hogging conditions compared to the hogging exhibited by the masonry facades.

**Influence of building stiffness**

Analysis OEB was repeated with the in-plane bending stiffness for the beams varied by using five different multiples as shown in table 8.10. These factors were used to multiply the relevant in-plane bending stiffness properties for the beams given in tables 8.3 and 8.4. Figure 8.38 shows the surface displacements for all beams after the final tunnelling stage.
As with the symmetric analyses, reducing the bending stiffness causes the profile to take a shape more like the greenfield profile, while the building weight acts to increase the magnitude of the displacements relative to the greenfield run. This is more evident in the rear facade (figure 8.38(b)) than the other walls.

For the rear wall in hogging, it can be seen that the elastic beams with bending stiffness reduced to 1% of their original value (analysis OEB5 with $EI_m/100$) most closely replicate the surface profile under the masonry facades (OMF), giving an excellent match. This is interesting in the context of the requirement to choose and appropriate stiffness reduction factor $f_b$ for the equivalent masonry beams as discussed in section 8.2.5 above. It is thus apparent that the appropriate factor is $f_b=0.01$. Further discussion of the choice of $f_b$ for equivalent masonry beams is given below.

All elastic beam results give similar profiles for the front wall, with no significant deviation from the masonry facade profile. For each of the end walls, runs OEB5 and 6 can be seen to have the most similar displacement magnitude to the OMF run.

For sagging regions it can be seen that beams used in OEB with the original properties give a good match to the masonry facade settlement profile. The properties assigned to run OEB are thus considered to be appropriate for areas of sagging, however the in-plane bending stiffness needs to be reduced to a value of 1% of the original value to give a similar displacement profile in hogging.

### 8.3.5 Analysis with equivalent masonry surface beams

Surface displacement results from run OMB21 with the building represented by surface Timoshenko beams with the equivalent masonry beam model are presented in figures 8.39 to 8.41. The parameters of critical curvature and residual bending stiffness factor, used in the equivalent masonry beam model were $\kappa_{\text{crit}}=1.0 \times 10^{-5}$ and $f_b=0.100$ respectively. It was originally intended to use the same values as the symmetric central run ($\kappa_{\text{crit}}=1.0 \times 10^{-5}$ and $f_b=0.010$) but this was not possible due to computer hardware failures limiting the number of runs able to be performed.
Figure 8.38: Equivalent elastic surface beams (Tunnelling Stage 8): Influence of bending stiffness
Figure 8.39: Equivalent masonry surface beams OMB: Surface displacement contours
Figure 8.40: Equivalent masonry surface beams OMB: 3D Surface profile
Figure 8.41: Equivalent masonry surface beams OMB: Surface displacements
Table 8.11: Parameters for equivalent masonry beams - oblique analyses

<table>
<thead>
<tr>
<th>Analysis</th>
<th>( k_{crit} )</th>
<th>( f_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>OMB11</td>
<td>( 1.0 \times 10^{-4} )</td>
<td>0.100</td>
</tr>
<tr>
<td>OMB12</td>
<td>( 1.0 \times 10^{-4} )</td>
<td>0.010</td>
</tr>
<tr>
<td>OMB13</td>
<td>( 1.0 \times 10^{-4} )</td>
<td>0.001</td>
</tr>
<tr>
<td>OMB21</td>
<td>( 1.0 \times 10^{-5} )</td>
<td>0.100</td>
</tr>
</tbody>
</table>

The results again show the influence of the beams on the settlement profile and the additional vertical settlement due to the building weight. In areas where the beams are in sagging the settlement profile is the same as the OEB analysis described above. In areas of hogging, however, in particular the rear wall as shown in figure 8.41(b), there is greater hogging curvature (rear facade maximum \( \Delta/L=0.0038\% \)) than the response of run OEB (rear facade maximum \( \Delta/L=0.0017\% \)). This compares to the maximum hogging curvature of the masonry facade run with rear facade maximum \( \Delta/L=0.0269\% \). Charts with comparisons including the OMF runs are given below.

Influence of \( k_{crit} \) and \( f_b \)

The influence of the critical curvature, \( k_{crit} \) and residual bending stiffness factor, \( f_b \) used in the equivalent masonry beam model is investigated in this section. Analysis type OMB was repeated using the range of different parameters shown in table 8.11. All other properties for the beams remained the same as those given in tables 8.3 and 8.4. The base analysis for which results are presented above is OMB21.

A discussion regarding choice of properties of \( f_b \) and \( k_{crit} \) was given in section 8.2.5. From the oblique analyses described in this section it is apparent that the most appropriate value for \( f_b \) is 0.01. The discussion in section 8.2.5 also indicated that \( k_{crit} \) of \( 1.0 \times 10^{-6} \) was potentially too low as the equivalent masonry beams lost stiffness at too low a hogging curvature. This indicates that the optimum combination of \( f_b \) and \( k_{crit} \) based on these results is \( f_b=0.01 \) and \( k_{crit}=1.0 \times 10^{-5} \).

Section 8.2.5 also contains a discussion on the impact, in terms of requirement for different numbers of calculation steps, of the choice of \( f_b \) and \( k_{crit} \). As discussed above, limitations
Figure 8.42: Equivalent masonry surface beams (Tunnelling Stage 8): Influence of $\kappa_{\text{crit}}$ and $f_b$. 

(a) Front facade

(b) Rear facade

(c) North end wall

(d) South end wall
in the KONRAD hardware systems meant that it was not possible to obtain results for what would have been analyses OMB22 to OMB33 (in a similar fashion to the symmetric analyses). Repeated failures of the computing systems prevented the analysis of these models. It is recommended that these further analyses form part of future investigations.

The results in figure 8.42 show that varying $f_b$ and $\kappa_{\text{crit}}$ in the range shown has minimal effect on the displacement results with the OMB results generally plotting under the red OEB line. Run OMB21 can, however, be seen in figure 8.42(b) to hog more than the other runs, but not enough to replicate the response of the masonry facades.

### 8.3.6 Comparison of models

**Numerical models**

A clear comparison of each of the numerical models for the final tunnelling stage is given in figure 8.43. These charts show the main analyses without additional parametric results and are thus easier to compare. Discussion of the ability of the equivalent elastic and masonry beams to replicate the response of the masonry facades has been given in the preceding sections but figure 8.43 clearly shows the key conclusions. In sagging, the equivalent masonry surface beams replicate the response of the masonry facades very well, particularly for the front facade. In hogging, however, the masonry beams do not suffer the same relative displacement as the facades. This is further investigated using traditional response measures below.

**Traditional assessment measures**

Comparisons of the key responses of each of the masonry facade, elastic beam and masonry beam analyses from figure 8.43 are given using traditional building assessment parameters in table 8.12. The traditional assessment is undertaken using the methods described in section 2.5 including the Potts and Addenbrooke (1997) relative stiffness approach.

Following the approach of Augarde (1997) and Wisser (2002), the ground stiffness is given
Figure 8.43: Oblique analyses: Comparison of numerical models
a value of $E_s = 108 \times 10^3$ kPa, which is 80% of the Young’s modulus of the soil at a depth of $z/2$, where $z$ is the depth to the tunnel centreline. For this analysis the bending and axial stiffness of the facades is calculated using the equivalent elastic approach of section 4.3. In hogging sections, the bending stiffness is reduced to 1% of the value in sagging reflecting the loss of stiffness in hogging as discussed in section 8.3.4. Note that this differs from the approach taken by Augarde (1997) and Wisser (2002) where the stiffness was reduced by 10%. If the stiffness is not reduced in hogging, the Potts and Addenbrooke (1997) relative stiffness method predicts a very rigid facade in hogging which is unrealistic.

Key features of the traditional approach are compared to similar results implied by the finite element analyses. In particular, the deflection ratio modification factors presented by Potts and Addenbrooke (1997) can be compared to those inferred from the displacement results from the finite element runs.

Key items of comparison from table 8.12 include that the finite element greenfield analysis gives less severe relative displacements and overall displacement magnitude than the traditional approach. For the rear facade, the different hogging response of the methods is clearly shown with the equivalent masonry beams hogging more than the elastic beams, but less than the masonry facades. The implied modification factor for the masonry beams is, however in excellent agreement with the factor from the Potts and Addenbrooke (1997) approach despite the magnitude of hogging being too low. For the front facade none of the finite element approaches exhibited any sagging leading to implied modification factors of 0.0 in this mode. In hogging, the masonry facades and beams exhibited greater relative hogging displacement than the greenfield analysis leading to implied modification factors greater than 1.0. This indicates the relative susceptibility of masonry facades to hogging displacements. Differences between the traditional and finite element approaches are thought to result from the fact that the traditional approach involves projecting the building onto the transverse settlement trough, whereas for the finite element results, displacement profiles were taken exactly under the facade alignment. The fact that the greenfield finite element profile is shallower and wider than the Gaussian profile is also a differentiating factor.
Table 8.12: Oblique Analyses: Traditional Assessment and Finite Element Model Comparison

<table>
<thead>
<tr>
<th>REAR FACADE</th>
<th></th>
<th>Traditional</th>
<th>OGF</th>
<th>OMF</th>
<th>OEB</th>
<th>OMB</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Greenfield (prior to considering building)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Settlement (mm)</td>
<td>31.3</td>
<td>16.9</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Max $\Delta/L$ Hog ($DR_{hog}^g$)</td>
<td>0.0579</td>
<td>0.0136</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Max $\Delta/L$ Sag ($DR_{sag}^g$)</td>
<td>0.0025</td>
<td>0.0229</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
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<td><strong>Potts and Addenbrooke (1997) modification factors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$M_{D_{hog}}$</td>
<td>0.2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$M_{D_{sag}}$</td>
<td>0.1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Modified traditional</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max $\Delta/L$ Hog ($DR_{hog}^g$)</td>
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<td>-</td>
<td>-</td>
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<td>-</td>
</tr>
<tr>
<td>Max $\Delta/L$ Sag ($DR_{sag}^g$)</td>
<td>0.0003</td>
<td>-</td>
<td>-</td>
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<td><strong>Finite element results with building</strong></td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Max Settlement (mm)</td>
<td>-</td>
<td>-</td>
<td>24.9</td>
<td>21.9</td>
<td>21.7</td>
<td></td>
</tr>
<tr>
<td>Max $\Delta/L$ Hog (%)</td>
<td>-</td>
<td>-</td>
<td>0.0269</td>
<td>0.0017</td>
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</tr>
<tr>
<td>Max $\Delta/L$ Sag (%)</td>
<td>-</td>
<td>-</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
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</tr>
<tr>
<td><strong>Implied modification factors from finite element results</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Implied $M_{D_{hog}}$</td>
<td>-</td>
<td>-</td>
<td>1.97</td>
<td>0.13</td>
<td>0.28</td>
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<tr>
<td>Implied $M_{D_{sag}}$</td>
<td>-</td>
<td>-</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td><strong>FRONT FACADE</strong></td>
<td></td>
<td>Traditional</td>
<td>OGF</td>
<td>OMF</td>
<td>OEB</td>
<td>OMB</td>
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<tr>
<td><strong>Greenfield (prior to considering building)</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Max Settlement (mm)</td>
<td>31.3</td>
<td>16.9</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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<tr>
<td>Max $\Delta/L$ Hog ($DR_{hog}^g$)</td>
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<td>0.0018</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Max $\Delta/L$ Sag ($DR_{sag}^g$)</td>
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</tr>
<tr>
<td>$M_{D_{hog}}$</td>
<td>0.2</td>
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<td>-</td>
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<td>$M_{D_{sag}}$</td>
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<td><strong>Modified traditional</strong></td>
<td></td>
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</tr>
<tr>
<td>Max $\Delta/L$ Hog ($DR_{hog}^g$)</td>
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<td>-</td>
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<td>-</td>
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<td>Max $\Delta/L$ Sag ($DR_{sag}^g$)</td>
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<td>-</td>
</tr>
<tr>
<td><strong>Finite element results</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max Settlement (mm)</td>
<td>-</td>
<td>-</td>
<td>28.2</td>
<td>27.2</td>
<td>27.5</td>
<td></td>
</tr>
<tr>
<td>Max $\Delta/L$ Hog (%)</td>
<td>-</td>
<td>-</td>
<td>0.0082</td>
<td>0.0006</td>
<td>0.0013</td>
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</tr>
<tr>
<td>Max $\Delta/L$ Sag (%)</td>
<td>-</td>
<td>-</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td><strong>Implied modification factors from finite element results</strong></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Implied $M_{D_{hog}}$</td>
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<td>-</td>
<td>4.53</td>
<td>0.31</td>
<td>0.71</td>
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<tr>
<td>Implied $M_{D_{sag}}$</td>
<td>-</td>
<td>-</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>
8.3.7 Summary

The comparison of the response of equivalent elastic and masonry beams to that of masonry facades in analyses of tunnelling under a building oblique to the centreline results in the following key points.

The response of the equivalent elastic beams shows good agreement with the profile of the masonry facades for sagging regions. In hogging regions, however, the EEBs are too stiff. Analyses of EEBs with a range of reduced in-plane bending stiffnesses leads to the conclusion that a reduction in bending stiffness to 1% of the original value is required to match most closely the response of the masonry facades in hogging.

The equivalent masonry beams respond in a similar fashion to the elastic beams in sagging as expected. In hogging regions with the parameters investigated, the beams can be seen to lose stiffness and hog more than the elastic beams, but not to the degree necessary to replicate exactly the masonry facade response. More combinations of suitably low values of critical curvature ($\kappa_{\text{crit}}$) and residual bending stiffness ($f_b$) should be investigated as part of future work in order to improve the response of the equivalent masonry beams.

8.4 Conclusion

The numerical models of building response to tunnelling presented in this chapter were undertaken to evaluate the three-dimensional surface beam approach for modelling masonry buildings developed in earlier chapters of this thesis. In particular, to assess the use of 3D Timoshenko beam elements and the equivalent elastic and equivalent masonry beam models to replicate masonry buildings by comparison with finite element analyses. The discussion and summaries presented in this chapter lead to a number of conclusions in this respect.

It is apparent that the stiffness of surface beams representing masonry facades in sagging should be determined by using the procedures accounting for the openings and geometry of the individual facades developed in this thesis. It is also clear that a reduction of the in-plane bending stiffness for surface beams in hogging is required to reflect properly
the response of masonry facades. Beams with properties in sagging as described by the equivalent elastic beam method are thus recommended, but in hogging the in-plane bending stiffness should be reduced by a residual stiffness factor, $f_b=0.01$.

The rate at which the equivalent masonry beams lose their stiffness is governed by the critical curvature. The most appropriate value of the critical curvature, is recommended as $\kappa_{\text{crit}}=1.0 \times 10^{-5}$. This recommendation is made subject to the requirement for further investigation of the parameters $f_b$ and $\kappa_{\text{crit}}$ in analyses involving different amounts of hogging curvature.

Overall, the use of equivalent masonry beams to model masonry facades in tunnelling analyses using the methods developed in this thesis has been shown to be a useful and successful alternative to modelling the full building in a finite element analysis.

Numerical modelling of masonry building response to tunnelling using equivalent masonry beams with properties as determined in this chapter will be evaluated by comparison to tunnelling case studies in Chapter 9.
Chapter 9

Numerical Modelling of Building Response to Tunnelling: 3D Case Studies

9.1 Introduction

Numerical models in civil engineering are developed as tools for use in the design and analysis of engineering problems. A key part of this development is the establishment of the validity of new methods by comparing their numerical output (whether this is predictions of loads, movements, damage or other results) with case study data. Unfortunately, access to high quality case study data for this purpose is often difficult to obtain, particularly in the geotechnical field. For the analysis of building response to tunnelling in soft ground, however, a significant amount of data have recently become available. These data were collected during the construction of the Jubilee Line Extension (JLE) project on the London Underground as part of a research project specifically focused on building response to tunnelling. The JLE works were completed in 1996 and an extensive two volume record of the research undertaken throughout the construction: *Building Response to Tunnelling: Case studies from the construction of the Jubilee Line Extension, London* (edited by
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Burland, Standing and Jardine) was published in 2001. The published research includes
27 case histories, 21 of which detail the response of buildings or groups of buildings, with the
remainder describing greenfield sites. From this source, the buildings chosen as case studies
for this thesis are Murdoch, Clegg and Neptune Houses in Moodkee Street, Rotherhithe.
These buildings, and the tunnelling beneath them, are described in detail in this chapter
along with an outline of the JLE and the associated research projects. Details of the
JLE construction and research are from the Burland et al. (2001) volumes; the authors of
individual chapters of which are cited where appropriate. The numerical methods described
in preceding chapters are used to develop a numerical model of tunnel construction at
the Moodkee Street site to investigate the response of the three buildings to tunnelling.
Two models are analysed, one greenfield model for comparison with observed greenfield
settlements (at sites near the chosen buildings) and one with the buildings represented by
equivalent masonry beams. The predictions of building response from the finite element
model are compared with the observed response from the case history.

9.2 The Jubilee Line Extension and associated
research projects

9.2.1 Overview of the JLE construction project

The JLE project comprised 15.5km of new twin-tube railway and 11 stations built between
1994 and 1996 to extend the Jubilee Line on the London Underground network from the
existing Green Park station in the West End to Stratford in East London. For a distance
of approximately 11.5km, the western portion is in twin tunnels of 4.4m inner diameter
located from 20 to 30m below ground (Jardine, 2001).

The ground conditions along the route vary from London Clay in the west to the Lambeth
Group further east. London Clay is a stiff to very stiff, heavily overconsolidated clay
with a typical undrained shear strength in the region of 200kPa. The Lambeth Group
consists of interbedded layers of stiff clays, sands, silts and gravels (Withers et al., 2001).
Figure 9.1: **Geological plan of the JLE route (after Withers et al., 2001)**

A description of the Lambeth Group and its engineering properties is given in section 9.3 of this thesis. Figure 9.1 shows the route outline over a plan of the geology of London and also indicates the area between Green Park and Canada Water in which case studies were conducted.

Tunnelling methods employed along the route differed with the ground conditions. In the London Clay the running tunnels were constructed using open-faced tunnelling shields with a backhoe or roadheader excavator mounted within the shield. The sprayed concrete lining method was used for the construction of the Waterloo and London Bridge station tunnel enlargements, also in the London Clay. In the section to the east of London Bridge, tunnelling was in the Lambeth Group, a much more difficult medium in which to tunnel due to its waterbearing nature and the different properties of each of its component units. Tunnelling here was undertaken using closed-face shields of both slurry and earth pressure balance (EPB) types (Mair and Jardine, 2001). Further details of the soil conditions and tunnel construction at the Moodkee Street case study site are given in section 9.3 below.
9.2.2 Overview of the JLE related research project

The two volumes of *Building Response to Tunnelling* (Burland et al., 2001) record the results of three related research projects. The main project focused on the response of buildings to the tunnelling works and was funded under the Construction Maintenance and Refurbishment (CMR) programme of the LINK scheme sponsored by (what are now called) the Department of the Environment Transport and the Regions (DETR) and the Engineering and Physical Sciences Research Council (EPSRC). The other two research projects were funded by an EPSRC grant and London Underground Limited (LUL) sponsorship and were concerned with the monitoring of surface and subsurface displacements at the St James’s and Southwark Park greenfield sites (Jardine, 2001a).

The research was a collaborative effort between the scientific partner, Imperial College, the lead partner, the Construction Industry Research and Information Association (CIRIA), and initial industry partners, LUL and the Geotechnical Consulting Group. Other industry partners included: AMEC Piling, Arup, Mott MacDonald, TRL, Union Railways and Trafalgar House Technology.

The objectives of the research were to use field measurements to improve the understanding of building response to ground movements due to tunnelling, damage resulting from such movements and measures for the protection of buildings (Jardine, 2001a). Measurements were made of surface and sub-surface settlements, observed cracking and damage to buildings, horizontal strains, and full facade monitoring. The measurements were undertaken using precise surface levelling, precision tape extensometers, inclinometers and total surveying stations. A full description of the methodology is contained in Jardine (2001a).

The case study sites lie between Green Park and Canada Water as shown in figure 9.1. Among the 21 case histories of buildings in the project, a range of building and footing types are represented. Fourteen buildings are load-bearing masonry; five are reinforced concrete; and two steel and masonry framed structures. The most common footing types are pad or strip footings, with five buildings on piles and five on a raft footing. At many of the sites, compensation grouting was employed to protect the buildings. The ground
conditions can be generally categorised as either London Clay or the Lambeth Group.

The range of data collected and reported thus provides researchers with the opportunity to select appropriate case study sites for further research from the large number of case histories reported.

9.3 Moodkee Street case study buildings

9.3.1 Choice of buildings for case study

The choice of case study buildings for this thesis from the range of case histories available was made using criteria based on the objectives of the current research (as described in Chapter 3) and consideration of both the numerical methods to be validated and those in use at Oxford University. These criteria included that the buildings should be:

- Constructed of load bearing masonry to enable the use and validation of the equivalent masonry beam model for surface beams representing masonry facades;
- Built on simple strip or spread footings to avoid the modelling of pile footings which are not part of this research;
- Unprotected by compensation grouting as the modelling of this protection method is not part of this research; and
- Oriented asymmetrically with respect to the tunnel to test fully the 3D capabilities of the numerical methods developed.

The buildings chosen for this case study, based on the above criteria, are Murdoch, Clegg and Neptune Houses, a complex of three buildings in Moodkee Street, Rotherhithe. These buildings are described in detail in section 9.3.2 below and were chosen because, of the JLE case histories (and indeed, of all the published case histories reviewed by the author), they best met the criteria outlined above.
Figure 9.2: Plan of Moodkee Street case study buildings, Rotherhithe (after Mair and Taylor, 2001)

9.3.2 Description of buildings

Murdoch, Clegg and Neptune Houses are located at the corner of Neptune and Moodkee Streets, Rotherhithe, approximately 200m west of Canada Water station. They were constructed in 1931 for the employees of the South Metropolitan Gas Company. The layout of the buildings is shown in figure 9.2 including the location of the JLE tunnels under the buildings. Each building is constructed of solid load bearing brick masonry (as opposed to cavity wall construction) and is three storeys high. Murdoch and Neptune Houses are approximately 40m by 8m in plan, with Clegg House being half as long. Photographs of the buildings taken by the author in December 2004 are shown in figure 9.3.

The footings for the buildings are shown in figure 9.4. These comprise strip footings with stepped brickwork under the wall to a depth of 450mm atop a 910mm wide mass concrete strip. The concrete strip is typically 430mm thick, except for a section of Murdoch House, where part of the footing was thicker (600mm) and was constructed on a number of concrete piers (approximately 3.2m deep) where the building outline crossed the site of an old pond.
Figure 9.3: Moodkee street buildings

Figure 9.4: Moodkee street building footings
9.3.3 Ground conditions

The ground conditions at the site consist of 2m of made ground overlying 4m of Thames Gravel. The gravel lies on top of the various units of the Lambeth Group, which extends to a depth of 20 to 25m, under which lies a bed of Thanet Sand to the top of the Upper Chalk formation at a depth of approximately 40m. Figure 9.5 shows a typical section near the site, with the various stratigraphic units and the location of the tunnels relative to the buildings. The soil layer information is from Mair and Taylor (2001) and Withers (2001b) with the material descriptions and layer depths taken from borehole 417 located approximately 20m south west of Neptune House. Geotechnical properties for each of the soil layers are given by Withers et al. (2001); the soil model and material parameters used in the numerical model are described in section 9.4.
9.3.4 Settlement and damage predictions

Prior to the observed site settlements being made available, predictions were made by the Geotechnical Consulting Group (GCG) regarding the expected greenfield surface settlements, the settlements under each of the buildings and the expected damage to the structures at the site. These predictions formed a report on the structures prepared at the time of the tunnel construction and are detailed by Mair and Taylor (2001). Each of the buildings was predicted to behave almost rigidly in response to the tunnelling-induced ground movements and incur negligible damage. The actual predictions made are detailed and compared to both the observed site settlements and the settlement results from the finite element analysis of this thesis in section 9.5.

9.3.5 Tunnel construction

The JLE running tunnels were constructed beneath the Moodkee Street site using earth pressure balance shields. The tunnels are located approximately 17m below Neptune House, rising towards Canada Water station in the East, with the shields passing under the site during February (Westbound) and July (Eastbound) 1996. The shields used were 7m long Kawasaki machines of 5m outer diameter. They erected 1.2m long, 0.25m thick precast concrete bolted ring segments at the rear of the shield as a permanent tunnel lining. The finished tunnels have an outer diameter of 4.9m and an inner diameter of 4.4m.

9.3.6 Observed settlements during tunnelling

During tunnelling, on-site measurements were made of surface settlements. Methods used for settlement monitoring included precise levelling on sockets in the building walls and tape extensometer measurements between the levelling sockets (Withers, 2001b). The locations of the various monitoring points are shown in figure 9.2. The observed site settlements are compared to both predictions the finite element results of this thesis in section 9.5.
Figure 9.6: Layout of Moodkee Street case study problem

9.4 Composition of numerical model

Model layout and tunnelling

The geometry of the case study problem assumed for the numerical model is shown in figure 9.6. Only the westbound running tunnel is modelled as this was the first tunnel constructed and individual (single tunnel) data are thus available.

Figure 9.7 shows a plan of the site used for the numerical model. The location of the building footprints on the plan is detailed in table 9.1. Tunnel construction is modelled in stages with the stage boundaries representing the location of the tunnel heading during actual construction at the dates shown. The tunnel was modelled at a constant 17m depth (to the centreline) with an inner diameter to the inside of the tunnel lining of 4.4m and a 250mm thick lining. The lining was modelled as a linear elastic concrete material with properties given in table 7.1.

The footing plan assumed is shown in figure 9.8 and is a simplified version of the actual footings shown in figure 9.4.

The mesh used for the case study finite element model is shown in figure 9.9. Boundary conditions applied to the soil mesh are fully fixed displacements \((x, y\) and \(z\) displacements)
Figure 9.7: Plan of Moodkee Street case study problem

(a) Simplified footing plans assumed for numerical model
(b) Section through footings assumed for all Moodkee Street buildings

Figure 9.8: Simplified footings for model of Moodkee Street buildings
fixed) at the base and horizontal displacements (x or y displacements as appropriate) fixed perpendicular to each vertical soil boundary.

### Soil model and material parameters

The soil profile assumed at the site is that shown in figure 9.5 above. This soil profile is approximated for modelling purposes by the profile shown in figure 9.6 comprising 25m of Lambeth Group soil overlying 15m of Thanet Sand with the base of the soil mesh taken at the top of the Upper Chalk formation at a depth of 40m.

The soil model used for both soil layers is the nested yield surface model described in section 7.4. This model was used as it is important to be able to model the non-linearity of the soil response at small strains. This model is the only soil model capable of capturing
Table 9.2: Material properties for Lambeth Group layer

<table>
<thead>
<tr>
<th>Property</th>
<th>Withers (2001)</th>
<th>Other calc’s/comments/ref’s</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>15-21kN/m$^3$</td>
<td>Made ground: 15-19kN/m$^3$, alluvium: 16-20kN/m$^3$, terrace gravel: 19-20kN/m$^3$, Lambeth Group: 19-21kN/m$^3$</td>
<td>20kN/m$^3$</td>
</tr>
<tr>
<td>$s_{u0}$</td>
<td>$s_{u0}=15-70$kPa</td>
<td>Range for made ground. Surface value chosen as approx average</td>
<td>40.0kPa</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$s_{u0}=100-400$kPa</td>
<td>Group range. Increase in $s_u$ with depth chosen appropriate for depth. At $z=20$m with $\mu=4.0$kPa/m, $s_{u0}=120$kPa thus within range</td>
<td>4.0kPa/m</td>
</tr>
<tr>
<td>$G_s^0$</td>
<td>$E_u/s_u=1500$</td>
<td>From Figure 5.9 of Withers et al. (2001). $G_s^0/s_u=1500/2(1+\nu)$ or $G_s^0=500s_u$ (rigidity value from Wisser (2002))</td>
<td>$2.0 \times 10^4$kPa</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Nil</td>
<td>$\omega=G_s^0/10$ (Wisser (2002))</td>
<td>$2.0 \times 10^3$kPa/m</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Nil</td>
<td>Nil</td>
<td>0.49</td>
</tr>
</tbody>
</table>

**Yield surface parameters**

<table>
<thead>
<tr>
<th>Surface</th>
<th>$c'_\alpha$</th>
<th>$g'_\alpha$</th>
<th>Surface</th>
<th>$c'_\alpha$</th>
<th>$g'_\alpha$</th>
<th>Surface</th>
<th>$c'_\alpha$</th>
<th>$g'_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.020</td>
<td>0.900</td>
<td>4</td>
<td>0.100</td>
<td>0.300</td>
<td>7</td>
<td>0.300</td>
<td>0.100</td>
</tr>
<tr>
<td>2</td>
<td>0.040</td>
<td>0.750</td>
<td>5</td>
<td>0.150</td>
<td>0.200</td>
<td>8</td>
<td>0.500</td>
<td>0.050</td>
</tr>
<tr>
<td>3</td>
<td>0.060</td>
<td>0.500</td>
<td>6</td>
<td>0.200</td>
<td>0.150</td>
<td>9</td>
<td>0.700</td>
<td>0.025</td>
</tr>
</tbody>
</table>

This small strain non-linear response available in the OXFEM program. The use of a model developed for cohesive soils for the non-cohesive layers at this site (the Glauconitic sand and the Thanet sand) was felt to give a more accurate response than a non-cohesive model (such as a linear elastic-frictional model) with no facility for modelling non-linearity at small soil strains. The properties used in the model for the Lambeth Group and Thanet Sand layers are described below.

The Lambeth Group properties are based on those given by Withers et al. (2001) for all components of the Group using the simplifying assumption that the upper 25m of model soil represents only the Lambeth Group and that this layer comprises only one material. The values selected were derived from the literature as outlined in table 9.2. The yield surface parameters are those presented by Houlsby (1999). The ratio of horizontal to vertical stress is chosen as 1.0 for all soil layers.

The properties chosen for the Thanet Sand model layer are derived from information presented by Kovacevic et al. (2001) and Powrie and Batten (2000). The use of a model
Table 9.3: Material properties for Thanet Sand layer

<table>
<thead>
<tr>
<th>Property</th>
<th>Reference value</th>
<th>Reference/calc’s/comments</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>22kN/m$^3$</td>
<td>Kovacevic et al. (2001)</td>
<td>22kN/m$^3$</td>
</tr>
<tr>
<td>$G_0^s$</td>
<td>$E' = 300$MN/m$^2$</td>
<td>Powrie and Batten (2000). $E_u$ assumed to be slightly higher in range 300-320 MN/m$^2$. Then $G_0^s = E_u/2(1+\nu)$</td>
<td>$1.05 \times 10^5$kPa</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Nil</td>
<td>$\omega = G_0^s/10$ (Wisser (2002))</td>
<td>$1.05 \times 10^4$kPa/m</td>
</tr>
<tr>
<td>$s_{u0}$</td>
<td>$G_0^s$</td>
<td>$s_{u0} = G_0^s/500$ (Wisser (2002))</td>
<td>210kPa</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Nil</td>
<td>$\mu = s_{u0}/10$ (Wisser (2002))</td>
<td>21kPa/m</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Nil</td>
<td></td>
<td>0.49</td>
</tr>
</tbody>
</table>

Yield surface parameters

<table>
<thead>
<tr>
<th>Surface</th>
<th>$c'_a$</th>
<th>$g'_a$</th>
<th>Surface</th>
<th>$c'_a$</th>
<th>$g'_a$</th>
<th>Surface</th>
<th>$c'_a$</th>
<th>$g'_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.020</td>
<td>0.900</td>
<td>4</td>
<td>0.100</td>
<td>0.300</td>
<td>7</td>
<td>0.300</td>
<td>0.100</td>
</tr>
<tr>
<td>2</td>
<td>0.040</td>
<td>0.750</td>
<td>5</td>
<td>0.150</td>
<td>0.200</td>
<td>8</td>
<td>0.500</td>
<td>0.050</td>
</tr>
<tr>
<td>3</td>
<td>0.060</td>
<td>0.500</td>
<td>6</td>
<td>0.200</td>
<td>0.150</td>
<td>9</td>
<td>0.700</td>
<td>0.025</td>
</tr>
</tbody>
</table>

dveloped for cohesive soils for such material is not the most realistic approach, but it is felt that this is worthwhile for the ability to include non-linear behaviour at small strains. The values selected were derived from the literature as outlined in table 9.3

Properties for beams representing buildings

The dimensions used for each of the facades to determine their equivalent beam properties are given in figures 9.10 to 9.13. These dimensions are based on site measurements and photographs (not construction drawings). In addition, it is assumed that the end walls of all buildings have the dimensions and layout shown in figure 9.13. In reality each building has one end wall of approximately 9.5m long and three sets of windows. It is not felt that this simplification significantly affects the results of the numerical model relative to the case study.

The properties for each of the walls modelled are given in table 9.4. The properties are derived using the methods described in Chapters 4 and 5 of this thesis where definitions of the various properties are also given. A smooth base is assumed for the determination of the properties as it is felt that strip footings will allow some movement between soil and footing at foundation level, unlike a piled raft for example. The short interior walls
Figure 9.10: Clegg House front dimensions (mm)

Figure 9.11: Clegg House rear dimensions (mm)
Figure 9.12: Neptune/Murdoch House front and rear dimensions (mm)
Figure 9.13: **All building end dimensions (mm)**

are assumed to have the same properties as the end walls and the long interior walls are assumed to have the same properties as the front facade of the respective building. The footings are included in the assessment of the properties of the beams to represent the walls. The \( \kappa_{\text{crit}} \) and \( f_b \) values for the beams are those deemed to be most appropriate from the discussions in Chapter 8.

The walls and the footings are assumed to have a self weight of 20\( kN/m^3 \) which contributes to the loads on the beams. In addition a dead load for floor and roof loading of 0.5\( kN/m^2 \) is assumed. Self weight and dead loads are calculated using the building dimensions and are applied at the nodal points on the surface beams. The loads differ from node to node due to openings in the facades.

**Calculation procedure**

The calculation is undertaken in stages in a similar fashion to the procedure described for the example analyses in section 8.2.1. The volume loss used in the excavation is 0.8%, which is the reported observed volume loss for the westbound tunnel (Withers, 2001b). Table 9.5 details the calculation stages for the analysis.
Table 9.4: Material properties for equivalent masonry surface beams

<table>
<thead>
<tr>
<th>Clegg House front wall and long interior wall</th>
<th>Clegg House rear wall</th>
<th>Murdoch/Neptune Houses front wall and long interior wall</th>
<th>Murdoch/Neptune Houses rear wall</th>
<th>All Houses end walls and short interior walls</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>$G$</td>
<td>$A^*$</td>
<td>$I_{in}^*$</td>
<td>$I_{out}^*$</td>
</tr>
<tr>
<td>$1.000 \times 10^7$ kPa</td>
<td>$4.167 \times 10^6$ kPa</td>
<td>1.336 m$^2$</td>
<td>13.631 m$^4$</td>
<td>$5.637 \times 10^{-3}$ m$^4$</td>
</tr>
<tr>
<td>$J$</td>
<td>$k$</td>
<td>$\gamma$</td>
<td>$\kappa_{crit}$</td>
<td>$f_b$</td>
</tr>
<tr>
<td>$2.117 \times 10^{-2}$ m$^4$</td>
<td>0.85</td>
<td>20.0 kN/m$^2$</td>
<td>$1.00 \times 10^{-5}$</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 9.5: Calculation stages for Moodkee Street case study analysis

<table>
<thead>
<tr>
<th>Stage</th>
<th>No. Steps</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15</td>
<td>Application of building weight and initial soil stresses</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>Reset displacements and strains to zero</td>
</tr>
<tr>
<td>3</td>
<td>15 GF / 30 EMB</td>
<td>Tunnel construction stage 1: 00.0 - 30.0m</td>
</tr>
<tr>
<td>4</td>
<td>15 GF / 30 EMB</td>
<td>Tunnel construction stage 2: 12.5 - 60.0m</td>
</tr>
<tr>
<td>5</td>
<td>15 GF / 30 EMB</td>
<td>Tunnel construction stage 3: 25.0 - 75.0m</td>
</tr>
<tr>
<td>6</td>
<td>15 GF / 30 EMB</td>
<td>Tunnel construction stage 4: 30.0 - 90.0m</td>
</tr>
<tr>
<td>7</td>
<td>15 GF / 30 EMB</td>
<td>Tunnel construction stage 5: 35.0 - 120.0m</td>
</tr>
</tbody>
</table>
9.5 Numerical modelling results

9.5.1 Introduction

Two finite element analyses are undertaken in this section using the model described above. A greenfield analysis is undertaken and the results compared to observed JLE project greenfield settlements at a number of sites near the Moodkee Street location. A combined analysis with the buildings included is then undertaken, with buildings represented by equivalent masonry beams. Observed building displacements are compared to the finite element results as well as the predicted building responses from the published JLE studies.

9.5.2 Greenfield analysis

The results of the greenfield finite element analysis are shown in figures 9.14 and 9.15 for each tunnelling stage as construction progresses. A transverse section of the settlement profile after the final tunnelling stage is shown in figure 9.16 along with observed settlements at a number of monitored JLE greenfield sites. Table 9.6 contains a description of each of the greenfield sites which were all monitored for settlements due to EPBM tunnelling through Lambeth Group soils and thus provide a good reference. The monitoring and the observed displacements at the sites are fully described by Withers (2001a).

The finite element displacements (obtained using an input volume loss of 0.8% as noted in section 9.4) can be seen to be shallower and wider than a Gaussian curve calculated using the same volume loss and a trough width parameter of 0.45. This is a common trait of finite element models of greenfield sites as discussed in sections 2.3.2 and 2.4. The maximum finite element settlement closely matches the maximum settlements from each of the observed case study greenfield sites, and in the central trough region exhibits a similar shape, in particular to the Old Jamaica Road settlements. The shape of the trough, however, is shallower and wider than the Niagara Court greenfield site. This site is the closest to the Moodkee Street buildings and the observed trough there closely matches the Gaussian curve. This indicates that the results from the combined finite element analyses
Figure 9.14: Moodkee Street FE Model Greenfield run: Surface displacement contours
Figure 9.15: Moodkee Street FE Model Greenfield run: 3D Surface profile
Figure 9.16: JLE observed greenfield displacements and finite element results (transverse slice at $Y=60m$)

Table 9.6: Observed greenfield sites presented in JLE case studies

<table>
<thead>
<tr>
<th>Site</th>
<th>Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Niagara Court</td>
<td>Withers (2001a)</td>
<td>Located 10m east towards Canada Water Station from Clegg House in similar ground conditions. Settlements were monitored at the base of a low (0.8m) brick wall.</td>
</tr>
<tr>
<td>Southwark Park</td>
<td>Withers (2001a)</td>
<td>Located 300m west of the Moodkee Street site. Ground conditions are generally similar to the site although the tunnel is slightly deeper in the Glauconitic Sand layer. Regularly spaced survey points located on open grass.</td>
</tr>
<tr>
<td>Old Jamaica Road</td>
<td>Withers (2001a)</td>
<td>Located approximately 1km west of the Moodkee Street site. Ground conditions are similar but with the level of the tunnels slightly deeper and in the lower clayey beds of the Lambeth Group. An irregular line of survey points was situated on asphalt and concrete pavement slabs.</td>
</tr>
</tbody>
</table>
with buildings will be based on corresponding greenfield finite element settlements which exhibits less severe curvatures than were observed in the field. It should also be noted, however, that the volume loss calculated for the observed JLE greenfield sites was in the range of 0.4-0.6%; the shapes of the observed troughs thus differ from the finite element trough which exhibits small displacements at locations remote from the tunnel centre line which contribute to its 0.8% volume loss.

It is interesting to compare the finite element results above to those published by Kovacevic et al. (2001) for a finite element model of the Southwark Park greenfield site. The authors present a range of displacements resulting from the use of volume loss values of 0.5, 1.2 and 2.0 per cent. For the westbound tunnel the maximum settlement values are 3.3mm, 9.3mm and 16.5mm respectively. The maximum settlement of 4.3mm for the Moodkee Street site finite element model presented in this thesis compares favourable with these Southwark Park results when considering that a volume loss of 0.8% was used here.

9.5.3 Combined analyses with buildings

The results of the finite element analysis including equivalent masonry beam models of the three buildings are shown in plan and three dimensions in figures 9.17 and 9.18 respectively. Figures 9.19 to 9.24 show comparisons of the finite element displacements along each of the building walls, with the observed as well as the predicted building displacements as reported by Withers (2001b). The predicted greenfield and the finite element greenfield settlements (along each building wall) are also shown for comparison.

The predicted settlements were made by the Geotechnical Consulting Group (GCG) as part of the JLE research project and are detailed by Mair and Taylor (2001). The method of predicting the settlements and building response followed the approach described in section 2.5 including the relative stiffness method (Potts and Addenbrooke, 1997). The predictions are reproduced in this thesis from the comparisons given by Withers (2001b) and the predictions by Mair and Taylor (2001). A summary of the building settlements and results from the finite element model and the observed data is included in table 9.7.
Figure 9.17: Moodkee Street FE Model with Equivalent Masonry Beams: Surface displacement contours
Figure 9.18: Moodkee Street FE Model with Equivalent Masonry Beams: 3D Surface profile
Figure 9.19: Predicted, Observed and Finite Element Settlements for Neptune House (Rear)

Neptune House

Figures 9.19 and 9.20 show the settlements of Neptune House. The building is wholly in hogging mode with a maximum observed settlement of almost 3mm. Neptune House was predicted to behave almost rigidly in response to the tunnelling using this method. The observed settlement magnitudes were very similar to those predicted, but the response of the building was more flexible. Both the front and rear facades acted more flexibly than predicted in hogging with the maximum deflection ratio ($\Delta/L$) value being 0.003%. The shape of the building displacement profile can be seen to follow a broadly similar shape to that of the assumed greenfield profile for the site.

The finite element model results using the equivalent masonry beams show the building acting flexibly in hogging, following the greenfield profile (of the finite element greenfield run). The curvature of the building ($\Delta/L$ of 0.001%) is less severe than the observed curvature of the building for both the front and rear facades, but the hogging nature of the response is very similar. The lower curvature in the FE building response is not surprising considering the difference between the finite element and the predicted greenfield
curves; the finite element greenfield response being less severe. This leads to the finite element model of the building also having a lower curvature. The finite element building response relative to the finite element greenfield response compares favourably, however, to the observed building response relative to the predicted greenfield settlement. The finite element model leads to a ‘negligible’ predicted damage category which agrees with the fact that there was no damage recorded as a result of the JLE tunnelling (Withers, 2001b).

Murdoch House

Murdoch House was also predicted to respond in a rigid manner as shown in figures 9.21 and 9.22. The observed building response was, however, more flexible than assumed although the magnitudes of the maximum settlement were as predicted.

The rear facade of the building behaved flexibly in hogging as shown, with a deflection ratio of 0.005%, closely following the assumed greenfield profile for the site. The finite element response for the rear facade was less severe, but still exhibited a hogging curvature ($\Delta/L$ of 0.001%), following a similar profile to the finite element greenfield prediction.
Figure 9.21: Predicted, Observed and Finite Element Settlements for Murdoch House (Front)

Figure 9.22: Predicted, Observed and Finite Element Settlements for Murdoch House (Rear)
Figure 9.23: Predicted, Observed and Finite Element Settlements for Clegg House (Rear)

The front of the building exhibited a slight hogging curvature in the final stage with a deflection ratio of 0.004%. This compares to the finite element response where the deflection ratio was 0.001%. The 0.007% maximum deflection ratio noted in table 9.7 was a temporary curvature during an intermediate tunnelling stage. This situation was not replicated in the finite element model with the maximum intermediate deflection ratio being 0.001%.

For both the front and rear facades, the finite element response can be seen to be less severe than the observed response. The building response does, however, follow a similar shape to the finite element greenfield profile in the same way that the observed building response was more flexible than predicted and followed the predicted greenfield surface profile in hogging. Both the observed and finite element building responses exhibited negligible horizontal tensile strains and damage.
The predicted and observed responses for Clegg House, along with the finite element model results are shown in figures 9.23 and 9.24. The observed response closely matched the rigid predicted response, with negligible building curvatures developing and with very similar maximum settlements. The finite element response for both front and rear facades was also essentially rigid, with negligible curvatures developing. The shape of the model results, however, differ from the observed profile, with the finite element building response being rigid, but in relation to the finite element greenfield settlement profile which differs from the predicted greenfield profile. Both the observed and finite element building responses exhibited negligible horizontal tensile strains and damage.

9.5.4 Numerical Modelling Summary

The finite element displacements agree reasonably well with the observed displacements. In all situations where the buildings were observed to exhibit hogging curvature in response
to the JLE tunnelling, the finite element model (with buildings represented by equivalent masonry beams) also predicted a hogging response. In sagging, the buildings in the finite element model acted in a rigid manner similar to the Moodkee Street buildings.

The severity of the hogging curvatures in the finite element model, however, were generally less than were observed in practice. The fact that the curvatures of the finite element greenfield profile are less severe than the observed greenfield settlements, means that the finite element building predictions could never exactly match the observed building predictions. However, when considering the finite element predictions with respect to the finite element greenfield curves, the equivalent masonry beam model can be considered to have worked very well.

As discussed in section 9.4, the soil model used for these analyses was developed for modelling cohesive soils rather than the non-cohesive soils that exist at the Moodkee Street site. The results presented should thus be considered with this in mind. Numerical analyses of this case study site with a more appropriate soil model could form part of future modelling work.

The case study site chosen could be said to be less useful than other alternatives, as the settlements and damage were not severe enough to test the numerical model fully. This is something of a ‘catch-22’ situation as the site was chosen partly for the fact that the
buildings were not protected by compensation grouting, but the reason protection was not required was that the predicted settlements and damage were considered to be negligible. Further case study verification of the equivalent masonry beams is required using cases with more severe surface settlements.

9.6 Conclusions

This chapter described the use of numerical modelling techniques to model the impact of tunnel construction on surface settlements and building damage in three dimensions. The numerical modelling techniques used are those developed as part of this thesis and described in previous chapters.

The results of the numerical models have been compared to the observed surface settlements from on-site monitoring of Neptune, Murdoch and Clegg Houses during the construction of the Jubilee Line Extension tunnels in 1996. This comparison shows that the building response predicted by the numerical model, agrees well with the observed response. In particular, the model accurately predicts a flexible response when the buildings are in a hogging mode of deformation and a stiff response when they are in sagging, as was observed during tunnelling on-site. The ability of the 3D model to capture the response of multiple buildings with differing asymmetrical orientations to the tunnel during a staged construction sequence which allows investigation of transitory effects was demonstrated.

The importance of modelling of both the three-dimensional aspects of building response to tunnelling and the different nature of the response of masonry buildings in hogging and sagging when making predictions regarding building response is thus reinforced.

The wide range of different case study sites in the published JLE material and the quality of detailed observations of building response to tunnel construction should encourage more researchers to make use of the data available in the future development of 3D numerical modelling of building response to tunnelling.
Chapter 10

Concluding Remarks

This thesis describes the development and use of surface beams to represent masonry buildings in three-dimensional numerical analyses of tunnelling. It is common practice to use linear elastic surface beams to represent buildings in 2D finite element analyses, or include full building models in complex 3D analyses. This thesis investigates the extension of the surface beam approach for use in 3D, and the development of beams which capture the behaviour of a masonry building. To achieve this, the research includes the implementation of 3D capable beams in the OXFEM finite element program; development of a method for determining equivalent elastic properties to more accurately model building response; and development of a non-linear equivalent masonry beam model to capture masonry specific responses in hogging and sagging. Three-dimensional numerical models of tunnelling using the equivalent surface beams to represent masonry buildings are compared to numerical models with full masonry buildings and with case study data from the Jubilee Line Extension project.

It is common practice to use elastic beams on the ground surface to represent a building in 2D finite element analyses. When representing masonry buildings using elastic surface beams, elastic properties based on the masonry material, are assigned to the beams. Tests show that this simple assignment of properties results in beams that do not respond to imposed base displacements in a manner similar to 2D facades. The test results show that the overall geometry of the facade \((L/H)\) ratio, and the amount and position of openings
affects the facade response. Generally, the more openings and the lower the $L/H$ ratio, the lower the stiffness of the facade compared to a traditional elastic beam. To model the facade using a surface beam, effective properties need to be used that account for these factors. The equivalent elastic beam procedure developed in this thesis is a geometric procedure to determine effective cross sectional area ($A^*$) and second moment of area ($I^*$) for assignment to equivalent surface beams by considering the amount and location of openings and the overall facade geometry. The procedure is shown to lead to predicted responses that are a significant improvement on a traditional deep beam and to lead to results that closely match the finite element response of facades to imposed displacements.

Further investigation of the equivalent elastic procedure for facades with more openings would be worthwhile as there is some deviation of the equivalent beam response from the observed finite element facade results in this region. Differences between the new theory and finite element results are also evident around the critical length to height ratio. Further investigation of the choice of this value may also lead to improvements in the procedure.

It is clear from the literature that different mechanisms operate in masonry facades in sagging and hogging leading to different facade responses in each of those modes. The suite of finite element analyses reported in this thesis confirm these differing responses, the key feature of which is the loss of flexural rigidity in masonry facades in hogging. This feature has previously not been captured in equivalent surface beams representing buildings in numerical analyses. The equivalent masonry beam model developed in this research is a non-linear elastic model including loss of stiffness in hogging. The parameters of critical curvature ($K_{crit}$) and residual bending stiffness factor ($f_b$) are introduced to control this loss of stiffness.

The implementation of 3D Timoshenko beam elements into the existing in-house finite element program OXFEM represents and enhancement of the finite element code. The equivalent elastic and masonry constitutive beam models implemented are also new additions. Tests of equivalent elastic Timoshenko beams confirmed their correct implementation and the response of the beams to imposed displacements conformed to the implemented theory. When subjected to Gaussian displacements the beam response mostly matched that of elas-
tic facades under the same conditions, however they displayed differing end effects. This should be the subject of further investigation. Such investigation could consider the use of higher order beams such as three-node beams with linear shear variation. Such beams could be combined with the use of 20 node tetrahedral soil elements (for combined soil-structure interaction analyses) for improved compatibility and accuracy. With advancing computer technology the additional run times inherent in such an approach should soon be only a minor consideration. Tests showed that the moment-curvature response of the equivalent masonry beams conformed to the implemented theory. Alternative implementations for the masonry beams could be considered as part of future research which might include a sudden reduction of stiffness instead of a gradual reduction or models which consider the variation of axial or shear stiffness as appropriate.

The use of the parallel computers at the Oxford Supercomputing Centre was a key feature of the time spent on this research. At the commencement of the project, the use of the parallel systems enabled runs to be completed significantly faster than on standard PC or Unix systems and the analysis of multiple runs to be undertaken simultaneously. For the type of 3D analyses under investigation, however, it is recommended that the use of a local high specification PC or cluster of PCs (combined with the ability to queue and batch jobs) be investigated. Hardware advances in recent years have rendered high end PCs capable of running these analyses in a reasonable length of time and their use would remove dependence on shared and at times unreliable university resources.

Three-dimensional finite element models were used to investigate the use of the equivalent elastic and masonry beams for modelling masonry buildings. The example analyses were chosen to be comparable with previous work carried out by members of the Civil Engineering Research Group at Oxford University. Greenfield analyses predicted settlements that were shallower and wider than a traditional Gaussian profile. This was not considered a problem for their use as controls for the testing of the equivalent beams. For both symmetric and oblique analyses the use of the equivalent elastic beams resulted in settlement profiles with excellent agreement to the response of full masonry facades in sagging regions. The beams showed less local settlement variation due to the lack of windows and their con-
sistent stiffness, but the magnitude and differential settlement of the beams was generally very close to that of the masonry facades. The equivalent elastic beams did not replicate the response of the masonry facades well in hogging zones. The beams suffered minimal curvature and deflection ratios compared to the facades. The affect of assigning different stiffness values was shown by the parametric stiffness study. Reducing the stiffness of the equivalent elastic beams to 1% of their original value was found to lead to a response very close to that of the masonry facade for hogging regions.

The equivalent masonry beams responded in the same manner as elastic beams in sagging, but were more flexible in hogging. They did not, however, respond in hogging in the same manner as the masonry facades. From the results of the oblique analysis, significantly less hogging was evident in the equivalent masonry beams with the range of $K_{crit}$ and $f_b$ values used than the masonry facades. This indicates that further work is required to investigate the interaction of the critical curvature and residual bending stiffness parameters with lower values of critical curvature thought to lead to more realistic responses in hogging. Limitations of availability of computing systems prevented further analyses from being carried out and these are recommended for the future.

It was satisfying to be able to utilise one of the published case studies of the Jubilee Line Extension project in London. The case study material is extremely thorough and further research using this material is encouraged. The case study analysis using equivalent masonry beams to represent three masonry buildings simultaneously gave good agreement with the observed settlements and damage. In particular, the flexible response of hogging sections of the building were replicated by the equivalent masonry beams. The response in sagging regions was essentially rigid as was observed during construction. Modelling in 3D of the staged construction underneath multiple buildings simultaneously was an efficient way to analyse this problem. The case study buildings chosen suffered from being subject to smaller magnitudes of displacement than might otherwise be desirable to fully test the masonry surface beams. The problem of selecting a site with no compensation grouting made this situation unavoidable. Further case study verification of the surface beam approach for modelling masonry buildings above tunnels in more severe situations
would be useful.

The developments of this thesis could be improved upon or used as a basis for future research. The most direct application of this research is that the equivalent elastic and masonry beam approaches can be used to determine properties for elastic and masonry beams for the modelling of masonry facades in two and three dimensions. A drawback to the simplification of a full masonry building as a surface beam is that no information regarding plane stresses and strains or crack profiles is available as a result of the simplified analysis. This is equally true for 2D analyses of buildings using surface beams, but these have been used extensively in previous analyses, in particular for parametric studies. Further investigations could make use of the 3D equivalent beams developed in this thesis for 3D parametric analyses of the interaction between buildings and tunnels. Such parametric studies would be useful for investigating the affect of variables such as the angle of an oblique building relative to the tunnel (and its eccentricity) and could assess: whether simply projecting the building onto the settlement profile (as is current practice) is appropriate; and the interaction between the building twist and damage. This could potentially lead to further extension of the relative stiffness approach to include oblique 3D considerations.

The use of the 3D surface beams as presented in this thesis would also be useful if a full model of a building is required where the building is part of a congested urban physical situation. Equivalent masonry surface beams could be used to represent buildings not of immediate interest but adjacent to the main building and thus important for correct modelling of the overall soil-structure interaction situation. This development would make full 3D models of buildings more accurate, by the inclusion of surrounding buildings in a simplified manner.

Soil structure interaction problems involve overlap between structural and geotechnical engineering. In the traditional structural engineer’s view, such problems often involve simplification of the soil and a detailed model of the structure. The approach adopted in this thesis turns the tables on this view with the use of a complex soil and tunnel model and the development of a simplified building.
However such problems are considered, it is clear that the need for assessment of the influence of urban tunnel construction on existing structures will continue to be important. The simplified beam approach developed here has room for improvement and further development, but it represents a new contribution to the ongoing development of numerical modelling of building response to tunnelling.
Appendix A

Facade meshes with windows

The facade meshes shown in this Appendix are those, with windows, described in tables 4.2 and 5.1. They are given here without the E or M descriptor which is simply used to designate the material used in each of the finite element analyses in Chapters 4 and 5.

Figure A.1: Meshes with $L=60\text{m}$, $H=8\text{m}$
Figure A.2: **Meshes with L=40m, H=8m**

Figure A.3: **Meshes with L=40m, H=12m**
Figure A.4: **Meshes with** $L=20\text{m}$, $H=8\text{m}$

Figure A.5: **Meshes with** $L=20\text{m}$, $H=12\text{m}$

Figure A.6: **Meshes with** $L=20\text{m}$, $H=16\text{m}$

Figure A.7: **Meshes with** $L=12\text{m}$, $H=16\text{m}$
Appendix B

Formulation of beam element stiffness matrix components

B.1 Axial stiffness contribution

For the two-node straight beam of length, $L$, with nodal axial degrees of freedom $u_1$ and $u_2$ shown in figure 6.3 the choice of a linear axial displacement field leads to the following relationship for axial displacement, $u$, between the nodes

$$u = N^e_a d^e_a$$  \hspace{1cm} (B.1)

where

$$d^e_a T = [u_1, u_2]$$  \hspace{1cm} (B.2)

$$N^e_a = [1 - \frac{x}{L}, \frac{x}{L}]$$  \hspace{1cm} (B.3)

are the nodal displacements and axial element shape functions respectively.

Using the strain-displacement relationship $B = dN/dx$ gives

$$B^e_a = \begin{bmatrix} \frac{1}{L} & 1 \end{bmatrix}$$  \hspace{1cm} (B.4)
where $B^e_a$ is the axial strain displacement matrix for an element. If $A$ represents the cross sectional area and $E$ is Young’s modulus we obtain the axial stiffness matrix $K^e_a$ for an element in the usual manner as

$$K^e_a = \int_0^L B^e_a [EA] B^e_a dx = \begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix}$$ (B.5)

### B.2 Torsional stiffness contribution

For the two-node straight beam of length, $L$, with nodal torsion degrees of freedom $\theta_{x1}$ and $\theta_{x2}$ shown in figure 6.3 the choice of a linear torsion displacement field leads to the following relationship for torsional rotation, $\theta_x$, between the nodes

$$\theta_x = N^e_t d^e_t$$ (B.6)

where

$$d^e_t = [\theta_{x1}, \theta_{x2}]$$ (B.7)

$$N^e_t = [1 - \frac{x}{L}, \frac{x}{L}]$$ (B.8)

are the nodal displacements and torsional rotation element shape functions respectively.

Using the strain-displacement relationship $B = dN/dx$ gives

$$B^e_t = \begin{bmatrix} \frac{1}{L}, \frac{1}{L} \end{bmatrix}$$ (B.9)

where $B^e_t$ is the torsional strain displacement matrix for an element. If $J$ represents the polar moment of inertia and $G$ is modulus of rigidity we obtain the axial stiffness matrix $K^e_t$ for an element in the usual manner as

$$K^e_t = \int_0^L B^e_t [GJ] B^e_t dx = \begin{bmatrix} \frac{GJ}{L} & -\frac{GJ}{L} \\ -\frac{GJ}{L} & \frac{GJ}{L} \end{bmatrix}$$ (B.10)
B.3 Derivation of bending shape functions

The bending shape functions for one independent plane of the two-node beam element shown in figure 6.3 are derived. The second plane in bending is independent and thus makes use of the same shape functions derived below but in the orthogonal direction. Shape functions for the axial degrees of freedom above are Lagrangian polynomials and are required here to derive the shape functions for the bending degrees of freedom as described in section 6.4.1. From equation 5.17, these are the order one Lagrange polynomials

\[ L_1(x) = \frac{x - x_2}{x_1 - x_2} \]  
\[ L_2(x) = \frac{x - x_1}{x_2 - x_1} \]  

(B.11)  
(B.12)

For the two-noded beam with nodes at \( x = 0 \), \( L \)

\[ L_1(x) = 1 - \frac{x}{L} \]  
\[ L_2(x) = \frac{x}{L} \]  

(B.13)  
(B.14)

Using the convention that \( \bar{x} = x/L \)

\[ L_1(\bar{x}) = 1 - \bar{x} \]  
\[ L_2(\bar{x}) = \bar{x} \]  

(B.15)  
(B.16)

and the derivatives are

\[ L'_1(\bar{x}) = -1 \]  
\[ L'_2(\bar{x}) = 1 \]  

(B.17)  
(B.18)
The bending shape functions are found using equations 5.15 and 5.16 as

\[ N_1(\bar{x}) = H_{01} \]  \hspace{1cm} (B.19)  
\[ N_2(\bar{x}) = H_{11} \]  \hspace{1cm} (B.20)  
\[ N_3(\bar{x}) = H_{02} \]  \hspace{1cm} (B.21)  
\[ N_4(\bar{x}) = H_{12} \]  \hspace{1cm} (B.22)

It thus follows that

\[ N_1(\bar{x}) = [1 - 2(\bar{x} - 0)(-1)][1 - \bar{x}]^2 = 1 - 3\bar{x}^2 + 2\bar{x}^3 \]  \hspace{1cm} (B.23)  
\[ N_2(\bar{x}) = L(\bar{x} - 0)[1 - \bar{x}]^2 = L(\bar{x} - 2\bar{x}^2 + \bar{x}^3) \]  \hspace{1cm} (B.24)  
\[ N_3(\bar{x}) = [1 - 2(\bar{x} - 1)(1)][\bar{x}]^2 = 3\bar{x}^2 - 2\bar{x}^3 \]  \hspace{1cm} (B.25)  
\[ N_4(\bar{x}) = L(\bar{x} - 1)[\bar{x}]^2 = L(-\bar{x}^2 + \bar{x}^3) \]  \hspace{1cm} (B.26)

and the second derivatives with respect to $\bar{x}$ required for the strain-displacement matrix are

\[ N''_1(\bar{x}) = -6 + 12\bar{x} \]  \hspace{1cm} (B.27)  
\[ N''_2(\bar{x}) = L(-4 + 6\bar{x}) \]  \hspace{1cm} (B.28)  
\[ N''_3(\bar{x}) = 6 - 12\bar{x} \]  \hspace{1cm} (B.29)  
\[ N''_4(\bar{x}) = L(-2 + 6\bar{x}) \]  \hspace{1cm} (B.30)
Appendix C

Example equivalent elastic beam properties calculation

C.1 Introduction

A fully worked example of the use of the procedures described in Chapter 4 for determining the effective cross sectional area, \( A^* \), and effective second moment of area, \( I^* \), for the beam approximation of a masonry facade is given in this section. The building facade analysed is the front facade of the building from the symmetric and oblique example problems described in Chapter 8 and shown here as figure C.1

C.2 Worked example

Part A - Shear

Take 11 vertical strips as shown and assume a wall thickness of 1m and thus a nominal cross section area, \( A \), of 8m\(^2\). Approximate the door as a window, thus making the assumption that all openings are windows of dimensions 1.5m wide and 2m high. The cross sectional area of each window is thus 2m\(^2\). Facade \( L/H=2.5 \) is less than the critical \( L/H=3.0 \), thus
Figure C.1: **Facade dimensions (metres) for calculation of $A^*$ and $I^*$**

use the linear depression method (see section 4.3.2). For each strip, $i$, calculate $L_i/A_i$, where $A_i$ is the cross sectional area net of windows, then sum and calculate $A^*$ using equation 4.23. A strip wise calculation is shown in table C.1.

**Table C.1: Calculation of effective cross sectional area $A^*$**

<table>
<thead>
<tr>
<th>Strip $i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_i$</td>
<td>2.25</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>2.25</td>
<td>2.00</td>
<td>2.25</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>2.25</td>
</tr>
<tr>
<td>$A$</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>No. windows</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$A_{\text{windows}}$</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>$A_i$</td>
<td>8</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>8</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>$L_i/A_i$</td>
<td>0.281</td>
<td>0.375</td>
<td>0.188</td>
<td>0.375</td>
<td>0.281</td>
<td>0.500</td>
<td>0.281</td>
<td>0.375</td>
<td>0.188</td>
<td>0.375</td>
<td>0.281</td>
</tr>
<tr>
<td>$\sum (L_i/A_i) = 3.500$</td>
<td></td>
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<tr>
<td>$A^* =$</td>
<td>4.762</td>
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</tbody>
</table>
Part B - Bending

Take five horizontal strips as shown and assume a wall thickness of 1m. Again, approximate the door as a window, thus making the assumption that all openings are windows of dimensions 1.5m wide and 2m high. The face area of each window is thus $3\text{m}^2$. Facade $L/H=2.5$ is less than the critical $L/H=3.0$, thus use the linear depression method (see section 4.3.2). For each strip, $j$, calculate the effective height of the strip $h'_j = A_{fj}/L$, where $A_{fj}$ is the face area net of windows. Calculate $I_j$ for each strip using equation 4.20, then sum and calculate $I^*$ using equation 4.24. A strip wise calculation is shown in table C.2.

Table C.2: Calculation of effective second moment of area $I^*$

<table>
<thead>
<tr>
<th>Strip, $j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td>$h_j$</td>
<td>1.00</td>
<td>1.25</td>
<td>2.00</td>
<td>1.25</td>
<td>1.00</td>
</tr>
<tr>
<td>$A_f$</td>
<td>20</td>
<td>40</td>
<td>40</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>No. windows</td>
<td>0</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>$A_{windows}$</td>
<td>0</td>
<td>15</td>
<td>0</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>$A_{fj}(m^2)$</td>
<td>20</td>
<td>25</td>
<td>40</td>
<td>25</td>
<td>20</td>
</tr>
<tr>
<td>$h'_j$</td>
<td>1.00</td>
<td>1.25</td>
<td>2.00</td>
<td>1.25</td>
<td>1.00</td>
</tr>
<tr>
<td>$b_j$</td>
<td>3.5</td>
<td>2.0</td>
<td>0.0</td>
<td>2.0</td>
<td>3.5</td>
</tr>
<tr>
<td>$I_j$</td>
<td>12.333</td>
<td>5.163</td>
<td>0.667</td>
<td>5.163</td>
<td>12.333</td>
</tr>
<tr>
<td>$\sum(I_j)$</td>
<td></td>
<td></td>
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<td></td>
<td>35.659</td>
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<tr>
<td>$I^*$</td>
<td></td>
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<td>29.716</td>
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</table>
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