The contribution of non-structural components to the overall dynamic behaviour of concrete floor slabs

Shahram Falati

New College, Oxford

Submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy at the University of Oxford.

Hilary 1999
To my beloved parents

Dr. Moussa Falati and
Mrs. Malaknaz Ghaemi
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Abstract

Prestressing and the advent of high strength materials enable the construction of more slender floor slabs with lower values of natural frequency and damping. Under certain circumstances, the vibrations due to forcing frequencies of normal human activities can be annoying to the occupants. Since the occupants are both the source and the sensor, the vibration cannot be isolated and must be controlled by the structural system. At present, there is a limited knowledge about the overall dynamic characteristics of such concrete slabs, including contributions from individual structural and non-structural components.

An extensive programme of modal testing on a slender one-way spanning 50% scaled post-tensioned concrete slab is described. Testing was performed using electromagnetic shaker, instrumented hammer, heeldrop, and walking excitations, to determine full floor dynamic characteristics. The tests investigated the effect on vibration performance of the level of prestress, and of various non-structural additions, including vibration absorbers and effect of occupants.

It was found that an increase in prestressing force increases natural frequency and decreases damping due to closing up of microcracks. A model is presented to reflect these changes in terms of effective second moment of area. Cantilever partition tests showed energy to dissipate by swaying, and full-height partitions were seen to act as line supports leading to a significant stiffening of the slab. Analytical models are derived for both forms of partitions. Tests with false floors showed a significantly higher increase in slab damping when the floor panels rested on the pedestals, as opposed to being rigidly fixed to them. Although the addition of viscoelastic screed layers were not seen to have great effect in damping, an analytical model is derived which shows the advantage of using such layers. A TMD system was designed and installed on the floor, using plywood sheets, which led to a reduction in vibration response by as much as 80%. A theoretical model is derived to represent the TMD results and a design criterion is suggested. Finally, the effect of human-structure interaction is investigated. An analytical model shows the natural frequency of the body to be 10.43Hz with a damping of 50%. Results are also reported of tests on a full-scale field slab, confirming some of the findings of the model slab experiments.

Broadly, the results show that contrary to popular belief, merely the presence of non-structural components does not necessarily enhance the dynamic behaviour of the system. The design of these components and nature of their installation are important factors affecting their contribution to the overall floor vibrational behaviour.
Acknowledgements

I would like to express sincere gratitude and appreciation to my supervisor, Dr M.S. Williams, for his generous support and guidance throughout the duration of my research. In particular, his encouragement, useful suggestions, help and advice have been invaluable and his assistance in obtaining the experimental equipment have made this research programme possible.

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Finally, I believe nothing in my life would have been achieved if it wasn’t for the continuous support, encouragement and prayers of my parents. They have sacrificed more than many a parent for me and my brother Shahrokhi, and I hope to be able to repay a fraction of their kindness during the rest of my life. It is to them that I dedicate this work.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Area</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Cross-sectional area of concrete</td>
</tr>
<tr>
<td>$A_0$</td>
<td>Initial peak acceleration (as determined by the heeldrop test)</td>
</tr>
<tr>
<td>$A_{rms}$</td>
<td>Root mean square acceleration</td>
</tr>
<tr>
<td>$A_s$</td>
<td>Cross-sectional area of steel reinforcement</td>
</tr>
<tr>
<td>$A_{st}$</td>
<td>Steady-state acceleration</td>
</tr>
<tr>
<td>$b$</td>
<td>Width of slab</td>
</tr>
<tr>
<td>$B$</td>
<td>Structural factor</td>
</tr>
<tr>
<td>$c$</td>
<td>Viscous damping constant</td>
</tr>
<tr>
<td>$c_c$</td>
<td>Critical viscous damping constant</td>
</tr>
<tr>
<td>$C_e$</td>
<td>Dynamic crowd effect</td>
</tr>
<tr>
<td>$C_z$</td>
<td>Intermediate variable in the $x$-direction (Concrete Society (1993))</td>
</tr>
<tr>
<td>$d$</td>
<td>Effective depth of beam or slab</td>
</tr>
<tr>
<td>$D$</td>
<td>Flexural rigidity of a plate ($=\frac{Eh^3}{12(1-\nu^2)}$)</td>
</tr>
<tr>
<td>$DAF$</td>
<td>Dynamic amplification factor for displacement</td>
</tr>
<tr>
<td>$DLF$</td>
<td>Dynamic amplification (load) factor for acceleration</td>
</tr>
<tr>
<td>$e$</td>
<td>Eccentricity of prestressing tendons</td>
</tr>
<tr>
<td>$E$</td>
<td>Modulus of elasticity (Young’s modulus)</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Modulus of elasticity of concrete</td>
</tr>
<tr>
<td>$E_s$</td>
<td>Modulus of elasticity of steel</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>$f_0$</td>
<td>Fundamental natural frequency of a system (Hz)</td>
</tr>
<tr>
<td>$f_{0j}$</td>
<td>Fundamental natural frequency of $j^{th}$ component of a system (Hz)</td>
</tr>
<tr>
<td>$f_{0u}$</td>
<td>Undamped fundamental natural frequency of a system (Hz)</td>
</tr>
<tr>
<td>$f_n$</td>
<td>$n^{th}$ linear natural frequency of a system</td>
</tr>
<tr>
<td>$f_{nyq}$</td>
<td>Nyquist frequency</td>
</tr>
<tr>
<td>$f_p$</td>
<td>Pedestrian activity rate (Hz)</td>
</tr>
<tr>
<td>$f_t$</td>
<td>Tensile stress of concrete cross-section</td>
</tr>
<tr>
<td>$f_x$</td>
<td>Natural frequency of slab in the $x$-direction</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>Frequency resolution</td>
</tr>
<tr>
<td>$F$</td>
<td>Force</td>
</tr>
</tbody>
</table>
$F^*$ Generalised force acting on a system
$F_0$ Force exerted by a single heeldrop (idealised as 2670N)
$F_p$ Force due to a single pedestrian activity
$F_g$ Force due to group activity
$g$ Acceleration due to gravity (9.81m/s$^2$)
$G$ Weight of a notional pedestrian (generally assumed as 800N)
$G_{d}$ Load density of a crowd (N/m$^2$)
$G_{aa}$ Force frequency spectrum of spectrum analyser
$G_{bb}$ Response frequency spectrum of spectrum analyser
$G_{ab}$ Cross-spectrum of $G_{aa}$ and $G_{bb}$
$h$ Thickness of a plate, slab or beam
$H(\omega)$ Complex frequency response (FRF) of a system to applied excitation of frequency $\omega$
$H_{ab}$ Transfer function (FRF) of spectrum analyser
$i$ integer(1,2,3,...) representing the $i^{th}$ harmonic frequency of a force
$I$ Second moment of area
$I_{cr}$ Cracked second moment of area of concrete slab cross-section
$I_e$ Effective second moment of area of concrete slab cross-section
$I_t$ Transformed second moment of area
$I_u$ Uncracked second moment of area of concrete slab cross-section
$I_x$ Second moment of area in the $x$-direction
$I_y$ Second moment of area in the $y$-direction
$J$ Number of discrete points in DFT
$k$ Stiffness of a system
$k_p$ Stiffness of gypsum plasterboard partition sheets
$K^*$ Generalised stiffness of a system
$KB$ DIN4150(1975) intensity of perception factor
$KE$ Kinetic energy
$l_x$ Span of one bay in the $x$-direction
$l_y$ Span of one bay in the $y$-direction
$L$ Span length
$L_x$ Span length in the $x$-direction
$L_y$ Span length in the $y$-direction
$m$ integer (1,2,3,...) representing $m^{th}$ mode of vibration of a system (orthogonal to the $n^{th}$ mode of vibration)
$M^*$ Generalised mass of a system
$M_a$ Gross maximum bending moment
$M_{cr}$ Bending moment at the onset of cracking
$N$ Total number of harmonics of forcing frequency considered
$N_x$ Intermediate variable in the $x$-direction (Concrete Society(1993))
n Integer $(1,2,3,\ldots)$ representing $n^{th}$ mode of vibration of a system
$n_x$ Number of bays on a floor in the $x$-direction
$n_y$ Number of bays on a floor in the $y$-direction
$p$ Loading on a system (varying with coordinates and time)
$P$ Impulse force due to a heeldrop (idealised as 70Ns)
$P_0$ AISC(1997) constant excitation force
$P_t$ Total prestress force due to all prestressing tendons
$PE$ Potential energy
$q_i$ $i^{th}$ generalised co-ordinate (varying with time)
$R_s$ Response factor given in the SCI(1989) guide
$R_x$ Response factor in the $x$-direction (Concrete Society (1993))
$R_y$ Response factor in the $y$-direction (Concrete Society (1993))
$R$ Total response factor in both directions (Concrete Society (1993))
$S$ Stored Energy
$t$ Time
$t_0$ Time to reach first maximum amplitude in a heel impact test
$t_d$ Duration of a single heel impact (idealised as 0.05 seconds)
$t_p$ Duration of contact of feet with ground when jumping
$T$ Period of a signal
$u$ Displacement of a vibrating system (varying with coordinates and time)
$\dot{u}$ Velocity
$\ddot{u}$ Acceleration
$w$ Mass per unit length ($=\rho bh$)
$W$ Mass per unit area
$WD$ Work done
$W_e$ Effective weight of a floor
$W_t$ Total weight of a floor plus contents
$x, y, z$ Cartesian coordinates
$X_a$ Force signal of spectrum analyser
\( X_b \)  Response signal of spectrum analyser
\( X \)  Amplitude of response of a system to dynamic excitation
\( X_{\text{peak}} \)  Peak amplitude of response of a system to dynamic excitation
\( X_I \)  Imaginary part of \( X \)
\( X_m \)  Modulus (magnitude) of \( X \)
\( X_R \)  Real part of \( X \)
\( y_t \)  Position of neutral axis
\( a_i \)  Fourier coefficient of the \( i^{th} \) harmonic of forcing frequency
\( \delta \)  Logarithmic decrement
\( \Delta_b \)  Static deflection due to bending and shear of beam
\( \Delta_c \)  Static deflection due to axial shortening of column
\( \Delta_g \)  Static deflection due to bending and shear of girder
\( \Delta_s \)  Total static deflection of a system
\( \Upsilon_{ab} \)  Coherence function of spectrum analyser between force \( (a) \) and response \( (b) \)
\( \phi_i \)  Phase lag of \( i^{th} \) harmonic relative to the first harmonic
\( \Phi_n \)  \( n^{th} \) mode shape of a system
\( \nu \)  Poisson’s ratio
\( \mu \)  Mass ratio
\( \nabla^2 \)  Laplacian operator
\( \rho \)  Density
\( \sigma_t \)  Concrete tensile strength
\( \sigma_{av} \)  Average prestress in slab
\( \lambda_x \)  Effective aspect ratio of slab in the \( x \)-direction
\( \omega_n \)  \( n^{th} \) angular natural frequency of a system \( (\omega_n = 2\pi f_n) \)
\( \tau \)  Shear stress
\( \zeta \)  Damping ratio (expressed as percentage of critical damping \( \zeta_c \))
\( \zeta_{eq} \)  Equivalent damping ratio of a structure
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Chapter 1

Introduction

Currently, there is a lack of knowledge about the dynamic behaviour of suspended concrete slabs. With the advent of more slender slab designs, especially with the use of prestressing, concrete floors are becoming more susceptible to vibrations. These vibrations are only a problem from a serviceability point of view. They have been encountered many times before, especially with long span floors of light construction, such as composite floors. Until now, there has been very limited research into the area of post-tensioned concrete floor vibrations, even though these types of floors can be very slender. Moreover, the knowledge of the dynamic properties of floors, particularly damping and natural frequencies, and contributions from non-structural elements is incomplete.

In this chapter, the problem of vibration susceptibility of floors to low level excitations is introduced. A brief account is presented on the merits of post-tensioned floors over ordinary reinforced slabs, and their possible drawbacks concerning vibration. The areas in which there is a need for further research are identified and the deficiencies of some current design codes are noted. The objectives and aims of the research programme are hence listed and an outline of the thesis structure is given.

1.1 Post-tensioned floors and their vibration susceptibility

Vibrations in a structure occur due to a multitude of causes, such as walking (inside a building) or wind loading (external to the building). Other forms of loading include blasts, seismic vibrations, construction loading, traffic, or human activities such as dancing and jumping [Sabnis (1979)]. While some of these vibrations can induce permanent damage, others (e.g. walking) lead to serviceability problems within the structure. Serviceability limit states in buildings are conditions in which the function of the building is disrupted because of local minor damage or deterioration of building components, or because of occupant discomfort. Vibration problems in structures are generally the most common serviceability issue faced by engineers today. Floor vibration is much
more likely to annoy people than to cause overloading or fatigue [Allen et al. (1985)]. While safety is generally not an issue with serviceability, the economic consequences can be substantial [ASCE (1986)].

With traditional construction and design, making use of allowable or working stress criteria, serviceability usually was not a problem. Generally, simple criteria, based on percentage of span or height, dealt with cracking and motion related problems. However, changing architectural and building use requirements, coupled with improved methods of construction and design, e.g. prestressing, have led to structural systems that are less stiff and massive. In addition, the use of materials with low weight-to-strength ratios (e.g. high strength steel and concrete), has meant that modern floor areas are increasingly large with low slab thicknesses. For these flexible and lightly damped floors, serviceability issues are of paramount importance, since they are susceptible to transient vibrations induced by small impacts, such as human footfalls. Under certain circumstances, these vibrations can be very annoying to occupants of buildings [Allen (1974), Allen and Rainer (1976), Vannoy and Heins (1979), Warwaruk (1979), Murray (1981), Bachmann (1992a)]. One of the most important advances in construction design has been the advent of prestressing, which has led to more slender, and longer span, structures.

Post-tensioning, which is a form of prestressing, has been in use in floor construction for several decades, particularly in the United States, Australia, the Far East, and to some extent in Europe. Its economic and technical advantages are being increasingly appreciated, and the proportion of concrete floors being post-tensioned is growing [Khan and Williams (1995)]. Similar to normal reinforced concrete floors, post-tensioned floors can be made in flat, ribbed, one-way with beams, or waffle slab configurations (see section 2.4.1).

There are many advantages of using post-tensioning over normal reinforced concrete members. These include (i) increased clear spans, (ii) thinner slabs, (iii) lighter structures, (iv) reduced cracking and deflections, (v) reduced storey height, (vi) rapid construction, and (vii) better watertightness [Concrete Society (1994), BCA (1992)]. These advantages become most apparent when longer spans are required. Essentially, post-tensioning is an economical and fast way of achieving slender floor slabs with large open areas. The main drawback of these floors is their susceptibility to vibration. With increased spans and thinner slabs, the natural frequencies of such floors can be reduced to within the range where human annoyance due to resonance with harmonics of activities, such as walking, is likely. Furthermore, due to reduced cracking and deflections, the damping capacities of these floors are small.
Serviceability vibration problems have been encountered in many types of floors, including those of composite steel-concrete, precast, and cast-in-place construction. References relating specifically to post-tensioned, prestressed floors are very few in number [Bachmann (1992b), Waldron et al. (1993), Caverson et al. (1994), Williams and Waldron (1994)]. As these slabs become more slender with larger spans, it is logical to anticipate vibration as a problem.

1.2 Overview of research needs

A common human response to motion in buildings is that the people become anxious about the safety of the structure, even to the extent of refusing to use it. Many examples of these cases have been reported such as the closure of a new department store because of uncomfortable floor motion, or of the complete loss of secretarial efficiency in a new office building due to occupant induced floor vibration [Murray (1981)]. In such cases, the actual danger of structural collapse is very small, the strains involved often being 10 to 100 times less than those which might initiate damage [Bachmann et al. (1995)]. Nevertheless, it is a serious matter for the designer and correcting such situations is usually very difficult and expensive [Murray (1981)]. Therefore, with ever increasing numbers of flexible floors, account must be taken of the human sensitivity to vibration at the design level.

Most structural designers are acutely aware of potential floor vibration problems, but until very recently, guidelines were not available to aid in the determination of the suitability of a proposed floor system.

"Current serviceability guidelines are not useful in many cases of new construction. The absence of meaningful criteria may be interpreted by some to mean that serviceability is not important, and encourage a casual attitude towards serviceability. With the continuing trend towards high-strength, flexible structures, efficient structural systems, and limit state design, this casual attitude will likely lead to costly problems in many buildings.... Additional research must be undertaken in numerous areas in order to develop or complete the data base needed to prepare rational and practical serviceability guidelines." [ASCE (1986)].

Increasingly, recommendations are being adopted concerning floor vibrations. SCI (1989) and AISC (1997) are two design guides specifically written for composite steel-concrete floors. For post-tensioned slabs, in recognising the need to consider vibrations, Concrete Society (1994) states:
"Post-tensioned flat slabs usually have greater mass and a lower fundamental natural frequency than slabs on profiled metal decking. Although increasing the mass of the floor reduces the dynamic response, floors of lower natural frequency are excited by much larger components of the walking force. It is therefore likely that dynamic response will control the thicknesses of some flat slabs". [Concrete Society (1994)].

At present, there is a lack of knowledge about the means of controlling floor vibrations in terms of added stiffness and damping. Estimations of dynamic parameters can be very inaccurate. In the case of natural frequencies, a particular area of difficulty is the characterisation of boundary conditions. Also, equivalent viscous damping ratios of floors in buildings vary in a wide range depending on non-structural elements, such as hung ceilings, false flooring, furniture, partition walls, etc., and cannot be generally stated. Although the effects of architectural components have been noted in the published estimates of fundamental modal damping, [Murray (1975), CSA (1989)], only their mass is considered in the beam formula for the calculation of fundamental frequency [Pernica (1987)].

A common practice amongst designers is to pre-suppose damping values based on past experience of similar structures. This is a very crude way of measuring a major structural parameter and tests have shown that two similar buildings can have floor damping ratios which are very different [Bachmann et al. (1995)]. Hence, it is evident that the damping ratio of a complete floor system is dependent upon the damping characteristics of all its various components, both structural and non-structural. To this end, there is a need to test concrete floors in order that their dynamic behaviour may be more fully understood and the existing base of knowledge be enlarged.

"Estimates of the stiffness, mass, and damping of building systems are needed to predict their dynamic response.... In the area of floor vibration, structural damping is an important factor in human reaction to transient vibrations that occur in lightly damped, long span floor systems. Damping depends on the level of vibration, and the contribution of non-structural elements and people. Additional field investigations of common floor systems exhibiting noticeable floor vibrations are urgently needed to determine appropriate construction systems and damping levels.... Simple and economical devices need to be developed to increase damping in floors." [ASCE (1986)].

To this end, a closer examination of vibration in concrete slabs, with specific regard to post-tensioned floors, is justified.
1.3 Research objectives

There are several very useful reports of field tests on various floor structures [Pernica and Allen (1982), Rainer and Swallow (1986), Pernica (1987), Osborne and Ellis (1990), Bachmann (1992a), Caverson et al. (1994), Pavic et al. (1994), Zaman (1996)]. However, very few laboratory experiments have been reported [Lenzen (1966), Pavic and Waldron (1996b), Pavic et al. (1997)]. While field testing provides valuable information on the behaviour of real structures, it gives little opportunity to investigate, in detail, the parameters governing vibrational behaviour, or the possible ways of improving performance. A programme of large-scale laboratory testing was therefore undertaken in this research project with the following aims:

i) to investigate the effect of the level of prestress on vibrational performance

ii) to assess the effects of various non-structural additions, including false floors and partitions, on the overall dynamic characteristics of a floor

iii) to evaluate the use of high damping admixtures, applied as layers of screed on the floor surface, in reducing floor vibrations

iv) to assess the effectiveness of tuned mass dampers (TMDs) in reducing floor vibrations and to develop a simple economical TMD in an effort to increase damping in a floor

v) to add to the understanding of human-structure interaction

vi) to investigate the most suitable and accurate method of vibration testing of real floors

vii) based on the test results, to recommend simple design guidelines for the most effective use of non-structural components on concrete floors

These objectives have been accomplished through a programme of modal testing in which a post-tensioned concrete slab strip was designed and built in laboratory conditions, at approximately 50% of full-scale. The slab was one-way spanning and simply supported, with a span to depth ratio of 38. This high slenderness was chosen deliberately so as to give a slab with a low fundamental frequency, which might therefore be expected to be prone to vibration problems. The model slab was then altered in configuration and tests were conducted with the following:

i) slab with bonded rebar only (reinforced state), with 57% of design post-tensioning force (partially tensioned state), and with full design post-tensioning force (fully tensioned state)

ii) simulated live loads applied to represent floor components such as office furniture, hospital equipment, or cars in a carpark
iii) addition of simulated cantilever partitions in three layouts
iv) addition of simulated full-height partitions in two layouts
v) addition of false flooring in two layouts
vi) addition of screed layers, treated with viscoelastic admixture, in two separate layers with differing admixture concentrations
vii) application of a simple TMD system consisting of plywood sheets, tuned with addition of known weights, in four layouts
ix) tests with the presence of one or two occupants standing stationary on the slab

The apparatus for the dynamic testing of the model slab was also designed and adapted to the specific tests. The excitations applied onto the model floor at each configuration were:

- shaker loading tests (two levels of sine-sweep and three levels of single sine wave excitations)
- instrumented hammer tests
- heeldrop tests
- horizontal walking tests
- vertical walking tests

In addition, field tests were conducted on:

i) an Edwardian building for assessment of vibration transmissibility
ii) a dance floor for assessment of vibration characteristics [Falati (1996)]
iii) an office floor for derivation of dynamic behaviour [Williams and Falati (1998)]

1.4 Scope of thesis

This thesis is restricted in scope to linear, elastic, dynamic behaviour of concrete slabs within serviceability limits. Vertical vibrations caused by regular occupancy are examined and thus the imposed excitations are of small amplitude only. Hence, it is assumed that at no stage of the research programme, damage to the model slab and inelastic structural effects have taken place. The research work and results are presented in ten chapters as described below.
The following chapter, Chapter 2, is a review of the current knowledge and research in the fields of human perception of vibration and the dynamic behaviour of floor slabs, in particular their natural frequency, damping, and acceleration responses. Various vibration design criteria are also presented.

Chapter 3 gives a detailed description of the dynamic testing apparatus used, including their design and construction. The design and casting process of the model concrete post-tensioned slab are also described. The latter parts of the chapter are concerned with the simulations of non-structural components on the slab, such as partitions and false floors.

In Chapter 4, the experimental procedure employed in the dynamic tests is explained. The chapter starts with an overview of the modal analysis technique of vibration testing, and continues to describe the use of each excitation source separately.

Chapter 5 presents the results of all the experiments for each slab configuration and Chapter 6 describes detailed analyses of the results of prestressing effects, false floors, partitions and damping screeds. For each component, analytical model approximations are presented and compared with experimental results. Any implications are also discussed.

In Chapter 7, a detailed account of tests with TMDs is presented including experimental results and analyses. An analytical model is derived from which the use of the simple plywood TMD system is extended to other slab types. The chapter concludes with a proposed guideline on the use of such simple and easy-to-install systems on floors.

Chapter 8 contains a detailed account of tests conducted to assess the interaction between a floor and its occupants. Simple models are derived, which show that the current treatment of human-structure interaction in the design codes is incomplete.

An account is given in Chapter 9 of a field test conducted to assess the dynamic characteristics of an office floor. The experimental procedure is presented and the results are analysed. The floor is then classified for its vibration perceptibility.

Finally, Chapter 10 lists the conclusions of the research programme, describes its main findings, and gives possible recommendations for future work.
Chapter 2

Background knowledge and literature review

2.1 Introduction

Vibration tests on slender suspended long span floors have been ongoing since the early 1950s, when serviceability problems were encountered with annoying vibrations of some slabs. Although some design guides mention floor vibrations as a serviceability design concern [DIN4150/2 (1975), BS6472 (1984), NBCC (1985), CSA (1989), ISO2631/2 (1989), SCI (1989), Concrete Society (1994), AISC (1997)], there are others that simply rely on limiting deflection values or span-depth ratios to control the problem [ACI (1965), ACI-ASCE (1974), FIP (1980), Freyssinet (1994)]. This approach would be adequate for stiff, heavy systems of the past, but is sometimes found to be unsuitable for slender, long span floors of recent years.

In this chapter, a detailed review of research on human perception of vibrations and floor dynamics is presented, including representations of typical live loads in buildings. In particular, various methods of estimating floor responses and the modal parameters of natural frequency and damping are described and the various vibration serviceability guidelines are presented.

2.2 Human perception of vibration

The human body can sense vibration displacement amplitudes as low as 0.001mm, whilst fingertips are 20 times more sensitive [Bachmann et al. (1995)]. Human perception of vibration is very complex and depends on many inter-related factors, such as the person's posture and level of concentration. The following is a review of current knowledge in this field and the development of criteria for
acceptability of vertical floor vibrations.

2.2.1 Vibrations and human beings

There are three main types of human exposure to vibration:

- vibrations applied to the whole body surface (e.g. high intensity sound)
- vibrations applied to particular parts of the body (e.g. to hands by an electric drill)
- vibrations which are transmitted to the body as a whole through the supporting surface (e.g. from floor to feet of a standing person)

It is also possible for an indirect vibration nuisance to be caused by the vibration of external objects in the visual or aural field (e.g. rattling of windows). The response of the person to the vibration is mainly dependent on:

- type and intensity of excitation (e.g. impulsive shock, machinery vibration, etc.)
- distance from the source and the duration of vibration
- place and frequency of occurrence, time of day, and expectancy level
- body orientation and posture (standing, sitting, lying down, stiff, relaxed, etc.)
- personal activity (resting, walking, running)
- other factors (e.g. age, sex, level of concentration, sharing of experience with others, etc.)

Man is sensitive to mechanical oscillations ranging in frequency from below 1Hz up to at least 100kHz. This range of sensitivity is much broader than the range of human hearing [Guignard (1971)]. Oscillations at the lowest frequencies of around 0.1-3Hz are characteristic of large artificial structures which may transmit vibration to man (e.g. tall buildings or long suspension bridges). The range from about 3-30Hz is characteristic of ‘vibration’ in the everyday sense, occurring in vehicles, buildings or near machinery. It is also the range in which vibration is induced in many building components such as walls, floors and house frames. Such vibration may also be caused by disturbances within the building itself, including footfall. At frequencies much above 30Hz, typical of the response of lighter building elements such as windows and room fittings, sensitivity to vibration merges with and generally becomes secondary to the response to audible noise.

Human response to vibration and its influence on a person’s performance has been widely studied using both subjective and objective methods [Grether (1971), Guignard (1971)]. Many studies
have been very narrowly focused on a particular form of transport or a particular aspect of human response, such as visual impairment. Also, much of the published work has been concerned with relatively intense and continuous vibration, such as is felt in vehicles or industrial machinery.

A common approach has been to use subjective experiments as a guide to tolerable exposure levels. A typical graph from such an investigation is shown in figure 2.1, where it can be seen that maximum sensitivity occurs at around 4-8Hz. This agrees with research, which has established whole body resonance in this range [Dieckmann (1958), Herterich and Schnauber (1992), Fairley and Griffin (1989)]. As is usually the case, the vibration amplitude in figure 2.1 is given as an acceleration, expressed as a fraction of the acceleration due to gravity, $(g)$. The lines in the figure indicate acceptability limits, where the region above a line denotes unacceptable vibrations. The direction of the motion relative to the human body is significant and in the case of vibrating floors, vertical vibrations are most prominent.

![Figure 2.1: Magid et al.(1960) human subjective tolerance to whole body vertical sinusoidal vibrations [from Grether (1971)]](image)

### 2.2.2 Perceptibility guidelines

In this section, attention is focused on the perception of vibration by occupants of buildings, in particular long span floors. As this is a comparatively recent problem, it is not addressed fully in many design codes, but certain guidelines have been proposed which will be reviewed.

Here, a common representation of the various plots is adopted, as the guidelines proposed by each source vary widely using different axes and units. In the following discussion, each source's original
graph is converted into the common graphical representation (CGR) of metric root mean square (RMS) acceleration versus frequency. The CGR is consistent throughout with the ISO/BSI/ANSI scales and some suggested limits, as marked on the graphs. These scales and limits are discussed later in this section. Each line shown on the graphs represents a constant level of human reaction and is known as an iso-perceptibility line. The area above any line represents a reaction greater than the area below. Caverson (1992) used CGR with peak accelerations versus frequency.

The first comprehensive research on human sensitivity to vibration was carried out by Reiher and Meister (1931). They subjected a representative group of people to vertical and horizontal vibrations of five minutes duration. The amplitudes and frequencies of vibration were varied and the subjects rated the motion in one of six categories: imperceptible, slightly perceptible, distinctly perceptible, strongly perceptible, disturbing or very disturbing. The CGR plot of their results for vertical vibration are shown in figure 2.2. Note that damping is not mentioned in these investigations and that the duration of the vibrations is long enough to be regarded as continuous, rather than transient, excitation. The limits above distinctly perceptible are not included in figure 2.2 as they fall above serviceability conditions.

![Figure 2.2: CGR of Reiher-Meister(1931) and Lenzen(1966) modified Reiher-Meister](image)
Perhaps the first study to address human perception of floor vibrations in particular, was carried out by Lenzen (1966). He identified the main source of floor vibration as the occupants themselves impacting the floor through normal usage. As a result, he modified the Reiher-Meister graphs by scaling the amplitude axis up, by a factor of ten, to account for the reduced human sensitivity to transient vibrations, figure 2.2. Citing results from several floor tests, Lenzen concluded that if damping reduces the vibration to a negligible quantity in five cycles, the human would not respond, whereas if it persists beyond 12 cycles, he will respond as if to steady-state vibrations.

DIN4150/2 (1975) (as reported by Bachmann et al. (1995)) was one of the first design codes to consider structural vibrations. The limits given are based on a parameter known as the *intensity perception factor* ($KB$), which is calculated empirically in terms of frequency and displacement and is applied to frequencies within the range 1-80Hz. The formula for $KB$ is shown in equation 2.1 in terms of peak acceleration, $A_0$, and its CGR in shown in figure 2.3. In using these codes, a designer would have to ensure the calculated value of $KB$ falls below the limits of table 2.1.

$$KB = \frac{0.8f^2}{\sqrt{(1 + 0.032f^2)}} \cdot \frac{A_0}{4\pi^2f^2}$$  \hspace{1cm} (2.1)

![Figure 2.3: CGR of German standard DIN4150/2 (1975) perceptibility graph](image-url)
Table 2.1: DIN4150/2 (1975) acceptable KB values for buildings [after Bachmann et al. (1995)]

<table>
<thead>
<tr>
<th>Building type</th>
<th>Time</th>
<th>Continuous or intermittent vibration</th>
<th>Transient vibration (several occurrences per day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>rural, residential and holiday resort</td>
<td>day</td>
<td>0.2</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>night</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>small town and mixed residential</td>
<td>day</td>
<td>0.3</td>
<td>8.0</td>
</tr>
<tr>
<td></td>
<td>night</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>small business and office premises</td>
<td>day</td>
<td>0.4</td>
<td>12.0</td>
</tr>
<tr>
<td></td>
<td>night</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>industrial</td>
<td>day</td>
<td>0.6</td>
<td>12.0</td>
</tr>
<tr>
<td></td>
<td>night</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Following tests on 42 slabs, Allen and Rainer (1976) proposed a vibration criterion for long span floors, which was incorporated into CSA (1989). The data for each test floor included initial amplitude from a heeldrop impact (see section 2.7.1), as well as frequency, damping ratio, and subjective evaluation by occupants. By correlating their results, they arrived at a continuous and walking vibration criterion, the CGR of which is shown in figure 2.4. The continuous vibration iso-perceptibility line is due to a person walking, who causes between 10 and 30 cycles of vibration. One limitation of the criterion is that it can only be applied to quiet occupancies (i.e. residential, school and office floors), whereas the scales given in DIN4150/2 (1975) take account of the usage of the structure. Pernica and Allen (1982) modified the criterion from quiet to active occupancies by increasing the limits by a factor of three to account for usage such as shopping centres or carparks.

![Figure 2.4: CGR of Allen and Rainer (1976) and CSA (1989) annoyance criteria for floor vibrations](image-url)
The International Standards Organisation (ISO) revised its design codes concerning vibrations in 1985 [ISO2631/1 (1985)] and again in 1989 [ISO2631/2 (1989)] in order to encompass a broader review of motion in structures within the frequency range of 1-80Hz. These codes apply to vibrations in both vertical and horizontal directions and deal with random and shock, as well as harmonic excitations. The guideline is very similar to [BS6472 (1984)] and [ANSI S3.29 (1983)]. These guidelines differentiate vibrations into three distinct categories:

**continuous excitation:** steady-state excitation induced by machinery, etc. with typically more than 30 cycles of vibration.

**intermittent vibration:** lasts a few seconds but is characterised by a build-up to a level that is maintained for several cycles of vibration. Examples include traffic vibration and humans walking across a floor.

**transient excitation:** characterised by a rapid build-up to a peak followed by a decay. Examples include blasting or human jumping on a floor.

![Graph showing RMS Acceleration vs Frequency for ISO/BSI/ANSI vibration curves](image)

**Figure 2.5:** CGR of ISO2631/2 (1989), BS6472 (1984) and ANSI S3.29 (1983) building vertical vibration curves

The CGR form of the three guides is given in figure 2.5. There is a base curve which applies to floor vertical vibration of a person standing or sitting. This is the boundary at which the comfort level of a person starts to reduce as a result of vibrations (commonly known as *reduced comfort*)
level basis curve). This base curve can be adjusted by applying a weighting factor to the allowable accelerations. Variations in this weighting factor account for the type of excitation, occupancy, and the time of day. Table 2.2 shows these weighting factors for the various conditions.

<table>
<thead>
<tr>
<th>Place</th>
<th>Time</th>
<th>Continuous or intermittent vibration</th>
<th>Transient vibration (several occurrences per day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>critical working areas</td>
<td>day or night</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>residential</td>
<td>day</td>
<td>2 to 4</td>
<td>60 to 90</td>
</tr>
<tr>
<td></td>
<td>night</td>
<td>1.4</td>
<td>20</td>
</tr>
<tr>
<td>office</td>
<td>day or night</td>
<td>4</td>
<td>128</td>
</tr>
<tr>
<td>workshop</td>
<td>day or night</td>
<td>8</td>
<td>128</td>
</tr>
</tbody>
</table>

Following several experiments on concrete floor slabs, Waldron et al. (1993) and Caverson et al. (1994) assessed various acceptability criteria and discussed the ways in which the guidelines, drawn normally for composite slabs, could be applied to post-tensioned suspended concrete floors. They concluded that many of the scales of human perception of vibration are very similar in form, but they can yield different results because of small numerical differences in the borderline regions. However, these scales are seen to be difficult to use in design because of the need to calculate the frequency and acceleration response of slabs to human activities.

Bachmann and Ammann (1987) argued that due to the lack of exact knowledge about various floor parameters and their inter-relations, and also because of lack of simple design methods, it would be more practical to have limits by which engineers can design various floors. They presented acceleration limits based on past experience and measured values of similar structures. The use of acceleration limits has also been described by others [Ellingwood and Tallin (1984); Allen (1990b)], as shown in table 2.3. These are plotted in figures 2.2 to 2.5 and are seen to be generally quite high compared to the other guidelines. Also, there is a high degree of variability in the suggested acceleration limits, particularly for sensitive environments, such as offices.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>pedestrian structures</td>
<td>0.5-1.0</td>
<td>-</td>
<td>0.5-2.0</td>
</tr>
<tr>
<td>office buildings</td>
<td>0.2</td>
<td>0.04-0.37</td>
<td>0.5</td>
</tr>
<tr>
<td>gymnasium &amp; sports halls</td>
<td>0.5-1.0</td>
<td>0.4-0.7</td>
<td>-</td>
</tr>
<tr>
<td>dancing and concert halls</td>
<td>0.5-1.0</td>
<td>0.4-0.7</td>
<td>-</td>
</tr>
</tbody>
</table>

In summary, the human perception to vibrations is very complex and depends on many factors. Most perceptibility scales can reasonably predict human response given an accurate definition of the vibration, though some uncertainties and inconsistencies still remain. The difficulty arises at the design stage, where predicting the vibration characteristics is uncertain, as discussed in the subsequent sections. Some recent codes of practice, [BS6472 (1984), ISO2631/2 (1989) and DIN4150/2
introduced weighting factors, used to make the design more conservative according to the activity and the type of occupancy. Presently, this is the most commonly used approach.

2.3 Induced live loads on floor slabs

In this section, the knowledge on the forces induced by human activity on floors is reviewed. Live loads are imposed on structures by use and occupancy. Serviceability design in most building codes is based on statically applied live loads and aims to limit cracking and deflection. As such, vibrations and time-varying occupant induced loads have not normally been considered.

There are a number of possible causes of dynamic excitation on floors. It is widely accepted that continuous vibrations caused by machinery, road and rail traffic, or wind, should be dealt with at the design stage. Forces exerted by human activities on a floor are more difficult to design against. The loading induced by human movements may be categorised as static, impulsive or continuously changing, depending upon the duration in which a motion is achieved. Impulsive loads may be caused by motions such as a jump, dropping into a seat, or stopping suddenly. Movements such as walking, swaying, or rhythmic jumping, generate intermittent or continuous loading.

Perhaps the most universal human-induced excitation is the effect of walking on the floor. Walking is complex to represent mathematically as it involves the parameters of amplitude and position, both time-dependent, and the fact that people walk in different ways. Several studies of forces exerted by normal walking have been conducted [Wheeler (1982), Ellingwood and Tallin (1984), Mouring and Ellingwood (1994), Ebrahimpour et al. (1996)]. The starting point in developing an accurate model is the representation of the force due to a single human footfall. Footfall forces measured during several different walking activities were evaluated by Wheeler (1982) and are shown in figure 2.6. He found that the pacing frequency ranged between 1.7Hz and about 3.2Hz, from slow walking to running respectively. In terms of velocity, a slow walker travels at around 0.75m/s, a fast walker at about 1.75m/s, beyond which a person breaks into jogging and then running.

Walking as an activity is characterised by the heel and toe contact peaks and by the overlapping of succeeding paces; figure 2.6 shows an overlap of approximately 0.1 seconds for normal walking. Since the loading is periodic, it can be represented by a Fourier series of the form:

\[ F_p(t) = G \left\{ 1.0 + \sum_{i=1}^{N} \alpha_i \sin (2\pi f_p t - \phi_i) \right\} \]  \hspace{1cm} (2.2)
where $G$ is the weight of the person, $a_i$ is the Fourier coefficient of the $i^{th}$ harmonic and $N$ is the total number of contributing harmonics. Usually, a reasonable approximation can be obtained using three to six Fourier terms [Allen et al. (1985), SCI (1989), Mouring and Ellingwood (1994), Ji and Ellis (1994a), Bachmann et al. (1995), Ebrahimpour et al. (1996)]. Values of the Fourier coefficients have been calculated analytically for different contact ratios [Ji and Ellis (1994a)], or have been derived experimentally for different activities [Bachmann et al. (1995), Ebrahimpour et al. (1996)]. Typical measurements are shown in table 2.4.

Table 2.4: Bachmann et al (1995) normalised dynamic forces of human activities

<table>
<thead>
<tr>
<th>Type of activity</th>
<th>Activity rate (Hz)</th>
<th>Fourier coeffs &amp; phase lag</th>
<th>Design density (persons/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$a_1$</td>
<td>$a_2$</td>
</tr>
<tr>
<td>walking (continuous ground contact)</td>
<td>vertical</td>
<td>2.0</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>forward</td>
<td>2.4</td>
<td>0.2</td>
</tr>
<tr>
<td>running (discontinuous ground contact)</td>
<td>2.0-3.0</td>
<td>1.6</td>
<td>0.7</td>
</tr>
<tr>
<td>jumping (simultaneous ground contact of both feet)</td>
<td>normal</td>
<td>2.0</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>high</td>
<td>3.0</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.0</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.0</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dancing (equivalent to brisk walking)</td>
<td>2.0-3.0</td>
<td>0.5</td>
<td>0.15</td>
</tr>
</tbody>
</table>

As the frequency of walking increases, a threshold is reached where the activity is no longer classified as walking, but as jogging or running. In these modes, no more than one foot is in contact with the floor at any time and there is no separate heel and toe contact. As a result, the force-time graph
consists of near-sinusoidal peaks, as shown in figure 2.6. The running and jumping cases constitute the most severe loadings, as can be seen from the high values of $\alpha_i$ in table 2.4.

Recently, researchers have considered the effect of groups of people walking about a floor at random [Ellingwood and Tallin (1984), Allen and Murray (1993), Mouring and Ellingwood (1994), Ebrahim-pour and Fitts (1996)]. Generally, the excitation lacks coherence unless the group is walking in step and so a single pedestrian footfall provides an appropriate excitation amplitude for research purposes. However, in situations such as lively music concerts or aerobics, dynamic loads will be significant when any crowd movement (dancing, jumping, rhythmic stamping) is synchronised. In such cases, equation 2.2, describing the load, $F_p$, due to one person, can be replaced by the load due to a group, $F_g$: [BRE 426 (1997)]

$$F_g(t) = G_d \left\{ 1.0 + C_e \sum_{i=1}^{N} \alpha_i \sin (2\pi f_p t - \phi_i) \right\} \quad (2.3)$$

Here, the weight of one person, $G$, is replaced by the load density of the crowd, $G_d$, and the factor $C_e$ is introduced to represent the dynamic crowd effect, accounting for the fact that the crowd movement will not be perfectly synchronised [Ji and Ellis (1993)]. Although problems relating to crowd events had been recognised for many years, the first mention of such loading in British codes was in BS6399 (1996), relating to loads produced at pop concerts [Ji and Ellis (1997)].

Much larger impulsive loading can arise as a result of the heeldrop test. This is performed by a person of average weight ($\approx$800N) rising to the balls of his feet ($\approx$60mm from the floor), and then suddenly dropping his weight on his heels striking the floor. The standard graphical approximation of a heeldrop is shown in figure 2.7, with an initial peak load of 2.67kN and a duration of 0.05 seconds [Lenzen (1966), Murray (1975), Allen et al. (1979), Becker (1980)]. Its relative simplicity and ease of use as a standard and practical method has meant that many researchers have relied on heel-impact in their experiments, including for derivation of some human perceptibility guidelines [Lenzen (1966), Murray (1975), Allen and Rainer (1976), Wiss and Parmelee (1974), Foschi et al. (1995), Becker (1980)]. However, while the test may be meaningful for evaluating the response of floor systems to activities that cause impact forces, it produces an isolated transient vibration, which is not a good simulation of vibrations due to walking [Ellingwood and Tallin (1984), Onysko (1986)]. Furthermore, the heel-impact test indicates a stronger dependence on damping than has been found in other cases.

In summary, there are three main sources of human-induced loading. The most common is walking, which depends on many factors such as position on floor, stride length, step duration, and type of footfall function. These are functions of the pacing frequency and vary from person to person.
Figure 2.7: Approximation of heeldrop loading

Jogging and running are footfall activities, with the difference that at no point are both feet touching the ground. In some cases, dancing and jumping have similar representations to running and jogging, however, when a group is involved there is a likelihood of coherence of imposed forces. Factors to be considered here include the number of participants, the intensity of activity, and the positioning of the people with respect to each other. Impulsive loading is best represented by the heeldrop test, which depends on factors such as weight of the person and the duration of loading.

2.4 General dynamics of floor systems

Whether the loadings described in section 2.3 will cause unacceptable vibrations will depend on the dynamic characteristics of the floor system, particularly its natural frequency and damping. These parameters, and methods of their estimation, are discussed in subsequent sections. In this section, some general comments on post-tensioned floor types, and their dynamic behaviour, are given.

2.4.1 Types of suspended concrete slab

Post-tensioned concrete slabs are generally long span, slender structures which can be of many types, as shown in figure 2.8. Slabs are generally designed as either one or two-way spanning, depending on the way in which the floor load is to be transferred to the supports. Of the slab configurations in figure 2.8, ribbed and continuous band beam slabs are one-way spanning, because the load is principally supported by bending of the slab in one direction, perpendicular to the direction of bending of beams. Flat slabs, waffle slabs, or slabs with beams in two directions, behave as two-way systems. Note that the category of the slab (one or two-way spanning) is a
2.4.2 Introduction to vibration of slabs

Insight into the vibration of floor slabs can be gained by considering the dynamic characteristics of simple beams and plates. A continuous element, such as the simply supported beam of figure 2.9, will have a series of natural frequencies, each associated with its own mode shape. So long as behaviour is linear, the various modes are dynamically independent and response can be synthesised by adding modal solutions computed independently. The lowest frequency mode is known as the fundamental and the higher modes have shapes of increasing complexity.

A useful insight into the behaviour of floor slabs is given by the behaviour of an orthotropic plate, shown in figure 2.10. The fundamental mode shape resembles the corresponding beam mode shape.
in both directions, and this principle also applies to the higher modes. If the stiffness is highly orthotropic, the weak direction deformation has relatively little effect on the frequency and a basic family of modes retaining the fundamental shape in the strong direction is observed. One-way spanning slabs, such as beam-slab floors, behave in this way. Two-way slabs exhibit more isotropic behaviour. The vibration characteristics of such slabs are more complex and, for most boundary conditions, exact solutions of their natural frequencies and mode shapes are not available [Szlad (1974)]. It should be noted, however, that given the correct boundary conditions and material properties, the finite element (FE) method of analysis will estimate these parameters closely. The FE method is not used in this work but its merits are discussed in section 2.9.

![Figure 2.10: Modeshapes of orthotropic plate (from SCI (1989))](image)

### 2.4.3 Effect of prestressing on vibration

There is at present some doubt about the effects of prestressing on the dynamics of a system. Conventional analysis suggests that an axial load, applied along the centroidal axis of a beam, affects its vibration characteristics, since additional out-of-balance forces are generated as the beam deflects (see figure 2.11(a)). In a prestressed member, however, the load moves with the beam and no out-of-balance moments arise (see figure 2.11(b)). Therefore, the natural frequencies and mode shapes would be expected to remain unchanged, see section 6.2.

![Figure 2.11: Effect of prestressing on beam vibration](image)
Nevertheless, in practical cases, small increases in natural frequency are observed as a result of prestressing. In a reinforced beam, the concrete acts only in the area of the cross-section that is in compression, and would be cracked in the tension zone. In a prestressed beam, prestressing force imposes pre-compression in the tension zone so that, in most cases, the entire cross-section remains uncracked and elastic. This leads to a difference in stiffness between a normally reinforced and prestressed concrete element of similar geometry, due to the difference in the effective second moment of area. This in turn depends on the amount of cracking present in the section. Therefore, due to fewer cracks and greater effective section, it is likely that any prestressed member would be stiffer than one similarly reinforced, hence resulting in higher natural frequency. This theory is widespread among researchers [Dall'Asta and Dezi (1996), Deák (1996)], but yet to be fully proven.

2.4.4 Vibration of multiple spans or panels

For continuous floors, such as the one shown in figure 2.12, it is generally expected that the adjacent panels move in opposite directions (i.e. out of phase) when vibrating. The inertial forces due to the vibrations act in the senses shown and enhance the deflections. For static design, however, the self-weight effects of adjacent spans combine to reduce the corresponding stresses and deflections.

![Figure 2.12: Fundamental mode shape of a continuous beam](image)

2.5 Natural frequencies

The natural frequency of a floor slab is a key parameter in defining its dynamic behaviour. Concrete slabs are three dimensional and redundant in nature, making their behaviour complex and the theoretical estimation of their fundamental properties daunting. A particular area of difficulty is the accurate characterisation of the boundary conditions. In general, a floor is simplified theoretically and then its behaviour interpreted. In practice, this simplification is carried out in the design guides, where usually approximate formulae for the estimation of natural frequency and floor response, based on beam theory, are given. In the following sections, theoretical and practical approaches for evaluating floor natural frequencies are given, and the various methods are discussed.
2.5.1 General definitions

The simplest model of a dynamic system has one degree of freedom and is characterised by a mass, \( m \), supported by a spring with stiffness, \( k \). The undamped fundamental natural frequency of such a system is given by:

\[
f_{0u} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \tag{2.4}
\]

Taking the static deflection due to the self-weight of the mass, \( \Delta_s = \frac{mg}{k} \), this can be written as:

\[
f_{0u} = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta_s}} \tag{2.5}
\]

In real systems, any oscillations will cease with time, due to the dissipation of energy. This vibration decay, known as damping, is assumed to develop a force that opposes the direction of motion. The fundamental damped natural frequency of a SDOF system is:

\[
f_0 = f_{0u} \sqrt{1 - \zeta^2} = \frac{1}{2\pi} \sqrt{\frac{k}{m} (1 - \zeta^2)} = \frac{1}{2\pi} \sqrt{\frac{g}{\Delta_s} (1 - \zeta^2)} \tag{2.6}
\]

where \( \zeta \) is the damping ratio (see section 2.6 for a fuller definition of damping). The following sections describe the theory involved in calculating the natural frequencies of more complex systems.

2.5.2 Theoretical background and estimation methods

Vibration of a beam

Consider a simply supported beam with a mass per unit length \( w = \rho A \), where \( \rho \) is density and \( A \) is cross-sectional area. By taking an infinitely short elemental length of beam, figure 2.13, and applying Newton's second law of motion, the partial differential equation of motion of the beam can be obtained: [Rao (1995)]

\[
\frac{\partial^2}{\partial x^2} \left( EI(x) \frac{\partial^2 u}{\partial x^2} \right) + w \frac{\partial^2 u}{\partial t^2} = p(x,t) \tag{2.7}
\]

For unforced free vibration, separation of variables can be used by assuming a solution in the form \( u(x,t) = \Phi(x)T(t) \). The resulting natural frequencies and corresponding mode shapes are:

\[
f_n = \frac{n^2 \pi}{2} \sqrt{\frac{EI}{wL^4}} \quad \text{and} \quad \Phi_n(x) = C_n \sin \left( \frac{n\pi x}{L} \right) \tag{2.8}
\]

where \( n \) is the order of natural frequency and \( C_n \) is an arbitrary mode shape amplitude factor. There are infinite numbers of frequencies and orthogonal mode shapes and, so long as motion is linear, any deflected shape is the sum of these modes. Figure 2.9 shows the first three modes.
Vibration of a plate

A differential equation for the motion of a plate can be derived as in the case of the beam by adding an extra dimension. For a rectangular plate with mass per unit area, \( W \), simply supported on all four edges, the equation of motion is [Szilard (1974)]:

\[
D \nabla^2 \nabla^2 u(x, y, t) = p(x, y, t) - W \frac{\partial^2 u(x, y, t)}{\partial t^2}
\]

(2.9)

where \( D = \frac{Eh^3}{12(1-\nu^2)} \) is the flexural rigidity of the plate and \( \nabla^2 \) is the Laplacian operator. For free vibration assume a solution of the form \( u(x, y, t) = X(x)Y(y)T(t) \), where the time function \( T(t) \) is sinusoidal and \( \Phi(x, y) = X(x)Y(y) \) is a double Fourier series. The natural frequencies are:

\[
f_{mn} = \frac{\pi}{2} \left( \frac{m^2}{L_x^2} + \frac{n^2}{L_y^2} \right) \sqrt{\frac{D}{W}}
\]

(2.10)

These results can be interpreted as earlier in section 2.4.2 for an orthotropic plate. For more complex boundary conditions and shapes, exact solutions are not readily available [Szilard (1974)].

Rayleigh’s principle

This method provides a means of estimating an upper bound to the fundamental frequency of a system. It is based on energy concepts and states that the maximum kinetic and potential energies of a system are equal. The method is particularly useful for systems with many degrees of freedom. Consider a vibrating beam whose fundamental mode shape can be fully described by \( \Phi(x) \). By assuming the motion of the beam to be described as \( u(x, t) = \Phi(x) \cos \omega t \), the maximum kinetic and potential energies associated with this motion are: [Rao (1995)]

\[
KE_{\text{max}} = \frac{\omega^2}{2} \int_0^L \rho A(x)(\Phi(x))^2 dx
\]

(2.11)

\[
PE_{\text{max}} = \frac{1}{2} \int_0^L EI(x)(\Phi'(x))^2 dx
\]

(2.12)

Assuming conservation of energy (i.e. no damping), the maximum energies can be equated resulting in an estimate of the fundamental frequency, once the deflection \( \Phi(x) \) is known. Generally, the
static equilibrium shape is assumed for $\Phi(x)$. However, it should be noted that any assumed shape unintentionally introduces a constraint on the system (adding additional stiffness to the system) and so the estimated frequency is always greater than or equal to the exact value. Rayleigh's principle may be applied to slabs in the same manner as for beams but, due to more complex boundary conditions and mode shapes, it is not often used in practice. For higher frequencies, the Rayleigh approach can be extended by representing the total motion as the sum of several assumed mode shapes; this is known as the Rayleigh-Ritz method.

**Sodoia method**

This is an iterative static analysis method which provides an estimate of the fundamental frequency by an approximation of the dynamic mode shape. The estimated mode shape is first multiplied by the mass to derive a value of force proportional to the inertial force. This imaginary inertial force is then applied to the system and the resulting deflection, once normalised, is an updated better approximation of the mode shape. The procedure is continued until the mode shape no longer changes significantly between iterations. Convergence is normally achieved after two or three iterations. Hence, the square of the angular frequency is estimated by dividing the previous mode shape by the resulting displacement [Clough and Penzien (1993), Timoshenko et al. (1974)].

**Dunkerly's principle**

This method gives the approximate value of the fundamental frequency of a system in terms of the natural frequencies of the component parts. For an idealised floor system with the slab fundamental frequency, $f_{01}$, floor beam frequency, $f_{02}$, and main beam frequency, $f_{03}$, the fundamental system frequency, $f_0$, is obtained from: [Allen (1974)]

$$\frac{1}{f_0^2} = \frac{1}{f_{01}^2} + \frac{1}{f_{02}^2} + \frac{1}{f_{03}^2}$$

(2.13)

This method is more relevant to composite steel-concrete floors, which have separate components. For post-tensioned floors, it may be used to consider the main slab and its support beams.

**Continuous spanning slabs**

The simplest case of a continuous spanning system is a two-span beam. With equal spans, it is possible to consider only a single span since, in the fundamental mode, adjacent spans act in opposite directions due to inertial forces (figure 2.12). Making the spans unequal, would have
a stiffening effect, thus increasing the natural frequency. For multiple span beams, graphs are available to aid the calculation of natural frequencies [SCI (1989)].

CSA (1989) suggests the use of the simple beam formula to calculate natural frequencies of continuous beams. It suggests obtaining a dynamically equivalent simply supported beam from the span and restraint conditions of a continuous one. This requires an assumption of the mode shape, which in some cases may be very crude due to uncertainties in boundary conditions. Hence, care should be taken in decreasing the span length to obtain an *equivalent simply supported span*, as erroneous estimates of frequency may result [Caverson et al. (1994)].

### 2.5.3 Practical methods of frequency estimation

The theory introduced above can be used to estimate the frequencies of real floors in practice. However, the determination of the exact floor vibration natural frequencies is by no means straightforward, if all variables are considered. The complexity of the exact computation has prompted investigators to propose simplified approaches, three of which are discussed below.

**Equivalent beam method (EBM)**

This is the most commonly used method of predicting the fundamental natural frequency of a floor, as presented by Murray (1975) for composite steel-concrete slabs. The slab is approximated as a beam with transformed width and second moment of area, (figure 2.14), and the fundamental natural frequency is given as:

\[ f_0 = K \sqrt{\frac{EI}{wL^4}} \]  \hspace{1cm} (2.14)

where values of *K* depend on boundary conditions and are given as 1.57 for simply supported, 2.45 for fixed/simply supported, 3.56 for fixed/fixed, and 0.56 for a cantilever beam. For a simply supported beam, this formula is identical to the solution derived in equation 2.8. For the purposes

![Figure 2.14: Murray(1975) transformed beam](image-url)
of calculation of second moment of area, SCI (1989) suggests the effective width of slab to be the smaller of either \( L/4 \) or the beam spacing. Allen and Murray (1993) suggested the smaller of \( 0.4L \) or the beam spacing.

The EBM is seen by a number of researchers to give acceptable estimates of floor fundamental frequency [Chang (1973), Allen (1974), Murray (1975), Allen et al. (1979), Pernica and Allen (1982), SCI (1989)]. However, caution must be exercised in the application of this formula to two-way spanning floors. The limitations of this method are: (1) higher modes of vibration are not considered, (2) the state of the concrete cross-section (degree of cracking and modulus of elasticity) are uncertain, (3) for slabs with deep joists or trusses, expressing the natural frequency in terms of second moment of area can lead to substantial errors, mainly due to shear deformation effects of the joists [Allen et al. (1985)], and (4) its application to continuous slab systems and the determination of the dynamically equivalent simply supported span [CSA (1989)] are questionable. In a comparison carried out between predicted and experimental results, Williams and Waldron (1994) found that for post-tensioned concrete slabs, the EBM underestimated the real frequencies by between 17% and 54%. Hence, this method is unreliable for two-way spanning slabs.

**Static deflection method**

This approach uses the classical mass-spring system for calculating natural frequencies, as given in equation 2.6 in terms of static deflection [Allen et al. (1985), SCI (1989), Allen (1990b), Allen and Murray (1993), AISC (1997)]. Furthermore, for a composite one-way slab with uniform load, Dunkerly’s principle can be used to obtain the system static deflection: [Allen (1990b)]

\[
\Delta_s = \frac{\Delta_b + \Delta_g}{\lambda} + \Delta_c
\]  

(2.15)

where \( \Delta_s \) is the total static deflection and \( \Delta_b, \Delta_c \) and \( \Delta_g \) denote static deflections of beams, columns and girders respectively. \( \lambda \) is the transformation factor from real systems to equivalent SDOF systems. It takes the values of 1.3 for a simply supported beam and 1.5 for two-way slabs and fixed cantilevers. This method will give lower frequencies than the EBM, since it allows for shear and column deflections in addition to bending estimates. Since the EBM already underestimates values of frequency [Williams and Waldron (1994)], the effectiveness of the method on concrete floors is questionable. The main limitation of this method is that it requires the calculation of static deflections, which are difficult to carry out particularly for two-way slabs.
Concrete Society method

The Concrete Society (1994) proposed a calculation procedure for two-way slabs, in which vibration is assumed to occur in two independent orthogonal modes, in the two span directions. This method makes use of the EBM, but introduces modification factors to account for the increased stiffness of a two-way member. Account is also taken of the type of slab, aspect ratio, and boundary conditions. The method yields two natural frequencies, corresponding to independent modes in the two span directions. The formulae and procedures are given below in the x-direction for solid or waffle slabs only. The characteristics of the y-direction mode can be determined by interchanging the x and y subscripts in these equations.

Firstly, the effective aspect ratio of a slab panel is defined as:

\[ \lambda_x = \frac{n_x l_x}{l_y} \left( \frac{EI_y}{EI_x} \right)^{\frac{1}{4}} \]  

(2.16)

where \( n_x \) is the number of bays in the x-direction and \( l_x, l_y \) are bay spans in the x and y directions. This is used to calculate a modification factor \( k_x \):

\[ k_x = 1 + \frac{1}{\lambda_x^2} \]  

(2.17)

For slabs with perimeter beams, the natural frequency is then:

\[ f'_x = \frac{k_x \pi}{2} \sqrt{\frac{EI_y}{W' t_y^3}} \]  

(2.18)

For slabs without perimeter beams, an intermediate frequency is first calculated, which is used to modify equation 2.18:

\[ f_b = \frac{\pi}{2} \sqrt{\frac{EI_y}{W' l_y^3}} \frac{1 + \frac{EI_x l_x^4}{EI_y l_y^4}}{\sqrt{1 + \frac{EI_x l_x^4}{EI_y l_y^4}}} \]  

(2.19)

The natural frequency is then:

\[ f_x = f'_x - (f'_x - f_b) \left[ \frac{1}{2n_x} + \frac{1}{2n_y} \right] \]  

(2.20)

Similar calculations for the other slab types (e.g. ribbed slabs) are also given. As can be seen, this method is highly complex involving many parameters and equations. Furthermore, the mode in the x-direction is seen to depend on y-direction bending properties (equation 2.18), the reasons for which are not explained. Williams and Waldron (1994) found little agreement between their field results and predictions of natural frequency using this method. It was seen to under-predict the actual frequencies in all their cases, sometimes by as much as 77%.
2.6 Damping

Damping is a mechanism by which vibrational energy is gradually converted to heat or sound, resulting in a slow decrease of the response of a system. It can arise from almost any structural or non-structural element, and thus, is very hard to predict theoretically. The causes of damping are also difficult to determine, hence it is modelled as one or more of the following types:

**Viscous damping:** This is a common form of damping, which depends on the material properties of the structure. Here, the damping force is proportional to the velocity of the vibrating body, and always opposes the motion, resulting in an exponential decay of oscillations.

**Coulomb, or friction, damping:** Here, the damping force is constant in magnitude but opposite in direction to the motion of the vibrating body. It is caused by friction between rubbing surfaces that are either dry or have insufficient lubrication. This cause of damping is independent of frequency and the resulting decay of oscillations is linear. In concrete, Coulomb damping at the cracks and joints is one of the major causes of energy dissipation.

**Material, or hysteretic, damping:** When materials are deformed, energy is absorbed and dissipated by the material. The effect is due to friction between internal planes, which slip or slide as the deformations take place. When a body having hysteretic damping is subjected to vibration, the stress-strain diagram shows a hysteresis loop. The area of this loop denotes the energy lost per unit volume of the body, per cycle, due to damping. In concrete, hysteresis damping is most likely to occur at the cracks.

**Radiation damping:** This is the amount of energy that radiates outwards from the floor into the surrounding structural components, such as columns.

2.6.1 General definitions

In most mathematical models, the combined damping effect is assumed to be in the form of equivalent viscous damping, as this involves linear equations, which are relatively easy to solve. A viscous damper can be constructed using two parallel plates separated by a distance, $h$, with a fluid of viscosity, $\mu$, between the plates (see figure 2.15). If the top plate moves with a velocity $v$, there is assumed to be a linear variation of the velocities of intermediate layers between 0 and $v$, the velocity gradient being: $\frac{dv}{dy} = \frac{v}{h}$. The shear or resisting force, $F$, developed at the bottom surface of the moving plate is:

$$F = \tau A = \frac{\mu v}{h} = cv$$  \hspace{1cm} (2.21)
where $\tau$ is shear stress and $c$ is called the *viscous damping constant* given by:

$$c = \frac{\mu A}{h}$$

(2.22)

Figure 2.15: Example of viscous damping mechanism

In theoretical analyses, a damper is assumed to have neither mass nor elasticity and damping force exists only if there is relative velocity between the two ends of the damper. It is illustrated in figure 2.16(a) by a dashpot, which is usually in parallel to a spring representing the stiffness of the material. This is known as the Kelvin-Voigt model [Bert (1973)].

![SDOF Kelvin-Voigt spring-mass-damper model](image)

Figure 2.16: SDOF Kelvin-Voigt spring-mass-damper model

The application of Newton’s second law to figure 2.16(b) yields the equation of motion:

$$m\ddot{x} + c\dot{x} + kx = 0$$

(2.23)

By solving in the usual way, the roots of the characteristic equation are expressed as:

$$-\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

(2.24)

The *critical damping*, $c_c$, is defined as the value of the damping constant, $c$, for which the radical in equation 2.24 becomes zero, hence making the vibration non-oscillatory:

$$c_c = 2m\sqrt{\frac{k}{m}} = 2m\omega_n$$

(2.25)
For any damped system, the damping ratio, $\zeta$, is defined as:

$$\zeta = \frac{c}{c_c}$$

(2.26)

This is the conventional way of expressing damping in civil engineering structures.

### 2.6.2 Methods of structural damping measurement

The determination of the damping capacity of a system usually depends on the method of measurement employed. It can be derived from: (a) energy dissipation, (b) decay of free vibrations, (c) reduction of resonant response, or (d) phase difference between force and displacement [Ungar (1963)]. Here, two common methods are described in the time and frequency domains.

**Logarithmic decrement method**

The logarithmic decrement represents the rate at which the amplitude of free damped vibration decreases. It is defined as the natural logarithm of the ratio of any two successive amplitudes. In figure 2.17, if $x_1$ and $x_2$ denote the amplitudes corresponding to times $t_1$ and $t_2$, measured one cycle apart, the ratio of $x_1$ to $x_2$ is:

$$\frac{x_1}{x_2} = \frac{e^{-\zeta \omega_n t_1}}{e^{-\zeta \omega_n (t_1 + T)}} = e^{\zeta \omega_n T}$$

(2.27)

The logarithmic decrement is then:

$$\delta = \ln \frac{x_1}{x_2} = \zeta \omega_n T = \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}}$$

(2.28)

Similarly, if the displacements are measured over any number of complete cycles, $r$:

$$\delta = \frac{1}{r} \ln \left( \frac{x_1}{x_{r+1}} \right)$$

(2.29)

From equation 2.28, $\delta$ can be related to the damping ratio, $\zeta$, thus:

$$\zeta = \frac{\delta}{\sqrt{(2\pi)^2 + \delta^2}}$$

(2.30)

Hence, the damping in a system can be measured experimentally from the time-domain data of free damped oscillation. One draw-back is that $\zeta$ is often amplitude dependent, thus from different parts of the decay curve different damping quantities result [Ellis and Ji (1996)]. For purely viscous damping, the dotted envelope line in figure 2.17 is an exponential decay, and for pure friction damping it is a straight line decay. For real structures, the envelope line generally lies in-between these two cases [Bachmann et al. (1995)].
Halfpower bandwidth method

This method uses the frequency-domain response curve to determine damping from the amplification factor at a resonant mode (see figure 2.18). For a SDOF system, the dynamic amplification factor (DAF) is expressed as:

$$DAF = \frac{1}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}^{\frac{1}{2}}$$  \hspace{1cm} (2.31)

where $\omega_n$ is the natural frequency of the system and $\omega$ is the forcing frequency. At resonance, $\omega = \omega_n$ and equation 2.31 reduces to:

$$Q = \frac{1}{2\zeta}$$  \hspace{1cm} (2.32)

where $Q$ is the amplitude at resonance. The points $R_1$ and $R_2$, where the amplification falls to $\frac{Q}{\sqrt{2}}$, are called half-power points and the difference between $R_2$ and $R_1$ is called the bandwidth of the system. This can be found by equating the dynamic amplification of the system to $\frac{Q}{\sqrt{2}}$:

$$\frac{Q}{\sqrt{2}} = \frac{1}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}^{\frac{1}{2}}$$  \hspace{1cm} (2.33)

For small $\zeta$, the solution of equation 2.33 is:

$$\omega_{1,2} = \omega_n \sqrt{1 \pm 2\zeta} \approx \omega_n (1 \pm \zeta)$$  \hspace{1cm} (2.34)

Hence, if $\omega_1$ is the value of frequency at $R_1$ and $\omega_2$ is the frequency at $R_2$, $\omega_2 - \omega_1 \approx 2\zeta\omega_n$, and the damping ratio for the system is given by:

$$\zeta = \frac{\omega_2 - \omega_1}{2\omega_n}$$  \hspace{1cm} (2.35)

For complex structures with closely spaced modes, the resonance curves can be difficult to evaluate.
and curve-fitting techniques may be required to separate the individual peaks. Hence, the application of this method to complex systems can become cumbersome. Moreover, non-linear behaviour of a structure may lead to disturbances at the resonant peak, with the result that for a certain value of $\frac{\omega}{\omega_n}$, two (or three) different amplitudes may occur. This makes it difficult to evaluate the resonance curve. Hence, with this method, measuring errors can greatly affect the determination of the damping quantity.

2.6.3 Damping mechanisms in concrete

Material damping in concrete elements is mainly due to cracking and depends strongly on the stress level. Figure 2.19 shows the equivalent damping ratio of a bending element [Bachmann et al. (1995)]. For low stress, corresponding to the uncracked state, a relatively low damping ratio ($\zeta < 1\%$) exists. As cracks form, the damping ratio increases. In the cracked state, but still with relatively low stress, the damping ratio is relatively high, perhaps two or three times the value of the initial uncracked state. With a further increase in stress, the damping ratio decreases rapidly and may reach a value smaller than that of the initial uncracked state. This damping behaviour may be explained as follows:

- in the uncracked state, nearly pure viscous damping occurs in the concrete
- in the cracked state, two kinds of damping occur
  - nearly pure viscous damping in concrete uncracked compression zone
  - nearly pure friction damping, due to friction between concrete and reinforcing steel, in the cracked tension zone
The actual behaviour of the concrete can be represented by its equivalent viscous damping ratio, $\zeta_{eq}$, incorporating both means of energy dissipation:

$$\zeta_{eq} = \frac{1}{4\pi} \frac{\text{energy dissipation per cycle}}{\text{maximum potential energy}}$$  \hspace{1cm} (2.36)

Note that the maximum potential energy in a structure is proportional to the square of the stress intensity, as is the energy dissipation per cycle due to viscous damping [Bachmann et al. (1995)]. Hence, at a constant depth of the compression zone, a viscous damping component of $\zeta$ results, which is independent of the stress intensity (see figure 2.19). On the other hand, the energy dissipation per cycle due to friction is linearly proportional to the stress intensity and a friction damping component of $\zeta$ results, which decreases hyperbolically with increasing stress intensity (figure 2.19).

![Diagram](image)

Figure 2.19: Damping mechanisms in concrete with increasing stress intensity

**Overall damping of a floor structure**

Depending on the location of energy dissipation, the total overall damping of a floor structure is the sum of the contributions from the bare structure, non-structural elements, and energy radiation to surrounding structural components. Contributions by non-structural elements depend on their number, type, and relative dimensions. This contribution may be greater than the equivalent damping ratio of the bare structure alone. This explains why different structures or structural types, made of the same materials, may have very different overall damping ratios. The energy radiation to the soil, through walls or columns by travelling waves, may be significant but is difficult to quantify. The overall effect of all the above contributions to the damping of a floor can not yet
be stated with confidence. The estimation of the overall equivalent viscous damping ratio of a structure is largely based on the judgement and experience of the engineer.

2.6.4 Review of measurement practice and results

The first rigorous investigation into the damping characteristics of concrete was carried out by Penzien (1964), who tested a total of 20 prestressed concrete beams. He found that most of the equivalent viscous damping factors ranged from 0.5% to 7%, depending on the degree of cracking permitted in each test due to prestressing.

Lenzen (1966) tested actual concrete floors and found that floors for which vibrations were barely or not at all perceptible had damping exceeding 5%, and the vibrations were definitely perceptible in floors with damping less than 3%. The inclusion of partitioning and mechanical services increased damping to well above 6% for most floors.

Murray (1975) suggested values of floor damping ratios based on slab thickness. These are similar to Allen (1974), Allen and Rainer (1976) and CSA (1989) recommendations. Murray’s suggested values are: bare floor, $\zeta=1-3\%$ (depending on slab thickness and concrete mix design); ceilings, $\zeta=1-3\%$ (lower limit for hung ceiling and upper limit for ceilings attached to beams); ductwork and mechanical equipment, $\zeta=1-10\%$ (depending on amount); and partitions, $\zeta=10-20\%$ (if attached to the floor system). Murray also recommended that for floor systems with damping ratios more than 8-10%, there would be no need for vibration analysis. AISC (1997) recommends that these values be halved since they are derived using heeldrop tests, which include transmission of energy to other structural components, whereas modal damping excludes vibration transmission.

Farah et al. (1977) commented on the merits of using constrained viscoelastic materials to damp floor vibrations. This technique would be most useful in composite floors, where the viscoelastic layer would be applied to the bottom flange of the supporting beams and can be constrained using metal plates. Nelson (1968) reported three such applications with notable increases in damping capacity. Moiseev (1991) suggested using layers of concrete treated with viscoelastic admixtures to reduce vibrational response of floors. He described results of tests on two floors in which the addition of such damping layers reduced peak vibration velocities by as much as 60%, compared with predicted values of floors without the layers. A 20% increase was observed in the actual damping ratios of the floors and Moiseev commented that care should be taken with the use of the admixtures due to possible changes in the concrete properties.
Caverson et al. (1994) presented field test results, in which they found the mean damping ratios for prestressed floors and reinforced concrete floors to be similar at around 2.2%, and to be higher than the value for their composite floor at 1%. They also found that non-structural elements, such as false ceilings and false floors, significantly increased the damping ratios of floors to around 2.7-4.7%. These results agree well with Murray’s suggested values. Pernica (1987) tested a slab before and after installation of non-structural components and found a marked increase in damping.

In summary, the overall value of damping inherent in a structure is dependent on many factors, such as the structure’s geometry and non-structural components present, and is hence very difficult to estimate. Even now, engineers and designers select values of damping based on previous results of similar structures, or on their own past experience. It is hence commonly accepted that further research is required in this field to achieve better estimates of structural damping capacity.

2.6.5 Tuned mass dampers (TMDs)

A tuned mass damper (or vibration absorber) is a vibratory subsystem attached to a larger primary vibration system. The normal function of the absorber is to reduce resonant oscillations of the primary system. Accurate tuning of the frequency of the absorber results in induced inertia forces of the absorber mass, which counteract the forces applied to the primary system. While the vibration amplitudes of the primary system can be suppressed to a large extent, large displacement amplitudes must be accepted in the absorber system.

In most cases, vibration at one natural frequency of the primary system is troublesome and requires attenuation. When tuned to the troublesome frequency, an absorber can substantially reduce the maximum displacement amplitude of the primary system. The absorber hardly affects the response of the primary system away from the optimum absorber frequency.

Practical applications

The application of TMDs can be considered whenever critical dynamic forcing cannot be avoided and the structure cannot be stiffened economically. TMDs are only effective in certain types of structures, such as buildings and bridges, which demonstrate high displacement values. The first use of TMDs can be dated back to 1911 when they were used to reduce the rolling motion of ships, and also for the reduction in ship hull vibrations [Setareh and Hanson (1992a)]. For civil engineering applications, McNamara (1977) investigated the effectiveness of TMDs in buildings and found a
significant increase in their damping ratios. Bachmann (1992b) installed TMDs on footbridges and found that their inclusion reduced acceleration amplitudes by a factor as high as 20.

Until recently, the application of TMDs on floors was very rarely considered and mainly confined to laboratory experiments [Lenzen (1966)]. This was because most floors exhibit several closely spaced natural frequencies requiring an array of TMDs, making the operation uneconomical. Problems may also arise with their maintenance and re-tuning [Allen (1990a)], although recently adaptive TMDs have been developed with self-tuning capabilities [Rade and Steffen (1999)]. Some successful applications of TMDs to floors have been reported. Webster and Vaicaitis (1992) installed a TMD system in a composite floor and managed to reduce floor vibrations by as much as 60%. Setareh and Hanson (1992b) applied absorbers to a cantilever concert hall balcony, reducing vibration amplitudes by 78%. However, in both of these cases, the construction was of high displacement and low damping, which makes the use of TMDs more effective. Their use on normal simply supported two-way floors, in which the displacements are much smaller, will require more research.

2.6.6 Human-structure interaction

In this section, the interaction between a floor and its occupants is discussed as far as the response and energy absorption of a human body is concerned. This may be significant if the mass of the people is reasonably large in comparison with the mass of the structure. In this case, it is possible that the interaction between the people and the structure could effectively change the system characteristics. Existing methods of considering people in structural vibrations are varied and sometimes inconsistent. Some design codes treat human action as simply a load [BS5400 (1978), CSA (1989)], whereas studies of the human body suggest that it should be modelled as a spring-mass-damper system [Ellis and Ji (1997), Foschi et al. (1995), Folz and Foschi (1991)].

A simple model of human-structure interaction can be obtained by treating each as a SDOF system, so that they combine to give a 2DOF system. This is similar to the basic model of a structure-TMD setup, with some important differences:

- The mass ratio between humans and a structure is likely to be much higher than that of a TMD. This may affect the validity of the reduction of the structure to a SDOF system, which is based on the structure without the secondary system.
- For a TMD setup, the fundamental frequency of the absorber is a design variable that is tuned. The frequency of the human body, however, is constant for a given person and posture, typically between 4-6Hz when sitting and 8-10Hz when standing [Fairley and Griffin (1989)].
• The damping ratio of a TMD system is also a design variable, whereas that of a human body is a constant and can be as large as 47.5% at its sitting position [Ji and Ellis (1994b)].

• The human body is not always present on the structure and its effect as an energy absorber may be completely lost when it changes from a stationary state to a moving state.

A similar, but slightly more complex, representation of the human body is proposed by ISO2631/1 (1985), where the body is modelled as a 2DOF system with prescribed parameters. The two natural frequencies of this model are 5.03Hz and 12.49Hz. Folz and Foschi (1991) utilised this model in a numerical study supported by experiments on a composite floor and found good agreement. Foschi et al. (1995) stipulated that the second frequency does not have a significant influence on the response and so a SDOF system would suffice as a model. Other, more elaborate, representations of the human body include the 15DOF model introduced by Nigam and Malik (1987), and a continuous model of the body in the standing position proposed by Ji (1995).

Practical results

Currently, the subject of human-structure interaction is poorly covered in the literature and only a few notable findings can be recorded. Most investigations have involved structures such as grandstands, where the number of people present is high. Ellis et al. (1994) tested a cantilevered grandstand when empty and when occupied during a sports event. They found that with the crowd, an additional frequency was observed that did not exist when empty. They also noticed a significant increase of damping when people were involved. They concluded that the crowd can be modelled as a spring-mass-damper system, which interacts with the structure to form a 2DOF system, hence the appearance of two frequencies and increased damping. Pernica (1983) also observed increased damping with crowd involvement.

2.7 Floor accelerations

In general, there are two approaches to the design of structures to accommodate dynamic occupant loads. One relies on ensuring the fundamental natural frequency of the structural system is sufficiently high that resonances produced by occupant movements are avoided, the other provides a method for calculating structural response to dynamic loads so that a structural design can be checked. The approaches therefore are to avoid or design for the problem. In the following section, the second of these approaches is considered.
The response of a floor to occupant movements depends on the mass of the floor, its dynamic characteristics, such as natural frequency and damping, and on the nature of the input loading. The most common parameter used in the analysis of floor dynamic response is the value of the vertical acceleration of the slab caused by a given loading function. Most acceptability criteria for human perception of vibration use this parameter (see section 2.2.2). Here, methods of calculating acceleration amplitude are first discussed as a result of simplified loading functions. The various design guidelines in use, for limiting floor response, are described in section 2.7.2.

2.7.1 Estimation methods

To enable accurate estimation of the response of a floor, it is first necessary to consider the applied loading. Due to the complexity of analysing the response to a typical walking force, most design methods use a simplified empirical approach. The heeldrop test was developed as a result of this need and is used in most of the methods for estimating initial acceleration.

Allen and Rainer (1976) and Pernica and Allen (1982) suggested that for floors longer than 7.6m, with frequencies less than 10Hz, initial acceleration amplitude from a heeldrop can be estimated assuming it is suddenly applied to a simple spring-mass system giving:

\[ A_0 = \frac{2\pi f_0 P(0.9)}{M} \]  \hspace{1cm} (2.37)

where \( M \) is the equivalent mass of the oscillator, \( P \) is the impulse force due to a heeldrop (70Ns), and the value of 0.9 accounts for the loss of amplitude due to damping before reaching the first peak. SCI (1989) ignores this factor thus arriving at a marginally higher prediction.

CSA (1989) gives a formula for the peak acceleration amplitude of composite floors, assuming heeldrop loading and that a fixed width of floor, \( b \), participates in the response. Hence:

\[ A_0 = \frac{60 f_0}{w_L b L} \]  \hspace{1cm} (2.38)

where \( w_L \) is the weight of the floor plus contents. This method requires the estimation of natural frequency, which is calculated by EBM (section 2.5.3). Williams and Waldron (1994) found very poor agreement between acceptability ratings of this approach with experimentally measured parameters on prestressed concrete floors.

For rhythmic dance-type loads, in a departure from assuming the heeldrop, Ji and Ellis (1994a) used equation 2.3 (page 18) as the loading function. They hence presented an analytical solution for the forced vibration of simply supported floors, using plate theory, and considered several modes of
vibration. For this method, an assumption of the fundamental mode shape of the floor is required and the steady-state acceleration response, $A_{st}$, is given as:

$$A_{st} = B \frac{G_d}{W} (DLF)$$  \hspace{1cm} (2.39)

where $W$ is the mass of the empty floor per unit area, and $B$ is defined as the structural factor depending on type of structure and boundary conditions. $DLF$ is the acceleration dynamic magnification factor, for which the peak value of the $n^{th}$ Fourier component is defined as:

$$(DLF)_{peak} = \frac{\alpha_n n^2 \left( \frac{f_p}{f} \right)^2}{\sqrt{\left(1 - n^2 \left( \frac{f_p}{f} \right)^2 \right)^2 + \left(2 n \zeta \frac{f_p}{f} \right)^2}}$$  \hspace{1cm} (2.40)

where $\alpha_n$ is the $n^{th}$ Fourier coefficient of loading. In a verification of this method, Ellis and Ji (1994) found good agreement with experimental and numerical results. This solution is also recommended in BRE 426 (1997) and corresponds with ISO10137 (1992) and annex A of BS6399 (1996).

Note that the above methods necessitate the prediction of floor dynamic characteristics, which can be inaccurate. Murray (1975) observed that the calculated floor response is highly sensitive to the predicted natural frequency and to the amount of the floor which participates in response.

### 2.7.2 Design guidelines

Design methods have been developed to combine all relevant parameters of floor vibration, such as human perception, natural frequency, and dynamic loading, to limit floor vibration responses. This section presents four such guidelines.

**CSA (1980) Appendix G**

The CSA guideline for composite floors requires calculation of fundamental frequency by the EBM (equation 2.14), and estimation of the peak acceleration due to a heeldrop, using equation 2.38. The acceptability of the floor is then checked using the perceptibility diagram of figure 2.4. The drawbacks of this method include the use of heeldrop to predict walking vibration annoyance, and the estimations of parameters that can be inaccurate. In assessing its reliability for concrete floors, Williams and Waldron (1994) found very poor agreement. Hence, as it stands, this approach is unsuitable for concrete floors, unless the estimation of parameters is clarified for use with such floors.
SCI (1989) Guide

The SCI guide is based on the actual calculation of human response by introducing a new parameter called the response factor. Firstly, floors are separated into two categories: low frequency $\leq 7$Hz $\leq$ high frequency. Then, the response factors of the floors are estimated. For low frequency floors, resonance with the walking force ($\approx 2$Hz) is considered, whereas for high frequency floors, the response factor is based on the impulse response to a heel-impact, assuming no resonance effects. Empirical formulae are given to calculate the response factors that should not exceed tabulated limits. These limits account for three cases of general, special and busy office environments.

Concrete Society (1994) handbook

The Concrete Society present an elaborate method for calculating a similar response factor to the SCI (1989) guide. For concrete floors, the calculation of response factor, $R$, follows directly from the prediction of the natural frequencies by their method (see section 2.5.3). Firstly, two response factor coefficients, $N_x$ and $C_x$, are calculated. These equations apply to solid or waffle slabs only.

$$N_x = 1 + (0.5 + 0.1 \ln \zeta) \lambda_x$$  \hspace{1cm} (2.41)

$$C_x = \begin{cases} \frac{224.8}{(f_x \zeta)} & \text{if } f_x \leq 3 \text{Hz} \\ \frac{27.2}{\zeta} & \text{if } 3 \text{Hz} \leq f_x \leq 4 \text{Hz} \\ \frac{(83.2 - 14f_x)}{\zeta} & \text{if } 4 \text{Hz} \leq f_x \leq 5 \text{Hz} \\ \frac{0.88(20 - f_x)}{\zeta} + 2(f_x - 5) & \text{if } 5 \text{Hz} \leq f_x \leq 20 \text{Hz} \\ 30 & \text{if } 20 \text{Hz} \leq f_x \end{cases}$$  \hspace{1cm} (2.42)

The response factor in the $x$-direction is then given by:

$$R_x = \frac{C_x N_x}{W n_x n_y l_x l_y}$$  \hspace{1cm} (2.43)

Application of equations 2.41 to 2.43 is repeated for the slab spanning in the $y$-direction, simply replacing all the $x$ subscripts by $y$; the overall response factor is then $R = R_x + R_y$. For an acceptable floor, $R$ must not exceed 4 in a sensitive environment, 8 in a general environment and 12 in a busy environment. Williams and Waldron (1994) found generally good agreement of this method with their experimental results, although in some cases, the calculated response factor was excessively conservative.
AISC (1997) design guide

This is the most recent design guide for composite floors. The approach it recommends was developed by Allen and Murray (1993) and is based on the ISO2631/2 (1989) and BS6472 (1984) acceptability scales (figure 2.5). The excitation force is taken as the first harmonic of pedestrian activity (equation 2.2), and the fundamental frequency of the composite floor is derived by the static deflection method (section 2.5.3), where Dunkerly’s principle is used to account for the deflections of the beam and the girder (equation 2.15). The guideline then differentiates between walking and rhythmic excitations. For walking excitation, the response is given in terms of induced acceleration:

\[
A_{\text{rms}} = \frac{gP_0e^{-0.35f_0}}{\sqrt{2}\zeta W_e}
\]  

(2.44)

where \(P_0\) is a constant force representing the excitation (given as 0.29kN for floors) and \(W_e\) is the effective weight of the floor supported by the beam, girder, or combined panel (calculated from the effective width). The value of \(A_{\text{rms}}\) is then checked and should be less than the relevant ISO/BSI/ANSI curve (figure 2.5), or the limits of 0.5%g for offices or 1.5%g for shopping malls. Values of damping are given as \(\zeta=2-5%\) for offices and 2% for shopping malls. Furthermore, the guide suggests that for fundamental frequencies greater than 9Hz, the minimum stiffness criterion of 1kN/mm should be used in addition to the walking excitation criterion. Murray (1999) comments that in using this guideline, careful estimates of the actual live load on the floor during normal occupancy, and of the damping ratio, are required for a successful design.

The AISC criterion of equation 2.44 is a modified version of the one developed by Allen and Murray (1993). They started by defining an acceleration limit and used it to determine a minimum acceptable floor fundamental frequency:

\[
f_0 \geq 2.86 \ln \left[ \frac{K}{\zeta W_i} \right]
\]

(2.45)

Here, \(W_i\) is the total weight supported by one beam and \(K\) is a force constant. Comparing with equation 2.44, \(K = \frac{gP_0}{\sqrt{2}A_{\text{rms}}}\), so that it accounts for both the exciting force and the acceleration limit. \(K\) is specified as 58kN for offices and 20kN for shopping malls. This relaxation for more active occupancies is similar to increasing the acceleration limit by a factor of three, as recommended by Pernica and Allen (1982), see section 2.2.2. In comparison with other criteria, generally good agreement in the frequency range of around 5-8Hz was found, beyond which the CSA and SCI methods become more conservative.
For rhythmic activities, AISC (1997) assumes the structure to have only one mode of vibration and derives the peak acceleration of the floor as:

\[
A_{rms} = \frac{J \alpha_1 \frac{w_p}{w_h}}{\sqrt{2} \left[ \left( \frac{f_j}{f} \right)^2 - 1 \right]^2 + \left( \frac{2f_j}{f} \right)^2}
\]

(2.46)

where \( \alpha_1 \) is the dynamic coefficient of the forcing function, \( f \) is the forcing frequency, \( w_p \) is the effective weight of participants per unit area, \( w_h \) is the effective weight of floor and occupants per unit area, and \( J \) is a constant depending on activity (taken as 1.3 for dancing, 1.7 for sport events, and 2.0 for aerobics). The resulting acceleration should again be smaller than the relevant ISO/BSI/ANSI curve or the limits of 0.4-0.7\%g for offices and 1.5-2.5\%g for dining halls.

### 2.8 Remedial measures for problematic floors

While research is being conducted to introduce design codes for serviceability vibrations, floors are still being built, which have vibration problems. Remedial measures are needed for controlling undesirable vibrations in existing structures. These generally fall into four categories.

**Frequency Tuning**

Here, the fundamental frequency of the structure is measured and the structure is stiffened in such a way that the possibility of resonance is eliminated. For a satisfactory design, the ratio between the floor fundamental frequency and the disturbing frequency should be outside the range 0.5 to 1.5 [Fowler and Karabinis (1979)]. It is highly recommended that this ratio should be on the high side of the range so that the floor responds to the applied dynamic force as though it was a static force. The stiffening process can range from adding extra columns to beam stiffening. Guidelines have also been presented giving limits on the minimum value of natural frequency for satisfactory vibration design of floors, table 2.5. For synchronised dance-type loads, BS6399 (1996) recommends a minimum fundamental frequency of 8.4Hz for vertical vibrations. Note that other important factors, such as floor span-depth ratios and damping, are not considered in these criteria.
Table 2.5: Bachmann (1992a) recommended natural frequencies of structures

<table>
<thead>
<tr>
<th>Structure type</th>
<th>Construction type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>reinforced concrete</td>
</tr>
<tr>
<td>gymnasiums and sports halls</td>
<td>&gt; 7.5</td>
</tr>
<tr>
<td>concert halls, theatres, and spectator galleries with fixed seating.</td>
<td>&gt; 3.4</td>
</tr>
<tr>
<td>Classical or soft pop music concerts</td>
<td></td>
</tr>
<tr>
<td>concert halls, theatres, and spectator galleries with fixed seating.</td>
<td>&gt; 6.5</td>
</tr>
<tr>
<td>Hard pop music concerts</td>
<td></td>
</tr>
<tr>
<td>dance halls and concert halls without fixed seating</td>
<td>&gt; 6.5</td>
</tr>
<tr>
<td>note: for footbridges avoid 1.6-2.4Hz (with low damping also 3.5-4.5Hz)</td>
<td></td>
</tr>
</tbody>
</table>

Limiting vibration amplitudes

Here, the vibration amplitudes are measured and the structures are upgraded by adding extra stiffness, such as columns, so as to ensure that certain allowable values are not exceeded. In comparison to frequency tuning, this method is theoretically more exact as it also requires the calculation of a forced vibration response and the damping properties of the structure. This would lead to uncertainties as there are no accurate methods of predicting various structural dynamic parameters.

Increasing damping

There are various methods of increasing the damping ratio of a structure. In cases where existing floors have vibrational problems, the addition of partitions either above or below the floor tends to increase damping to as high as 14% of critical [Allen (1974)]. For a planned structure, the use of non-composite construction tends to increase damping by 1-2% over composite construction. The use of damping posts, viscous dampers, TMDs, and active vibration absorbers is also possible.

Isolation of source

This form of vibration treatment is mainly concerned with steady-state vibrations caused by heavy machinery on a floor. There are various methods of isolating vibrating machinery, including heavy base blocks, rubber-in-shear mountings, air springs, and chemical grouting. It must however be noted that even where isolators are used, resonance between the disturbing force and the supporting structure should be avoided. For a more economical solution, the resonance effect could be avoided by a small alteration in the running speed of the machine [Steffens (1966)].
2.9 Recent developments

Finite element modelling of slab vibrations

The finite element method (FEM) of analysis was first introduced in the 1960s and its use has become widespread with the advent of faster and more efficient computer technology [Zienkiewicz (1992)]. The main feature of FEM is that it allows the division of a structure into many smaller components (elements) of various geometries which, when assembled together, can represent the whole structure. The assembled elements must not only reflect the shape of the structure but also ensure the correct flow of stress from the loaded zones to the reaction zones, hence the position of the nodes and the manner of their connection become very important [Carlton (1993)]. The steps involved in a typical finite element analysis are:

i) conversion of actual structure into an idealised structure
ii) conversion of the idealised structure into a geometrical model
iii) interpretation of the geometrical model as a meshed model of elements
iv) conversion of the meshed model into a mathematical model
v) running of the mathematical model to create a numerical solution
vi) using the numerical solution to perform a results interpretation
vii) characterisation of the behaviour of the real structure and the derivation of changes to the desired structural parameters, subject to the given loading conditions

The FEM can be used to model complex geometries easily and can solve problems which previously could be tackled only by the use of physical models or by prototype construction. It also allows a quick and inexpensive opportunity to evaluate several alternative solutions to a problem. In general, the use of the FEM can be categorised into four distinct applications: (1) prediction of complete structural behaviour, (2) verification of analytical methods, (3) comparison with experimental observation, and (4) in combination with theory, experiment and analytical results.

Reports of finite element investigations into the dynamics of floor slabs are few in number [Ellis and Ji (1996), Pavić et al. (1994), Pavić and Waldron (1996b), Ji and Ellis (1995), Zaman and Boswell (1996)]. Of these studies, most include a full experimental investigation coupled with a finite element model. The main drawback for the method is the level of accuracy of the final solution, which depends on the representation of the structure in terms of its boundary conditions,
material properties, geometry, and modelling of the excitation processes. If these factors in a model closely represent the actual conditions, then FEM can be the most accurate method of estimating structural response. The uncertainties involved in the representation of a structure by finite elements, and the moves to tackle the problem, are discussed briefly below.

With today's finite element systems and cost effective computing, the models being analysed are becoming increasingly complex with varying degrees of accuracy. However, until recently there were no means of coordinating the knowledge to achieve improved modelling of structures. To this end, the National Agency for Finite Element Methods and Standards (NAFEMS) was formed in 1983, which has developed a series of benchmarks for improved accuracy. The NAFEMS guidelines point out that an FEM is only as good as: (i) the model of the structure (mesh and elements), (ii) assumptions embedded in the properties used for each element, and (iii) the representation of the external loads and constraints in terms of the discrete boundary variables. For concrete or composite structures, the accuracy of an FE analysis can be influenced by: the type of element used (solid, shell, plate), concrete and profiled decking material properties (Young's modulus and material strength), discontinuities within the structure (cracking, construction joints, support continuity), discrepancies between the design and actual values for slab thickness and concrete density, boundary conditions (fixed, pinned), and soil and foundation properties.

A major advantage of FEM is that it allows a structure to be modelled completely, hence giving a full set of solutions for a given loading function. For vertical floor vibration analysis, it would be wasteful in time and effort to model a whole structure when only the behaviour of a single component (e.g. a floor) is of interest. Hence, attempts have been made at modelling the floor by itself and introducing boundary conditions to represent the connections to columns, beams, etc. This technique has proved useful in some cases giving close estimates of dynamic behaviour.

Pavic and Waldron (1996b) and Reynolds et al. (1999) have investigated floor dynamics using FE model updating. This is a technique developed in the context of mechanical engineering, in which the output from an FE analysis is systematically correlated with test data and the system parameters tuned to optimise the agreement. They found that a pre-test FE model did not give a very close prediction of the actual floor behaviour, but when the model parameters were updated from experimental results, a very close estimate of floor response was possible. They concluded that the improvements to a dynamic FE model of a floor slab could be achieved by: (i) explicit modelling of columns (rather than the common practice of idealising columns as pin supports), (ii) explicit modelling of slab ribs (rather than approximation of the ribbed slab as an orthotropic structure), (iii) modelling deep beams as shell elements, and (iv) improvements on geometry, boundary condi-
Hence, it can be concluded that FE modelling can be a very useful tool in the evaluation of floor dynamic behaviour, given correct idealisation of the structure. The experience gained in modelling one structure accurately will be invaluable in improving the accuracy of FE models of other floors. To this end, experimental data from dynamic testing are required to enable correlation with FE models. Ji and Ellis (1995) suggest that a combined theoretical and experimental study on the structure is important because an experimental study provides accurate but incomplete information (depending on variables measured), while a theoretical study supplies complete but potentially inaccurate results, which can be improved. The use of finite element modelling will become even more important in the future with increasing use of computer models in design [Ellis and Ji (1996)].

Active control approach for reducing floor vibrations

Active control technology is used in many disciplines to improve the response of dynamic systems. For instance, its application in the field of earthquake engineering is currently attracting major interest. Its usefulness in reducing floor vibrations has also been studied recently. Hanagan and Murray (1997) reported a successful implementation of such a system on a problematic floor slab.

In their control scheme, the movements of an experimental floor were measured and utilised in a feedback configuration to supply a control force that resulted in a reduction of vibration levels. The control force was supplied by an electromagnetic actuator mounted on the floor. Their experimental tests agreed well with the analytical studies and showed that for heeldrop excitation the floor damping increased from 2.2% to 9.7%. The damping became as high as 40% with less severe impulse forces, which were within the most useful range of the controller. In addition, walking excitation tests showed a reduction of peak velocity amplitudes to 12% of those recorded for the uncontrolled system.

While the experimental results are encouraging, the system has potential drawbacks, such as its disruptiveness to building functions, the complexity of its design, its control when in function, its high cost, and maintenance/reliability issues. Such systems are in their early stages of development and there is currently no record of their utilisation on real floors.
Chapter 3

Design and construction of experiment

3.1 Introduction

The objectives and nature of the research are stated in section 1.3. The purpose of this chapter is to describe the preparation phase of the laboratory tests. First, the design and building stages of the test apparatus are described. Casting of the model floor slab and the various non-structural additions are then explained. The calibration procedures and accuracy of the apparatus are checked in section 3.7. Discussion of the test procedures and data collection methods is given in Chapter 4.

3.2 Outline of tests

The overall layout of the experimental apparatus, used in the modal testing of the model floor, is shown schematically in figure 3.1. The sources of loading were in four forms: impulse hammer, electromagnetic shaker, a person performing heelpad loading, and either walking horizontally or on the spot. For the hammer and shaker tests, the actual applied load was measured and recorded. The tests with human activities were attempted to be as representative to real-life loading as possible. The actual experimental procedures for each loading type are explained in detail in Chapter 4.

Figure 3.1: Layout of modal testing experiments on model post-tensioned slab

The slab response to each of the imposed dynamic loads was measured using a piezoelectric ac-
celerometer, which was placed in a closely spaced grid on the slab. The output of the accelerometer was amplified and recorded in a spectrum analyser, where data analysis was performed.

The model slab was designed as a 5.5×1×0.125m post-tensioned concrete slab with design concrete strength of 40N/mm², using Ordinary Portland Cement. The design was in such a way that the slab would represent a simple idealised one-way spanning floor, which could easily be altered and non-structural components easily fixed. Sufficient bonded steel was included to enable testing of the slab without any tensioning of the tendons. The tendons were then tensioned to 57% of their design stress, at which stage more tests were carried out, and finally to 100% of the design force. Once the model floor was post-tensioned to its fall design stress, loading and response tests were carried out on the following slab stages: bare slab, slab with dead load, with partitions, with false floor, with standing human, with TMD, and with viscoelastic screed. Various combinations of these components were also investigated.

At regular intervals during the experimental investigation, cube and cylinder tests were carried out to determine the material properties of the concrete. In addition, 1500×200×150mm beams were cast and tested for the original slab concrete and the two viscoelastic screed layers.

3.3 Description and design of apparatus

The experimental apparatus consisted of two main elements: the loading equipment and the measurement instrumentation. Some items, for example the impulse hammer, are standard testing equipment developed previously by specialist firms. For other items, e.g. the shaker, specific design and modifications were necessary in order to adapt them to the requirements of the research programme. Details of individual testing apparatus and any necessary modifications form the bulk of the following sections.

3.3.1 Instrumented impact hammer

This instrument represents a relatively simple way of exciting the structure into vibration. The equipment consists of a 5.4kg impact hammer [Dytran Instruments Inc. (1993)] with a set of different tips for the head, which are used to vary the frequency and force level ranges. Integral with the impact head, there is a quartz piezoelectric force transducer, which detects the magnitude of the input force felt by the hammer head, assumed to be equal and opposite to that experienced by the structure. The hammer testing kit also includes a charge amplifier/power supply unit that
outputs the force signal at a level of 1V per 1000lbf (4448N). The magnitude of the impact force is determined by the mass of the hammer head and the velocity with which it is moving when it hits the structure. The frequency range of excitation is controlled by the stiffness and mass of the impact head. The stiffer the material, the shorter will be the duration of the pulse and the higher will be the frequency range covered by the impact. Figure 3.2 shows the hammer excitation equipment and figure 3.3(a) shows the frequency and magnitude ranges covered by each tip. With regards to concrete floor slabs, which exhibit low natural frequency, it is good practice to use a soft tip to input an impact of a shorter frequency range with more energy into the lower frequencies. For this research programme, different hammer tips were experimented and the brown tip made of polyurethane plastic was seen to be most suitable and was used throughout the testing programme. Figure 3.3(b) shows a typical hammer signal using the brown tip. The hammer testing procedure is described in chapter 4 and shown schematically in figure 4.4 on page 75.

![Instrumented impact hammer testing kit](image)

Figure 3.2: Instrumented impact hammer testing kit

![Frequency range and magnitude of impulse hammer tips and a typical hammer signal using the brown polyurethane tip](image)

Figure 3.3: Frequency range and magnitude of impulse hammer tips and a typical hammer signal using the brown polyurethane tip
It is important to recognise that the impact hammer is only in contact with the structure for a short period while the excitation is being applied. This type of instrument is categorised as a non-contacting vibration source and imposes transient vibration on the structure. The contacting type of excitation is the shaker, which is described below.

3.3.2 Electromagnetic shaker

This type of excitation source is more bulky and is connected and remains attached to the structure throughout the test at all frequencies and amplitudes of excitation. It is categorised as a contacting source and imposes a continuous oscillation on the structure.

The electromagnetic shaker consists of a coil inside an electromagnetic field, whose input signal is supplied by an external signal generator. The coil in turn is connected to the drive part of the device, which is attached to the structure. In this case, the frequency and amplitude of the excitation are controlled independently of each other, giving more operational flexibility as compared to the impact hammer. It is hence possible to excite the floor at a particular frequency and obtain a more accurate response measurement at that frequency. As such, the results derived from a floor excited by a shaker have distinct advantages over those derived using an impact hammer [Falati (1996)].

The shaker used for this research programme was a Ling Dynamic Systems Vibrator model 410 [Ling Dynamic Systems Ltd. (1972)], which is capable of applying 178N through its drive head when force cooled by air. A 300W output valve power amplifier was used to amplify the signal from the signal generator on route to the shaker coil. A special frame was designed and built in order to carry the shaker and modify it for use in the research programme. This was a major part of the apparatus design and is explained in some detail below.

Design of shaker frame

The shaker, as supplied, was designed for horizontal applications or to be used in cases where the drive head was required to act upwards. Hence, a frame was designed and built in order to contain the shaker such that the drive head would be facing downwards and would be in contact with the floor slab continuously. As the shaker had a mass of approximately 30kg, a substantial frame was required to hold it in an inverted position and also to resist any movement upwards while it was operating. Figure 3.4 shows the designed frame with the shaker installed and individual components marked. A brief description of these is given below.
**40mm Square Hollow Sections:** Fillet welded together to construct the main frame.

**500×76×38mm Channel Section:** spanning the top of the frame to accommodate the brass bearing holding the shaker.

**26mm diameter Brass Bush Bearing:** consisting of a brass block which was machined to have a bearing of 26mm through which the rod holding the shaker would move.

**25mm diameter Drive Rod:** welded at bottom to a 13mm thick steel plate holding the shaker. The surface of the rod was constantly smoothed and lubricated for ease of movement through the bearing during shaker operation.

**28.5mm diameter Compression Spring:** resisting the weight of the shaker. The spring properties were: load capacity=1023N, spring constant=24.87N/mm, inside hole diameter=28.5mm, and free length=88.9mm. The height of the shaker and hence the compression of the spring could be altered by way of a nut at top of the drive rod.

**13mm thick Steel Plate:** welded at the top to the moving rod and at the bottom bolted to the shaker using fatigue resistant bolts.

**1000N maximum capacity Load Cell:** used to measure the force output of shaker drive head onto the floor (see section 3.3.4).

**25mm diameter Ball-Bearing:** joining the load cell to the floor via a steel base. Used for reducing error effects and preventing damage to the shaker due to lateral loading.
In operation, an initial pre-compression of 50N was applied to ensure the ball-bearing was fully in contact with the floor. The shaker would then impose a continuous sinusoidal loading onto the floor and its subsequent upward motion would be resisted by the compression spring. The shaker frame, with the components installed, is shown in figure 3.5 and its experimental setup is explained in chapter 4. The test procedure is also shown schematically in figure 4.4 on page 75.

3.3.3 Piezoelectric accelerometers

The accelerometers used were model 4370 manufactured by Brüel & Kjær [Brüel & Kjær (1982)]. A B&K charge amplifier type 2635 was used and the acceleration signals were amplified to 1V per g. These signals were then fed into the spectrum analyser for initial frequency domain analysis. Piezoelectric accelerometers work on the basis that the piezoelectric material inside, usually quartz, generates a small electric charge in response to applied accelerations. Once amplified and calibrated, this signal can be used as a measure of acceleration response of the structure. Due to the sensitive nature of the accelerometers, the measured data was constantly monitored and the accelerometer sensitivities altered accordingly. Figure 3.6 shows the B&K accelerometers used in testing.
3.3.4 Load cells

The load cell used for the shaker test was an external Wheatstone Bridge type 1000N ultimate capacity transducer manufactured by Schlumberger. It was placed between the shaker drive head and the floor and would measure the force exerted onto the slab. This load cell was powered by an RDP instruments E307 'black box' power supply. The electrical output of the transducer was amplified by the 'black box' and then fed to the spectrum analyser for time and frequency domain analyses. The load cell calibration procedure is described in section 3.7.2.

3.3.5 Spectrum analyser

The spectrum analyser used in the Oxford University civil engineering group is an Advantest model R9211C [Advantest Corp. (1989)]. This is a very powerful signal processor with signal generating capabilities. Both these parts of the analyser were utilised in the experimental programme.

The signal generating function was used to provide the sinusoidal wave forms which, after amplification, would be fed to the electromagnetic shaker and ultimately control the movement of the shaker drive head. The frequency, amplitude and type of the input signal could be controlled and altered independently. The signal processing capabilities of the spectrum analyser were widely used in the initial data analysis. As shown in figure 3.7, the equipment has two input channels. Channel A was connected to the output of the load cell, either from the hammer or shaker, and would provide the force information for the eventual transfer function. Channel B was connected to the acceleration transducer on the slab and would read the amplified acceleration response to the imposed force. Digital signal processing was carried out by the analyser and the results were
obtained in the form of a frequency response function (FRF) describing the relative magnitudes of the range of frequencies present in the signals. Also at this stage, the coherence function was checked and the accuracy of the experiment evaluated. The underlying theory of signal processing is presented in section 4.2.

As the analyser was not equipped with an IEEE488/RS332 interface circuit, it was not possible to transfer the data directly to a computer for storage and further analysis. Hence, the data saving facility of the analyser was also very widely used, where the information was saved on floppy disks and subsequently transferred into a PC. Furthermore, as the analyser did not save the information in a *DOS* or *Windows* compatible format, a substantial amount of time was spent in converting the saved data from the analyser format to a format recognisable by a PC, using a translation program written in *QBASIC*.

### 3.4 Design of the model floor slab

#### 3.4.1 Objectives of the design

Since the main purpose of the research programme was to investigate vibration problems of post-tensioned floor slabs due to resonance with human activity, it was essential that the test floor's fundamental natural frequency fell within one or more of the harmonics of man-induced excitation. For an effective experimental programme, the model slab must therefore possess a fundamental natural frequency in the range 2-9Hz. This became the first priority of the model slab design.
Only the most slender post-tensioned floors are likely to experience vibrational serviceability problems. Current construction practice is to use post-tensioning for spans in the range 7-20m, where solid slabs are used in the lower end of the range and waffle slabs used for longer floors. For solid slabs, thicknesses are usually 200-300mm, resulting in span-depth ratios between 30 and 45. Average prestress levels are typically 3-5MPa. All of these facts were taken into account to produce a realistic 50% scaled model post-tensioned floor.

Other priorities were: the concrete strength to represent real cases, post-tensioning tendons used, and the overall weight of the slab, which had to be below the safe working load of the laboratory lifting equipment.

3.4.2 Dimensions and limitations

In arriving at a suitable size for the experimental slab, the space available in the laboratory became the most important factor. The dimensions of the final design were 5.5m long by 1m wide and 125mm deep, giving an initial span-depth ratio of 44. Ideally, this gave a calculated fundamental frequency of 5.6Hz (using equation 2.8 on page 23), which was the lowest possible under the laboratory space limitations. After construction of the slab and its placement on the supports, the length between the supports became 5.1m with an average thickness of 135mm, giving a span-depth ratio of 38 and a fundamental frequency of 7.96Hz (see appendix D). This would mean that a condition of resonance could be achieved with the third harmonic of walking frequency. It would have been desirable to have had a longer and wider slab, exhibiting two-way spanning behaviour and lower fundamental frequency, but the laboratory conditions did not allow this.

3.4.3 Design of slab as a reinforced beam

After deciding the dimensions, the slab was designed in a reinforced and post-tensioned state. The design of the model floor as a reinforced beam was to ensure enough bonded steel was included in the concrete so that it would withstand its own self-weight without any tensioning of the pre-stressing tendons. The design followed the usual British Standards [BS8110 (1985)] specifications with the following design values and reinforcement bar sizes: \( f_{cu} = 40 \text{N/mm}^2 \), cover=20mm, \( f_y = 460 \text{N/mm}^2 \) and 10mm diameter reinforcement bars. The design dead load was 4.5kN/m\(^2\) and the design live load was 3kN/m\(^2\) to account for people standing on the slab.
3.4.4 Design of slab as a post-tensioned floor

The floor was designed as a flat slab according to BS8110 and with the guidelines specified in Concrete Society (1994). Prestress was provided by four 15.7mm diameter PSC Freyssinet unbonded super-strands, with a characteristic breaking load of 265kN and characteristic strength of 1770N/mm². The sheathing used for these tendons was of plastic type. The design eccentricity was 30mm.

The slab was designed as a Class 1 member with maximum tensile stress under service conditions $f_t=0$N/mm², and maximum service compressive stress $f_{cc}=15$N/mm². When fully stressed, each tendon provided a prestressing force of 175kN, giving an average prestress in the slab of 5.2MPa, which was high but within the acceptable range. Figure 3.8 shows the final reinforcement cage with the post-tensioning tendons included. Figure 3.9 shows photographs of the tendon anchors.

![Figure 3.8: Reinforcement cage of model experimental slab showing the bonded steel and unbonded tendons](image1)

![Figure 3.9: Details of live tendon anchors (left) and dead anchors (right)](image2)
3.4.5 Design of slab supports

In order to achieve as low a fundamental frequency as possible, simple supports were used at either end. These were in the form of 60mm diameter circular rods, which were welded to the top flanges of two 400mm deep steel girders. The height of the supports was deliberately large to allow access to the bottom of the slab for fixing partitions, as explained in section 3.6.3. The slab at either end would hence rest on the circular rods with a minimum point of contact, so that the rod acts as a line support. Figure 3.10 shows one of the designed simple supports with the slab resting on it via 3mm thick hard rubber sheets to eliminate uneven contact. The inclusion of the rubber did not affect the dynamic characteristics of the slab.

![Figure 3.10: Simple supports used for experimental slab](image)

3.5 Construction of the model floor slab

The dimensions of the experimental slab meant that 700 litres of concrete (0.7m³) were required for its casting. In addition, small scale test samples had to be taken. The following sections describe the processes involved before and during the casting of the slab and other concrete samples.

3.5.1 Concrete mix design

The design concrete strength was 40N/mm² with a permissible standard deviation of 3N/mm². The density was taken to be 2400kg/m³ and the concrete was designed for a slump test of 50mm. The
mix quantities per cubic metre were: coarse aggregate 955kg, fine aggregate 750kg, cement (OPC) 432kg, and water 205kg. Small scale cube and cylinder samples of the mix were taken to check the design specifications against the eventual concrete properties.

3.5.2 Concrete casting process

Since the laboratory concrete mixer had a maximum capacity of 50l, needing 14 mixes, special priority was given to the speed of casting of each mix in order to avoid cold joints in the slab. The casting process involved 10 volunteers, three of whom were casting and vibrating the wet concrete, two were mixing, one taking small scale samples, two weighing the mix constituents, and the remaining two responsible for the transport of the wet concrete from the mixer, on ground floor, to the formwork, on first floor. Figure 3.11 shows the final result after completion.

![Figure 3.11: Model floor slab after curing](image)

3.5.3 Small scale test samples

In total, 15 cubes of the mix were taken and two 300mm × 150mm diameter cylinder samples were also cast. A further 10 cubes and 3 cylinders were cast in the latter stages of the experiment, which included the viscoelastic screed mixture (see section 3.6.7).

In addition, four small beams of 1500 × 200 × 150mm were cast during the whole of the experimental procedure. These consisted of two beams representing the concrete mixture used in the experimental slab, and the other two representing the concrete mixture used for the viscoelastic screed layers. The beams were designed only to support their own self-weight and although they were doubly reinforced, the compression reinforcement was kept to a minimum and was only included to ensure safety during their movement.
3.6 Additions to the floor during testing

The following sections describe the various simulated non-structural components, which were added to the bare concrete, and the way each one was connected to the slab for testing.

3.6.1 Dead load on floor

The purpose of this addition was to simulate office furniture, which would be stationary on the floor, such as desks, cabinets, etc. Five 25kg Lead weights were used, which were placed at one meter spacings giving a notional dead-load of approximately 250N/m². The deflection of the slab due to this load was also measured.

3.6.2 Cantilever partitions

To simulate cantilever partitions, typical of those used to divide open-plan office space, three layouts were used. Firstly, five 9mm thick by 1.2m high gypsum plaster sheets were fixed perpendicular to the slab span at one meter spacings. Secondly, three sheets were installed parallel to the slab span, and thirdly, all eight were attached on the slab in both directions. 25mm equal sided steel angle sections were used to connect the cantilever sheets to the slab. The connection was made rigid by screwing angles on each side of the plaster sheets and bolting them together, and to the slab, while sandwiching the plaster in-between. Figure 3.12 shows an overview of the partition arrangement and figure 3.13 shows the slab with the first layout attached.

![Diagram of cantilever partitions]

Figure 3.12: Layout of cantilever partitions on model slab
3.6.3 Full-height partitions

As for cantilever partitions, 9mm thick Gypsum plasterboard was used for the full-height partitions with the difference that both ends of the partitions were rigidly fixed, one to the bottom of the slab and the other to the ground. The fixing to the slab was made possible by the use of cast-in sockets, on the underside of the floor, and 25mm square steel angle sections. The bottom of these partitions were rigidly fixed to the ground using angle sections, which were glued on one side to the ground and bolted to the plasterboard on the other. Figure 3.14 shows the slab with full-height partitions attached.
3.6.4 False flooring system

The false flooring system used for the experiments consisted of 150mm adjustable height pedestals with adhesive for fixing to the floor, and a number of basic 600mm square false flooring panels, with a mass of 11.5kg each. These were fixed to the bare slab in two separate layouts. Layout 1, shown in figure 3.15, consisted of one row of 7 panels, covering around 50% of the entire floor, which were rigidly fixed to the pedestals on all four corners using special screws. This is a common mode of installation of floor panels. In layout 2, two rows of 7 panels were used, covering approximately the entire floor (see figure 3.16). Panels were screwed down at all interior corners but at the outer edge of the group, they simply rested on the pedestals. This gave some scope for relative movement between panel and pedestal during vibration testing. This detail is commonly used for panels around the edge of a floor or for heavy timber panels.

Figure 3.15: False floor panels layout 1

Figure 3.16: False floor panels layout 2
3.6.5 Tuned mass damper (TMD) system

To produce a TMD system for the experimental slab, cast-in sockets in the bottom of the slab were utilised to which 25mm square angles were fixed. A 1200mm long sheet of plywood was then placed in the space between the angles and rested on either side on the horizontal angle face. The TMD tuning effect was achieved by placing different weights on top of the plywood sheets. This experiment was performed with two different widths of plywood and also with single and double layers of plywood separated by polythene and rubber, as described in chapter 7. Figure 3.17 shows the slab with the TMD system attached.

![Image of TMD on slab]

Figure 3.17: Application of TMD on slab

3.6.6 Standing human

Here, the effect of human-structure interaction was investigated, as described in chapter 8. A person of mass 75kg stood still on the bare slab during these tests. The position of the standing human was as near to the middle of the slab as possible. Figure 3.18 shows a test in progress with the human on the slab. Heeldrop, walking on-spot, and horizontal walking tests were also performed on the slab, as well as tests with two standing men.

3.6.7 Viscoelastic screed layers

This addition to the experimental slab was the only one which was irreversible and hence could be regarded as structural, rather than non-structural. It consisted of a 25mm layer of concrete, treated with a viscoelastic material, being added on top of the slab. Two layers of such additions were applied using different concentrations of viscoelastic additive. The viscoelastic material used was
Figure 3.18: Experiments with one stationary standing man

donated by Concredamp Inc. (1993). This a patented additive and details of its constituents are not publicly available. The following two sections describe the preparation of the layers of concrete treated with Concredamp.

Concredamp Mix Design

The concrete mix design used for the Concredamp layers was exactly the same as the one used for the main slab with the difference that the amount of water in the mix was reduced by 50% of the amount of Concredamp added. This was according to the specifications supplied by Concredamp Inc. The two layers were cast at an interval of approximately one month and the concentrations of Concredamp used were 50l/m³ and 85l/m³ respectively.

Concredamp casting process

This was similar to the casting process of the main slab concrete, explained in section 3.5.2. The first attempt at layer 1 was abandoned after a period of ten days, when it was observed that the new layer had not achieved a good bond with the main concrete slab. This layer was subsequently removed and at the second attempt a better adhesion between the layer and the slab was achieved. In each case, it was noticed that the Concredamp additive had very adhesive behaviour (rather like paint) and had the effect of reducing the workability of the concrete mixture very quickly, making
the speed of casting more critical. Figure 3.19 shows the slab with two layers of screed added. In each case, screws were used as studs on the main slab to act as shear connectors and aid bonding.

![Slab with both layers of viscoelastic screed added](image)

Figure 3.19: Slab with both layers of viscoelastic screed added

### 3.6.8 Some combinations

With the viscoelastic screed in place, it was possible to repeat the experiments with some combinations of slab additions. For example, tests were repeated with the screed layers and cantilever partitions or false floors and with standing human.

### 3.7 Accuracy of measurement systems

During the course of the experiments, it was necessary to check the reliability of the transducers involved, in order to ensure the accuracy of the measured data. The following sections give a brief description of the calibration and checking of the transducers.

#### 3.7.1 Accelerometers

Each accelerometer was supplied with a detailed calibration certificate, which was used in the initial calibration of the experimental results. However, a simple way of checking the accelerometers was to put them alongside each other and to plot a transfer function of their respective outputs. A straight horizontal transfer function would mean that both accelerometers give the same output. This test was repeated at regular intervals during the testing program. Typical transfer function and phase plots of such a test are shown in figure 3.20, which were deemed to be acceptable.
3.7.2 Load cells

Of the two load cells used, the one supplied integral to the hammer head had a factory calibration certificate, which was used throughout the experimental programme. The Wheatstone bridge load cell, however, had to be calibrated. This process involved putting known weights on top of the load cell and reading its output as a voltage. The applied load was increased in equal and unequal increments and the voltage plotted. The plot, as illustrated in figure 3.21, shows a linear increase in output voltage with added weight and hence this was used as the calibration plot for the load cell. This process was repeated in the middle and latter stages of the experimental programme with a similar calibration graph in each case.
3.7.3 Shaker loading

Here, it was desirable to see whether the sine-sweep loading, applied by the shaker onto the slab, closely simulated the sine-sweep signal produced by the signal generator. A transfer function was plotted between the signal processor wave-form and the output of the load cell attached to the drive head of the shaker. A straight horizontal line would mean a perfect representation of the wave by the shaker, whereas any deviations would signal an error at that particular frequency. Figure 3.22(c) shows a typical transfer function, which is satisfactory for frequencies above 6Hz and unsatisfactory for all lower frequencies. This was deemed acceptable for the purposes of this experiment, as the frequencies of interest were above 7Hz. Any noise in the signal was attributed to the loadcell readings and speed of data capture, neither of which were seen to affect the final results. This checking process was repeated throughout the course of the testing programme.

Figure 3.22: Accuracy of shaker loading compared with original signal
Chapter 4

Modal analysis technique and experimental procedure

4.1 Introduction

In this chapter, the general theory and techniques used in the modal analysis of structures are first described. The experimental work and procedures of obtaining the modal parameters of the model concrete slab are then explained in detail. The preparation of the experiments is described in Chapter 3 and the results are presented in Chapter 5.

4.2 Modal analysis technique

Modal testing and analysis can be used to achieve a complete dynamic description of a structure. The method involves exciting the structure and measuring its response in terms of displacement, velocity, or acceleration, the latter being the most common measurement. This and the force signal are then Fourier transformed into the frequency domain, from which frequency response functions (also called transfer functions) are established. The frequency response function (FRF) is analysed to find the natural frequencies, mode shapes, and the system parameters of equivalent mass, stiffness, and damping ratio. In this section, the underlying theory and signal analysis techniques used to obtain the FRFs are discussed.

4.2.1 Underlying theory

The frequency response of a system can be represented in terms of: (1) modulus and phase angle against frequency, (2) real and imaginary components of response with varying frequency, or (3)
vector diagram of the real component versus the imaginary component of the response. The first of these is most commonly used in modal analysis of structures, as described below.

A multi degree of freedom system can be represented as $n$ equivalent SDOF systems [Craig (1981)]. For a SDOF system, the equation of motion under harmonic force $F(t) = p e^{i\omega t}$ is given by:

$$m\ddot{u} + c\dot{u} + ku = p e^{i\omega t}$$

(4.1)

the solution of which can be derived in the usual way to give the amplitude of response as:

$$X = H(\omega) \frac{P}{k}$$

(4.2)

Here, $H(\omega)$ is known as the complex frequency response where:

$$H(\omega) = \frac{1}{1 - r^2 + i2\zeta r}$$

(4.3)

with

$$r = \frac{\omega}{\omega_n} \quad \omega_n = \sqrt{\frac{k}{m}} \quad \zeta = \frac{\omega_n}{2k}$$

The amplitude of the response, $X$, can be expressed as: [Rao (1995)]

$$X = X_m e^{i\phi} = X_m \cos \phi + iX_m \sin \phi \equiv X_R + iX_I$$

(4.4)

where $X_m$, $X_R$, and $X_I$ denote the magnitude, the real, and the imaginary parts of $X$ respectively. These are:

$$X_R = \frac{p}{k} \left\{ \frac{1 - r^2}{(1 - r^2)^2 + (2\zeta r)^2} \right\}$$

(4.5)

$$X_I = -\frac{p}{k} \left\{ \frac{2\zeta r}{(1 - r^2)^2 + (2\zeta r)^2} \right\}$$

(4.6)

$$X_m = \sqrt{X_R^2 + X_I^2} = \frac{p}{k} \left\{ \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}} \right\}$$

(4.7)

The phase angle $\phi$ can be obtained as:

$$\tan \phi = \frac{X_I}{X_R} = \frac{2\zeta r}{1 - r^2}$$

(4.8)

This implies that the response can be expressed as

$$u(t) = u_R(t) + iu_I(t) \equiv (X_R + iX_I) e^{i\omega t} = X_m e^{i\phi} e^{i\omega t} = X_m e^{i(\omega t + \phi)}$$

(4.9)

where $u_R(t)$ is in phase with the applied force and $u_I(t)$ lags behind the applied force by $90^\circ$. The total response lags behind the applied force by an angle $\phi$, as shown in figure 4.1 [Rao (1995)]. The response of the system can be represented in the time domain or the magnitude of the response
can be represented in the frequency domain. The phase angle can also be plotted as a function of frequency ratio. In the experimental testing, the peak amplitude of response occurs at the condition of resonance (i.e. \( r=1 \)) giving:

\[
X_{\text{peak}} = \frac{1}{2\zeta} \frac{p}{k}
\]

(4.10)

![Figure 4.1: Plot of the response lagging behind the force](image)

### 4.2.2 Fourier analysis of signals

Signal processing is required in modal analysis to represent the response of the system, under a known excitation, in a convenient form. Often, the time-response of a system will not give much useful information. However, the frequency-response will show one or more discrete frequencies, around which the energy is concentrated. The process of converting the analogue time-domain signal into a digital frequency-domain signal is carried out inside a spectrum analyser, where the energy of a signal is separated into various frequency bands through a set of filters. The method used is called the fast Fourier transform (FFT), which uses the Fourier analysis as explained below.

Any signal, \( x(t) \), which is periodic over an interval, \( T \), can be decomposed into a constant part and an infinite series of harmonic force contributions. When superimposed, these result in the original total time signal function. This harmonic decomposition results in a Fourier series for the signal as follows [Ewins(1984)]:

\[
x(t) = \sum_{-\infty}^{\infty} X_n e^{j\omega_n t}
\]

(4.11)

where

\[
X_n = \frac{1}{T} \int_{0}^{T} x(t) e^{-j\omega_n t} dt
\]

(4.12)

in which \( \omega_n \) is the fixed repetition frequency of the excitation corresponding to the period \( T \), and the integer \( i \) is the index number of the harmonic components. The frequencies of the harmonic
components are multiples of the frequency $\omega_n$. By letting $T \to \infty$, equation 4.11 becomes an integral, so that for a continuous function we get the Fourier transform pair:

$$x(t) = \int_{-\infty}^{\infty} X(\omega)e^{i\omega t} \, d\omega \quad (4.13)$$

and

$$X(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(t)e^{-i\omega t} \, dt \quad (4.14)$$

In most practical applications, a signal is discretised by taking a section and dividing it into $J$ discrete points (at $t = t_k$, $k = 1, J$). The Fourier representation of the section is then:

$$x(t_k) = x_k = \sum_{n=0}^{J-1} X_n e^{ik(2\pi/J)n} \quad (4.15)$$

where

$$X_n = \frac{1}{J} \sum_{k=1}^{J} x_k e^{-ik(2\pi/J)n} \quad n = 1, J \quad (4.16)$$

Equation 4.15 is known as the discrete Fourier series of the signal and equation 4.16 is known as its discrete Fourier transform (DFT). This is the form of Fourier analysis most commonly used in digital spectrum analysers [Ewins (1984)]. It can be computed very efficiently using the fast Fourier transform (FFT) algorithm, which eliminates most of the repetition in the calculation of a DFT and permits much more rapid computation [Oppenheim et al. (1983)].

For modal analysis tests, there will usually be two separate signals entering the analyser at two input channels. In the Advantest spectrum analyser, these time-domain signals will be of force, $X_a$, and response, $X_b$, measurements. FFT is then carried out in the analyser to give the auto-power spectra, $G_{aa}$, for force and, $G_{bb}$, for the response, where:

$$G_{aa} = X_a^*(\omega) \cdot X_a(\omega) \quad (4.17)$$

$$G_{bb} = X_b^*(\omega) \cdot X_b(\omega) \quad (4.18)$$

Also, a complex function, $G_{ab}$, is calculated which is the cross-spectrum of channels $a$ and $b$:

$$G_{ab} = X_a^*(\omega) \cdot X_b(\omega) \quad (4.19)$$

From these calculations, three parameters can be plotted, which represent the modal analysis of the tested system.

- Transfer function (FRF) magnitude: The FRF is defined as:

$$H_{ab} = \frac{G_{ab}}{G_{aa}} \left( \frac{X_b(\omega)}{X_a(\omega)} \right) = X_R + iX_I \quad \text{(see equation 4.4)} \quad (4.20)$$
It is common to plot the magnitude of the transfer function \( |H| = \sqrt{X_R^2 + X_I^2} \) against frequency, from which values for peak frequency and damping can be derived (see equation 4.7).

- **FRF phase angle**: The phase plot is derived by:
  \[
  \phi = \tan^{-1}\left( \frac{X_I}{X_R} \right) \quad \text{(see equation 4.8)} \tag{4.21}
  \]

  This is used in conjunction with the transfer function magnitude. At points of resonance, a sharp change in phase angle is observed.

- **Coherence function**: This function is utilised to check measurement reliability. It is a measure of the amount of response which is directly due to the imposed excitation, given by:
  \[
  \Upsilon_{ab} = \frac{G_{ab}^* \cdot G_{ab}}{G_{aa} \cdot G_{bb}} \tag{4.22}
  \]

  Coherence always takes a value between 0 and 1. The closer the coherence function is to 1, the stronger the cause-effect relationship between input and output.

### 4.2.3 Related topics in signal analysis

In the Fourier transformation of any signal, there is a basic relationship between the *sampling duration*, \( T \), the number of discrete values, \( J \), the *sampling rate*, \( f_s \), and the *frequency resolution* \( \Delta f = \frac{1}{T} \). The range of the spectrum is 0-\( f_{nyq} \), where \( f_{nyq} \) is the *Nyquist frequency* given by:

\[
\Delta f = \frac{1}{T} \quad \text{and} \quad f_{nyq} = \frac{1}{2T}.
\]

As the size of the transform, \( N \), is generally fixed for a given analyser at a power of 2 (256, 512, 1024, etc.), \( f_{nyq} \) and \( \Delta f \) are determined solely by the sample time length, \( T \). This fact introduces constraints and discretisation approximations, which may lead to errors. Hence, the need to limit the length of the time history. Some important features used to reduce these errors are:

**Aliasing**: This is the misrepresentation of the original analogue signal during digitisation. If the sampling rate is too slow, the digital representation will cause high frequencies to appear as low frequencies (figure 4.2). The problem can be avoided by maintaining the sampling rate below the Nyquist frequency, which is twice the highest frequency of interest.

**Leakage**: This problem arises if the finite length of the time-history does not coincide with the assumption of periodicity of the signal. Figure 4.3 illustrates this fact, where for 4.3(a), the signal is perfectly periodic in the time window, giving an accurate spectrum at the frequency of the sine wave. In figure 4.3(b), for the same wave, this periodicity assumption is not valid.
and the implied discontinuity causes the spectrum to be inaccurate. Energy has ‘leaked’ into a number of spectral lines close to the true frequency. Leakage can be corrected by the use of a window function as described below.

**Windowing:** This involves multiplying the original signal by a prescribed time function prior to performing the Fourier transform. This forces the signal to be zero outside the sampling period, hence reducing leakage. There are many types of windows and their use depends on the original signal. Hanning or rectangular windows are commonly used for continuous signals, while exponential windows are most effective for transient vibration applications.

**Averaging:** Generally, it is necessary to perform an averaging process, involving several individual time records, before a result is obtained which can be used with confidence. The two major considerations, determining the number of required averages, are the statistical reliability and the removal of external noise from the signals, hence improving coherence.

### 4.3 Review of modal tests on floors

This section reviews techniques and types of floor vibration tests that have been conducted, which are seen to become more complex with the improvements in technology. Initially, the most popular and easy method of exciting a floor was by the use of a heeldrop (see section 2.3), [Wiss and

Osborne and Ellis (1990) performed tests on a composite floor before and after installation of services and false floors. Their tests were in the form of heel drop, walking, and jumping, where the floor performance was also subjectively rated. In addition, they used a rotating-mass shaker [Hudson (1964)], in which the forcing frequency could be controlled and the force input measured.

There have also been reports of experiments where the responses of structures have been measured while in use. Pernica (1983) measured the vibration response of a grandstand in a sports arena during a three hour rock concert, in order to determine the effects of audience loading, such as dancing and hand-clapping. Several studies have also been conducted on footbridges while in use [Wheeler (1982), Bachmann (1992a)]. Ellis et al. (1994) measured the response of a grandstand during a football match, in which the severe vibrations could be observed visually.

Recently, there has been a trend towards more accurate means of vibration testing, where the force input can be measured. Maguire and Severn (1987) and Caverson et al. (1994) used impact hammer testing to excite various structures. A spectrum analyser was used to obtain the transfer function, from which accurate measurements of frequency and damping were made. Pavic and Waldron (1996a) presented guidelines for the use of instrumented hammers to achieve more accurate results. Falati (1996) and Pavic et al. (1997) used electromagnetic shakers to excite floor slabs, where excitation frequency and amplitude could be controlled. Here, it was possible to impose continuous dynamic loading of known amplitude, giving better response and coherence.

The following sections outline the experimental testing programme on the model post-tensioned concrete floor, in which most of the above mentioned tests: hammer, shaker, heel drop, and walking, were used and results analysed accordingly.

### 4.4 Initial setup procedure

Before commencement of vibration testing on a floor slab, careful preparation is needed. Certain quality assurance guidelines are available [DTA (1993); ISO7626/5 (1994)] of which some have been modified for civil engineering use [Pavic et al. (1997)]. The experiments explained hereina follow these guidelines closely, which are described in order as: the preparatory phase, the exploratory phase, the measurement phase, and the data analysis and modal parameter estimation phase. The preliminary
preparation and exploratory phases are outlined in this section. Figure 4.4 shows an overview of the complete experimental setup, including the loading and measurement instrumentation for the shaker and hammer testing techniques.

![Diagram of experimental setup for shaker and hammer tests]

Figure 4.4: Overview of experimental setup for shaker and hammer tests

### 4.4.1 Location of grid points

In field testing, the floor slab under investigation is inspected and a representative panel on the structure is chosen, which exhibits the most symmetry and whose boundary conditions can easily be identified. In most cases, it may be necessary to test a few such panels and select the one that gives the best and most critical results. This test panel is then divided into a grid of equally spaced points, typically 1.5 to 2m apart. The experiment is then performed on the panel, the procedure being slightly different depending on the excitation source used, as described in section 4.5.

Under laboratory conditions, more control is available with regard to the symmetry, size, and boundary conditions of the test specimen. Also, the grid points can be much closer to each other, giving more accurate results of response and better mode shape representation. In the case of the
experimental model slab, the gridline layout is shown in figure 4.5, chosen to have the grid points closer together at the area of maximum response (i.e. the centre of the slab). The closeness of the grid points is designed to give more results, hence leading to a more accurate assessment of slab behaviour.

![Diagram of grid points layout]

Figure 4.5: Layout of experimental grid points on the model slab

### 4.4.2 Location of loading

In cases where the excitation source is stationary, it is very important to apply the imposed dynamic load at a point where all desirable natural frequencies of the structure are excited. For field experiments, this point can be found by trial and error and special care is taken not to apply the load at a node. With the experimental model slab, due to the simplicity of the setup, it was easy to predict where the nodes would occur. Hence, the shaker loading frame was placed at a point where the first three natural frequencies were excited (see figure 4.5).

### 4.4.3 Location of measurement instrumentation

As described in section 3.3, the vibration measuring instrumentation consisted of one accelerometer (two for measuring phase), one load cell, and a spectrum analyser. In the case of shaker testing, the loading frame was stationary on the slab and the accelerometer was moved from point to point throughout all grid points. During the hammer test, it was easier to move the hammer and keep the accelerometer stationary, where it was not near a node. With the other loading types, such as heeldrop, either option could be adopted. However, as the model slab was small, it was decided that the loading was to be applied at a stationary point near the slab centre, and the accelerometer
be moved through the grid points. The actual procedure for each of the loading types is described in more detail in section 4.5.

4.4.4 Preliminary tests

Before the core of the experimental programme on the model slab was carried out, two preliminary tests were conducted with the aim of equipment familiarisation. These tests were carried out on a dance floor and an Edwardian building, as described briefly below.

Tests on an existing dance floor

The floor under inspection was 'The Longroom' of New College in Oxford, currently being used as a dance floor. On its refurbishment into a dance-room in the early 1960s, the original floor was found to have vibration problems and limitations had been imposed on the number of people allowed to dance in the room. The tests carried out were aimed at determining the vibration characteristics of this floor. Full results of this investigation are explained in detail in [Falati (1996)]. The following is a brief summary of the test procedures.

Figure 4.6 shows the layout of the Longroom floor, including the timber beams supports and recent stiffening by a steel girder. A typical panel was chosen, which would exhibit the most symmetry, and the experiments were conducted on this panel. Limited tests were performed on the two adjacent panels in order to gain an idea of floor continuity.

![Figure 4.6: Plan of Longroom floor showing main test panel and grid points](image)

The main aims of these tests were the commissioning of the shaker loading frame and the overall sequence of the experimental procedure. Hammer testing was also carried out and comparisons
between the hammer and shaker results were made, in order to gauge the viability of the shaker loading frame. Figure 4.6 also shows the grid points on the main test panel. The shaker was positioned at point D3 and during hammer testing, the accelerometer was also stationary at D3. The tests were carried out at night to minimise vibration interference from outdoor activities.

<table>
<thead>
<tr>
<th>Modal property</th>
<th>Mode 1 Hammer</th>
<th>Shaker</th>
<th>Mode 2 Hammer</th>
<th>Shaker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural frequency (Hz)</td>
<td>25.94</td>
<td>25.68</td>
<td>31.83</td>
<td>32.00</td>
</tr>
<tr>
<td>Damping (% critical)</td>
<td>5.47</td>
<td>6.19</td>
<td>3.11</td>
<td>3.51</td>
</tr>
</tbody>
</table>

A summary of the damping and the natural frequency results for this floor is illustrated in table 4.1, which shows good agreement between hammer and shaker tests. Furthermore, the shaker tests were seen to give more accurate data with better coherence [Falati (1996)]. Overall, the tests were satisfactory and the shaker frame gave excellent preliminary results.

**Vibration transmission tests inside an old building**

The building under consideration was the 'Jenkin Building' of Oxford University, built in 1908. It consists of three floors and a basement with footing foundations on Oxford Clay. Figure 4.7 shows an outside view of this building. A proposal was made in 1996 to establish an earthquake vibration testing laboratory on the ground floor. It was hence necessary to determine the viability of such a proposal by testing the vibration transmission throughout the building, to gauge whether the ensuing vibrations would be annoying to the occupants. The tests were carried out using the shaker loading frame and the responses were measured at various points throughout the building.

![Figure 4.7: An outside view of Jenkin building](image)
The shaker was positioned directly above one of the brick piers supporting the suspended ground floor, so that the vibration was transmitted through the foundation to the underlying soil. This would accurately represent the load path when the proposed earthquake laboratory is in operation. The accelerometer positions were near offices and vibration sensitive areas of the building, as well as near walls and columns. Also, the response at the base of the pier, on which loading was being applied, was measured to gauge the proportion of the applied load which was actually being transmitted to the foundations. It was found that this fraction was very high. The tests were conducted at night when the building was empty of occupants, hence ensuring minimum interference.

In general, it was found that due to the low relative excitation amplitude imposed by the shaker, transmission levels were small, although at places just above and next to the loading, these responses were significant. The results were used in the design of the earthquake laboratory foundations, which took the form of a solid concrete block measuring $9.1\text{m} \times 4.2\text{m} \times 1.6\text{m}$ deep. The laboratory has now been commissioned and vibration tests are being conducted in the building with no subsequent report of annoyance to occupants. A detailed account of the laboratory can be found in [Williams et al. (1998)]. The shaker setup and experimental procedure were again found to be satisfactory.

4.5 Loading procedure

With the test equipment commissioned and the model slab designed and built, the main part of the experimental programme was carried out. The actual testing procedure was different for each type of loading, hence they are described separately below. The measurement and processing of raw data for each experiment were similar, as described in section 4.6. Note that the following would fall under the measurement phase of the quality assurance guidelines explained in section 4.4.

4.5.1 Instrumented impact hammer

In this test, the accelerometer was mainly attached to the slab at grid point $K2$, where good response in the first three natural modes was achieved (see figure 4.5). The impact hammer, containing the force transducer, was then moved successively from point to point and at each location the model floor was impacted seven times to obtain an average analysis for the particular grid point. It was found by preliminary trials that the average of the measured data became very stable after seven hammer blows and any further blows had little effect. Special care was taken to reproduce the same
force of impact for each hammer blow, although homogeneity tests showed that the magnitude of
the impact force had little or no effect on the overall results (see section 5.8). Transfer function,
coherence, and phase plots were obtained on the spectrum analyser for immediate analysis and
checking of results. The raw data were then recorded on floppy disks for post-processing.

4.5.2 Electromagnetic shaker

The essential difference between this test and the impact hammer test was that the accelerometer,
rather than the exciter, was moved from point to point on the grid, while the vibration source was
stationary. This was mainly due to the bulkiness and heaviness of the shaker test rig. Preliminary
reciprocity tests (see section 5.8) showed that the same results would be achieved whether the
exciter or the shaker is stationary. The position of the shaker is shown in figure 4.5.

The excitation imposed was of three distinct types. Firstly, at each experimental stage, a sine
sweep of 0-100Hz (resolution of 0.12Hz) was carried out and the response of the floor was measured
at the first three modes of vibration. Then, to achieve more accurate results at the first mode, a
sine sweep of 0-50Hz (resolution of 0.06Hz) was conducted and again the response was measured
at every grid point. The number of readings at each point was restricted to seven, which was seen
to give a good average of the data.

The shaker was then altered to produce a sine-wave excitation at a single frequency, equal to one of
the first three natural frequencies, hence exciting the slab at only that frequency. For measurements
of damping by the logarithmic decrement method, the shaker was suddenly switched off and the
subsequent free vibration of the floor was measured in the time domain. More emphasis was placed
on the first natural frequency of the slab, as it would generate the largest response and is within
the range of human annoyance. Hence, the slab was excited at its fundamental frequency with
three different loading amplitudes and the logarithmic decay at each of these was measured.

The effect of the mass of the shaker frame on the fundamental frequency of the slab was calcu-
lated to be of the order of 2%. However, to avoid any discrepancies between shaker and hammer
measurements, the hammer test was conducted with the shaker frame present on the slab.

4.5.3 Heeldrop loading

This test is described in section 2.3, but its popularity has declined recently as the drawbacks of
the approach have been recognised. Most obviously, the applied loading varies from test to test
and cannot be readily determined. In the case of the experimental slab, the person conducting the heeldrop weighed 75kg and was positioned on a fixed point near the centre of the slab (see figure 4.5). The accelerometer was moved from point to point and the time domain acceleration response of the floor was monitored at nine points near the slab centre. Figure 4.8 shows a typical response measured from a heeldrop loading. The effect of the mass of the person on the measured fundamental frequency was calculated to be of the order of 4%. This was reflected in the results, as shown in chapter 5.

![Graph](image)

Figure 4.8: Typical response to a heeldrop measured at point $H2$ in figure 4.5

### 4.5.4 Human walking

For each experimental condition, human walking tests on the slab were carried out and the response measured at nine points near the slab centre (gridlines $F$ to $H$ in figure 4.5). The ensuing acceleration readings were averaged and compared with human perceptibility guidelines for each slab configuration. Two types of tests were conducted, vertical and horizontal walking.

In vertical walking, also described as on-the-spot walking, the person was stationary with no horizontal motion and the frequency of walking was kept to normal realistic levels of around 2Hz. A typical time history of vertical walking is shown in figure 4.9.

Horizontal walking, also described as normal or forward walking, entails the person walking on the slab with horizontal motion and changing direction at the end of the slab. While turning, the subject would try to exhibit normal walking conditions. Figure 4.10 shows a typical horizontal walking time history. The irregular part is due to the person turning, hence losing the constant momentum in his walking rate.
4.6 Measurement procedure

The first indication of the experimental results is observed on the spectrum analyser, in the form of transfer function, phase, and coherence plots. A transfer function, as described in section 4.2, is a complex function which defines the output of the system given the input, and is basically the measured acceleration divided by the measured force at a particular point in the frequency domain. The phase plot describes the difference in phase between the force and acceleration signals. At a resonant frequency, this plot would be expected to undergo a sudden shift as the frequencies of the two signals become equal. The coherence is a measure of the accuracy of test data and a value of more than 95% is regarded as acceptable in these tests. It would be normal to accept a rather lower coherence in field testing conditions due to external vibrations other than the applied force.

Typical graphs for the transfer function, phase, and coherence, are shown in figure 4.11. The results were saved as $G_{aa}$, $G_{bb}$, and $G_{ab}$ in the frequency domain and as $X_a$, $X_b$ in the time domain for further analysis. As shown in figure 4.4, these data were converted and post-processed on a network of workstations, using a suite of programs written in Matlab. The derivation of the floor properties, in terms of natural frequencies, damping, and mode shapes, is described in the following sections.
4.6.1 Natural frequencies

As shown in figure 4.11, typical test transfer functions exhibit a varying number of peaks in the frequency range. In field tests, these peaks are generally closely spaced and sometimes difficult to identify. The model slab, however, is designed as a simple structure, hence exhibiting distinct peaks at each mode. For each grid location, the first peak represents the first mode, and so on. At the first peak amplitude, the value of frequency was noted and taken as the fundamental natural frequency of the floor. This procedure was carried out for the other modes of the floor system, where each mode was represented by a corresponding resonating peak. The overall natural frequency of each mode could hence be represented as the average value of all those extracted from each grid location for that mode. In the case of the model slab, this was in most cases an average between 39 values corresponding to the 39 grid points. The results are given in section 5.3 for each slab configuration.

From single sine-wave loading at individual modes of vibration, natural frequencies were estimated from time domain data by measuring the average time between zero crossings and calculating the average frequency, which is the reciprocal of the period. In addition, a Fast fourier transform (FFT) was also performed on each reading, which agreed very closely with the calculated values. Again, in most cases, these tests were performed for all grid points giving an average of 39 individual readings for each slab configuration. The results are illustrated in section 5.3.
4.6.2 Phase between two points

These tests were carried out to determine the direction of the mode shapes. Two accelerometers were used, where one was a reference placed in position L2, while the second was moved from point to point on the slab (see figure 4.5). At each point, the phase difference between the two accelerometers was deduced using the spectrum analyser. The phase value for each mode at each grid location was taken at the mode's natural frequency. When the two accelerometers were in phase, a reading of -90° to +90° would be observed, whereas a reading of -90° to -180° or +90° to +180° would signal the two points out of phase. A typical phase plot between two accelerometers is shown in figure 4.12.

![Figure 4.12: Typical phase plot with reference accelerometer on point L2 and second accelerometer on point C2 of figure 4.5](image)

4.6.3 Mode shapes

Since acceleration is proportional to the negative of displacement at any mode, the transfer function maximum amplitude for a particular natural frequency could be used to calculate the mode shape of the floor at that natural frequency. For each mode and grid location, the transfer function maximum amplitudes were normalised to the largest value, to obtain the magnitude of the mode shape co-ordinate for that particular mode and grid point. The phase differences between the points were then incorporated, which determined the sign of the mode shape co-ordinate at each grid point. After calculating all normalised mode shape co-ordinates and their directions, it was possible to plot the mode shapes of the floor. Generally, only the first two or three modes of vibration are significant, because they are the ones most likely to be excited by ambient excitations, such as walking and jumping. They are also the most perceptible to human beings. Typical mode shape plots for the experimental slab are shown in figure 5.4 (page 96) for the first three modes.
A second possible method of obtaining mode shapes would be to excite the floor at a natural frequency and plot the normalised acceleration responses of all points, with phase differences incorporated. This method was also carried out and identical results to the above were obtained.

4.6.4 Damping ratios

**Halfpower bandwidth**

This method is explained in section 2.6.2. In the case of the experimental results, *Matlab* programming was used to fit a smooth spline curve over the resonant peak at the fundamental frequency. The peak of the smooth curve was then measured and damping was calculated using equation 2.35 (page 32). A typical plot using this method is shown in figure 4.13. For higher modes, the data points were seen to exhibit more scatter and hence the spline fit was not a suitable option. Here, equation 2.31 (page 32) was used and a best fit SDOF visco-elastic curve was fitted to the data, which best presented the values of frequency, damping, and stiffness at the mode. Figure 4.14 illustrates this method at the third mode of vibration.

![Figure 4.13: Calculation of damping ratio in frequency domain by halfpower bandwidth method (spline curve fit)](image)

The logarithmic decrement, $\delta$, represents the rate at which the amplitude of free damped vibration decreases with time, as explained in section 2.6.2. In the case of the experimental slab, the time domain acceleration response of the floor was measured either after a heeldrop or after switching the shaker off. This resulted in a response, which was of the form of an exponential decay, as shown in figure 4.15. By fitting a smooth, linear viscoelastic, exponential curve and using the logarithmic
Figure 4.14: Calculation of damping ratio in frequency domain by halfpower bandwidth method (SDOF curve fit)

decrement method, the value of damping ratio at a certain grid point was obtained. It is assumed that the damping is uniform throughout the floor. To obtain better accuracy, this procedure was carried out on as many as 25 points for a given slab configuration and the average was taken, which was deemed to be the overall value of \( \zeta \) for the whole floor. To calculate the value of damping for each mode separately, it was not possible to use the heeldrop test as the frequency of the input force could not be controlled. With the shaker, however, it was possible to impose a single sine load equal to the relevant natural frequency of the slab. Hence, the shaker frequency was tuned to coincide with one of the slab resonant frequencies, exciting that particular natural frequency. On switching the shaker off, it was possible to obtain a damping value for that particular mode.

Figure 4.15: Damping ratio in time domain (logarithmic decrement method)
4.6.5 Maximum accelerations

After each alteration to the floor, a test was also carried out at which the shaker imposed a steady-state sinusoidal load at a slab natural frequency. The maximum acceleration amplitudes of the slab, at some grid points, were then measured. Responses were also measured as a result of heeldrop and walking excitations. Each acceleration reading was converted to its Root Mean Square (RMS) value for comparison with the guidelines mentioned in section 2.2.2. The results are presented in section 5.7.1 and analysed in section 6.6.1. Figure 5.5 (page 98) shows a sample acceleration of the slab when excited at its fundamental natural frequency.

4.7 Small specimen tests

During the casting process of the model slab and the visco-elastic screed, small sample specimens of the mix were taken in the form of cubes and cylinders (see section 3.5.3). The following describes the test procedure for determining the concrete material properties from these samples.

Determination of Young’s modulus

Axial compression tests were carried out on two 300mm long by 150mm diameter cylinders in accordance to standard guidelines given in BS1881: Part 121 [Kong and Evans (1989)]. In each case, five loading increments were used and each of these was repeated three times to account for hysteresis losses. The tests were performed at regular intervals throughout the research programme, to obtain a record of variations over the test period. Experimental results are described in section 5.2.1.

Determination of concrete strength

As mentioned in section 3.5.1, the concrete mix for the slab was designed to have a characteristic strength, $f_{cu}$, of 40N/mm². A total of fifteen 100mm concrete cube samples were taken to confirm the actual compressive strength of the slab. The cubes were crushed in the normal way and the results are described in section 5.2.2. All cubes were seen to undergo 'normal failure'.
Vibration tests on sample beams

Three doubly reinforced beams of $1500 \times 200 \times 150$mm were cast during the experimental programme, as described in sections 3.5.3 and 3.6.7. The beams were tested in the laboratory to find their vibration characteristics. The purpose of these tests was to identify the changes in damping behaviour of concrete, caused by the Concredamp visco-elastic additive [Concredamp Inc. (1993)]. Figure 4.16 shows the vibration tests carried out on these beams, which were impact hammer tests using a hard tip for a large frequency range. Response measurements were taken at eleven points along the beams. These points are illustrated in figure 4.17, which also shows the location of the hammer impacts. For each test point, ten hammer blows were conducted and from the average peak fundamental frequencies, inherent damping was derived using the halfpower bandwidth method. The results of the vibration tests on the three beams are given in section 6.5.1.

![Image](image.png)

Figure 4.16: Vibration testing of sample beams by impact hammer

![Image](image.png)

Figure 4.17: Test grid points on sample beams
Chapter 5

Experimental results

5.1 Introduction

In this chapter, the results of small specimen cylinder and cube tests are presented and the concrete properties are derived. The bulk of the chapter, however, presents summaries of the obtained values of fundamental mode natural frequency, damping, and peak accelerations for each slab stage, using the analysis techniques of chapter 4. Complete results of all tests at the first three modes are included in appendix A. In chapter 6, all the data presented here are discussed and analysed in relation to different slab conditions. Full results relating to tests with plywood TMDs and human-structure interaction are described in more detail in chapters 7 and 8 respectively.

5.2 Concrete properties

This section presents the results of tests carried out to verify the design concrete properties. The modulus of elasticity and concrete cube strengths, obtained in these tests, are compared with recommended values in BS8110 (1985), which show acceptable agreement. Factors such as creep, shrinkage, and thermal strains are ignored as the model slab is too small and the time period of testing too short for these to have any significant effect. As the properties of concrete are affected by time-dependent factors, such as stress, relative humidity, and temperatures, tests on sample cylinders and cubes were carried out regularly during the course of the experimental programme. The specimen preparation and testing techniques are described in chapters 3 and 4 respectively.
5.2.1 Modulus of elasticity

Young's modulus was derived from axial compression tests in the linear elastic region of the stress-strain graphs. As the research programme is solely concerned with serviceability loading, higher stresses at the non-linear region of the stress-strain curve were not attempted. Two cylinders were tested at regular time intervals using the procedure outlined in section 4.7. Figure 5.1 shows a typical experimental stress-strain curve. For each loading-unloading cycle, five values of Young's modulus were obtained, which over the three cycles were averaged to obtain an $E$ value for the particular test. The derived $E$ values for the two cylinders are plotted against time in figure 5.2. These are compared with BS8110: Parts 1 and 2 recommended values.

![Figure 5.1: Typical stress-strain curves over three loading-unloading cycles showing hysteresis](image)
The equation used in Part 2 for the modulus of elasticity at age \( t \) is:

\[
E_{c,t} = E_{c,28} \left( 0.4 + 0.6 \frac{f_{cu,t}}{f_{cu,28}} \right) \quad \text{for } t \geq 3 \text{ days} \tag{5.1}
\]

where the recommended values of \( f_{cu,t} \) in the codes suggest a gain of strength beyond 28 days, hence affecting modulus of elasticity. BS8110: Part 1, on the other hand, assumes no increase in strength beyond 28 days. From figure 5.2 it is apparent that the experimental results fall below these two versions of the codes, but are still within the recommended range given as 22-34kN/mm². From the test data, a value for Young's modulus of 25.6kN/mm² was used in all subsequent calculations. This is the mean of the test results, all of which lie within ±5% of this value.

![Graph showing experimental values of Modulus of Elasticity](image)

Figure 5.2: Experimental values of Modulus of Elasticity
5.2.2 Concrete cube strengths

At regular time intervals during the experimental programme, 100mm cubes were crushed to obtain values of cube strength, $f_{cu,t}$. These are plotted against time in figure 5.3. Notice that at the early stages of curing, more than one cube was crushed on the same day. Also in figure 5.3, plots of recommended values of cube strength with time from BS8110: Parts 1 and 2 are illustrated. Again, Part 1 assumes no gain in strength after 28 days. Satisfactory agreement exists between experimental values and those from the codes, particularly around the region where the design strength is between 40 and 50N/mm$^2$. This shows that the concrete mix has a slightly higher strength, which is close to the design target mean strength of 45N/mm$^2$. This is common in laboratory conditions. The average cube strength of all samples tested after 28 days was 44.8N/mm$^2$.

![Figure 5.3: Comparison of experimental concrete cube strength with BS8110](image)

Figure 5.3: Comparison of experimental concrete cube strength with BS8110
5.3 Natural frequencies

Having derived the concrete properties, the remainder of this chapter is concerned with tests on the model slab itself. As described in chapter 4, the model slab natural frequencies were derived from both frequency and time domain data for comparison. The frequency data were in the form of FRF plots and time data were in the form of time histories.

Tables 5.1 to 5.6 show the average fundamental frequency results, obtained from frequency domain and time domain analyses, at different slab stages. Detailed results and values at higher modes are presented in appendix A, which show that the frequency domain tests were carried out at two frequency resolutions of 0.12 and 0.06 Hz for accuracy (tables A.1-A.9). In general, the derived values agree well at the two frequency resolutions. Similarly, in the time domain, the slab response was measured at various loading amplitudes, excited at the natural frequencies, and the results show minimal sensitivity of natural frequency to this variation (tables A.10-A.17). Note that the results for full-height partitions are not given in the time domain, as the modes are very closely spaced making the derivation of a single natural frequency very difficult and inaccurate.

In general, it can be observed that the values obtained at the two domains agree well for a particular slab configuration. Here, second and third harmonics are considered to be too high to cause vibration problems. Hence, all further analyses are mainly concerned with the fundamental frequency of the slab, which is within the range of human sensitivity.

5.4 Damping ratios

As for natural frequency, the damping values of the slab were derived from both frequency domain and time domain measurements. The halfpower bandwidth method was used to derive the damping in the frequency-domain and the logarithmic decrement method was used for the time-domain results of shaker cut-off and heeldrop. Both these methods and their application to the experimental results are described in chapter 4. Full damping results of all tests are included in appendix A.

Tables 5.1 to 5.6 show the average fundamental mode damping results obtained using the two methods on each slab configuration. Again, in the frequency domain, two different frequency resolutions were used and the averages given were taken over a large number of points tested (tables A.18-A.26). In some cases, noticeable differences could be observed between readings from the two resolutions. Here, the value of the smaller frequency resolution is deemed to be more accurate.
as the experimental points are closer together before commencement of curve-fitting. However, in most instances the readings were close and the average values taken are therefore from readings of both frequency resolution tests.

The damping values obtained from the logarithmic decrement method were seen to be influenced to a great extent by the amplitude of the applied load (tables A.27-A.34). Here, the linear SDOF visco-elastic curve, representing damping and natural frequency, was fitted at all values of amplitude between 95% and 5% of maximum response. This would give a degree of consistency to the presented results and a means of comparison between different loading amplitudes.

From tables 5.1-5.6 it is observed that the values obtained from frequency and time-domain measurements are generally in reasonable agreement for each particular slab stage. The frequency-domain results are influenced by frequency resolution and the time-domain results by amplitude of loading. It is generally agreed that with a higher frequency resolution, the results become more accurate. However, as explained in section 4.6.4, the measurement of damping in the frequency domain is prone to non-linear behaviour of the structure, making the evaluation of resonance curves difficult. In these tests, minimal non-linearity was observed at the fundamental mode and the results could be viewed with confidence (see also section 5.8). The measurement of damping in the time-domain, however, is the most common method and generally recommended [Bachmann et al. (1995)].

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Fund. Frequency (Hz)</th>
<th>Damping (% critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>freq. domain</td>
<td>time domain</td>
</tr>
<tr>
<td>reinforced without post-tension</td>
<td>7.54</td>
<td>-</td>
</tr>
<tr>
<td>100kN post-tension force</td>
<td>7.63</td>
<td>7.62</td>
</tr>
<tr>
<td>175kN post-tension force</td>
<td>8.01</td>
<td>8.03</td>
</tr>
<tr>
<td>detensioned before demolishing</td>
<td>9.16</td>
<td>8.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Fund. Frequency (Hz)</th>
<th>Damping (% critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>freq. domain</td>
<td>time domain</td>
</tr>
<tr>
<td>100kN post-tension force with dead load of 0.25kN/m²</td>
<td>7.22</td>
<td>7.13</td>
</tr>
<tr>
<td>175kN post-tension force with dead load of 0.25kN/m²</td>
<td>7.70</td>
<td>7.71</td>
</tr>
</tbody>
</table>
### Table 5.3: Effect of false flooring on fundamental frequency and damping

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Fund. Frequency (Hz)</th>
<th>Damping (% critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>freq. domain</td>
<td>time domain</td>
</tr>
<tr>
<td></td>
<td>freq. domain</td>
<td>time domain</td>
</tr>
<tr>
<td>floor layout1 (see fig. 3.15, page 62)</td>
<td>7.87</td>
<td>7.89</td>
</tr>
<tr>
<td>floor layout2 (see fig. 3.16, page 62)</td>
<td>7.61</td>
<td>7.53</td>
</tr>
</tbody>
</table>

### Table 5.4: Effect of stationary man on fundamental frequency and damping

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Fund. Frequency (Hz)</th>
<th>Damping (% critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>freq. domain</td>
<td>time domain</td>
</tr>
<tr>
<td></td>
<td>freq. domain</td>
<td>time domain</td>
</tr>
<tr>
<td>one man standing near slab centre</td>
<td>7.92</td>
<td>7.65</td>
</tr>
<tr>
<td>equivalent mass of man near centre</td>
<td>7.69</td>
<td>7.66</td>
</tr>
</tbody>
</table>

### Table 5.5: Effect of partitions (see figure 3.12, page 60, for layout) on fundamental frequency and damping

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Fund. Frequency (Hz)</th>
<th>Damping (% critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>freq. domain</td>
<td>time domain</td>
</tr>
<tr>
<td></td>
<td>freq. domain</td>
<td>time domain</td>
</tr>
<tr>
<td>five cantilever partitions</td>
<td>7.76</td>
<td>7.69</td>
</tr>
<tr>
<td>perpendicular to slab span</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(nos. 1-5 in fig. 3.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>five cant. parts. perpendicular</td>
<td>7.81</td>
<td>7.77</td>
</tr>
<tr>
<td>and 3 transverse to slab span</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(nos. 1-8 in fig. 3.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>three parallel to slab span</td>
<td>8.00</td>
<td>-</td>
</tr>
<tr>
<td>(nos. 6-8 in fig. 3.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>three full-height partitions</td>
<td>20.66</td>
<td>-</td>
</tr>
<tr>
<td>perpendicular to slab span</td>
<td></td>
<td></td>
</tr>
<tr>
<td>one full-height partition</td>
<td>20.94</td>
<td>-</td>
</tr>
<tr>
<td>perpendicular to slab span</td>
<td></td>
<td></td>
</tr>
<tr>
<td>five cantilever and one</td>
<td>20.31</td>
<td>-</td>
</tr>
<tr>
<td>full-height partitions</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 5.6: Effect of visco-elastic screeds on fundamental frequency and damping

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Fund. Frequency (Hz)</th>
<th>Damping (% critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>freq. domain</td>
<td>time domain</td>
</tr>
<tr>
<td></td>
<td>freq. domain</td>
<td>time domain</td>
</tr>
<tr>
<td>25mm thick screed of low</td>
<td>8.82</td>
<td>8.72</td>
</tr>
<tr>
<td>Conceadamp concentration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(see section 3.6.7)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>additional 25mm thick screed of</td>
<td>10.14</td>
<td>10.15</td>
</tr>
<tr>
<td>high Conceadamp concentration</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5.5 Mode shapes

The mode shapes for each slab configuration were derived as explained in section 4.6.3. As the model slab is one-way spanning with simple supports, it exhibits beam like behaviour and its mode shapes resemble those for a simple beam. Figure 5.4 shows typical mode shapes for the first three modes of the slab when fully stressed. The non-zero end readings suggest some vertical flexibility at the supports, and may also be because the end points of measurement were not exactly on the support lines. In most instances, the addition of non-structural components did not affect the mode shapes and for each stage similar results as in figure 5.4 were obtained. The only addition which significantly affected slab stiffness, and hence mode shape, was when full-height partitions were installed. This is discussed in detail in section 6.4.1.

![Mode shapes of slab](image)

Figure 5.4: Typical first three mode shapes of slab

5.6 Heeldrop tests

Table 5.7 shows the derived natural frequency and damping values obtained from the exponential decay of vibrations following heeldrop loading. Time-domain frequency analysis and logarithmic decrement damping methods were used in these cases and the results agree well with comparable ones of shaker cut-off and FRF tests. Note that the damping values obtained due to heeldrop loading are high, as the decay occurs while the body of the applier is stationary on the slab. Hence, these results should be compared with those of shaker cut-off and FRF tests with a standing human present on the slab (see table 5.4 and appendix A). Also included in table 5.7, are the average peak
acceleration readings of the slab following heeldrops. These are used in some acceptability criteria, as explained in section 6.6.3.

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Fundamental frequency (Hz)</th>
<th>Damping (% critical)</th>
<th>Peak accel. (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bare slab fully prestressed</td>
<td>9</td>
<td>7.68</td>
<td>3.65</td>
<td>0.53</td>
</tr>
<tr>
<td>false floor layout 2</td>
<td>18</td>
<td>7.44</td>
<td>3.51</td>
<td>0.52</td>
</tr>
<tr>
<td>first screed layer</td>
<td>9</td>
<td>8.65</td>
<td>4.00</td>
<td>0.88</td>
</tr>
<tr>
<td>first screed layer and five cantilever partitions</td>
<td>9</td>
<td>8.59</td>
<td>3.72</td>
<td>0.97</td>
</tr>
<tr>
<td>second screed layer</td>
<td>13</td>
<td>10.02</td>
<td>3.22</td>
<td>0.85</td>
</tr>
<tr>
<td>second screed layer and false floor layout 1</td>
<td>12</td>
<td>9.95</td>
<td>3.18</td>
<td>0.67</td>
</tr>
<tr>
<td>second screed layer and false floor layout 2</td>
<td>8</td>
<td>9.82</td>
<td>3.30</td>
<td>0.83</td>
</tr>
</tbody>
</table>

5.7 Acceleration responses

As explained in chapter 4, the acceleration response of the slab was measured due to human walking, and due to a steady-state sine-wave excitation at the slab resonance frequency. These readings could then be checked against acceptability guidelines and the level of vibration could be rated. This procedure is described in section 6.6. In the following sections, the results obtained from the two types of loading tests are presented.

5.7.1 Excitation at resonance frequency

In this case, the acceleration response of the slab was measured as a result of imposing a steady-state excitation at the slab fundamental frequency. The acceleration was measured at the centre of the slab, where maximum response was expected. Readings were taken at nine points and converted to the Root Mean Square (RMS) values for comparison with the acceptability curves. Figure 5.5 shows a typical resonant response and its RMS acceleration value. For each case, the loading was applied at two different amplitudes for comparison (see table A.35 in appendix A). The loading amplitudes were generally small but since they were being applied at the resonant frequency, high acceleration responses were observed.

Table 5.8 shows the RMS acceleration responses, obtained for selected configurations at the fundamental mode, normalised over the loading amplitude. Similar results at the second and third natural frequencies are also given in tables A.36 and A.37 of appendix A. For the second mode, the presented values were averages of readings at 1/4 and 3/4 of slab span, where maximum response at this mode was expected. Responses obtained at the third mode were taken at slab midspan.
Table 5.8: Summary of RMS steady-state accelerations at slab fundamental frequency

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>RMS accel. ( (\text{m/s}^2) )</th>
<th>Slab configuration</th>
<th>RMS accel. ( (\text{m/s}^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100kN post-tension force</td>
<td>0.013</td>
<td>5 cant. &amp; one full-height partitions</td>
<td>0.002</td>
</tr>
<tr>
<td>175kN post-tension force</td>
<td>0.063</td>
<td>first screed layer (screed 1)</td>
<td>0.033</td>
</tr>
<tr>
<td>detensioned for demolishing</td>
<td>0.017</td>
<td>second screed layer (screed 2)</td>
<td>0.030</td>
</tr>
<tr>
<td>100kN force with dead load</td>
<td>0.040</td>
<td>screed 1 + man</td>
<td>0.028</td>
</tr>
<tr>
<td>175kN force with dead load</td>
<td>0.058</td>
<td>screed 1 + 5 cant. partitions</td>
<td>0.022</td>
</tr>
<tr>
<td>false floor layout 1</td>
<td>0.041</td>
<td>screed 1 + man + 5 cant. parts.</td>
<td>0.009</td>
</tr>
<tr>
<td>false floor layout 2</td>
<td>0.017</td>
<td>screed 2 + man</td>
<td>0.011</td>
</tr>
<tr>
<td>one man near slab centre</td>
<td>0.006</td>
<td>screed 2 + equiv. mass of man</td>
<td>0.026</td>
</tr>
<tr>
<td>equiv. mass of man</td>
<td>0.030</td>
<td>screed 2 + 5 cant. partitions</td>
<td>0.027</td>
</tr>
<tr>
<td>five perpendicular cant. parts.</td>
<td>0.034</td>
<td>screed 2 + man + 5 cant. parts.</td>
<td>0.012</td>
</tr>
<tr>
<td>5 perp. &amp; 3 transverse parts.</td>
<td>0.048</td>
<td>screed 2 + floor layout 1</td>
<td>0.023</td>
</tr>
<tr>
<td>one full-height partition</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.7.2 Walking excitation

As explained in section 4.5.4, excitation due to normal human activity was performed by a person walking at a medium pace along the slab (horizontal walking), and also as a result of a person walking on the spot near slab centre (vertical walking). Here, the results of these two walking excitations are presented.

Horizontal walking

Table 5.9 shows the RMS acceleration responses, averaged over nine points at the slab centre, as a result of normal human walking. Fast fourier transform (FFT) analysis was also carried out at each point to obtain the dominant frequency. Figure 5.6 shows a typical acceleration response to a walking excitation, and its derived frequency spectrum. Typical responses due to walking on
different slab configurations are shown in figure 5.7 and discussed in chapter 6.

Table 5.9: RMS accelerations due to horizontal walking along slab

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>RMS accel. (m/s²)</th>
<th>Dominant freq. (Hz)</th>
<th>Slab configuration</th>
<th>RMS accel. (m/s²)</th>
<th>Dominant freq. (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bare slab (fully stressed)</td>
<td>0.13</td>
<td>7.75</td>
<td>second screed layer (screed 2)</td>
<td>0.04</td>
<td>9.85</td>
</tr>
<tr>
<td>false floor layout 2</td>
<td>0.09</td>
<td>7.50</td>
<td>screed 2 + floor layout 1</td>
<td>0.04</td>
<td>9.75</td>
</tr>
<tr>
<td>first screed layer (screed 1)</td>
<td>0.06</td>
<td>8.75</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.6: Response and frequency spectrum of normal walking along slab with both screed layers

Vertical (on the spot) walking

The RMS acceleration responses of these tests are presented in table 5.10, where average values are given of tests on nine points at the slab centre. Similarly, the FFT frequency spectrum of each point was also taken and averaged for the dominant frequency. Typical responses at different slab configurations are shown in figure 5.8. Notice that in general, the accelerations due to this type of loading are more severe compared to normal walking. This is to be expected, as in this case the loading is purely vertical without any horizontal component. This leads to an increase in loading amplitude, as the whole weight of the person is applied vertically onto the floor.

Table 5.10: RMS accelerations due to vertical walking at slab centre

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>RMS accel. (m/s²)</th>
<th>Dominant freq. (Hz)</th>
<th>Slab configuration</th>
<th>RMS accel. (m/s²)</th>
<th>Dominant freq. (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bare slab (fully stressed)</td>
<td>0.14</td>
<td>7.75</td>
<td>second screed layer (screed 2)</td>
<td>0.05</td>
<td>10.13</td>
</tr>
<tr>
<td>false floor layout 1</td>
<td>0.14</td>
<td>7.50</td>
<td>screed 1 + 5 cant. parts.</td>
<td>0.08</td>
<td>8.50</td>
</tr>
<tr>
<td>false floor layout 2</td>
<td>0.11</td>
<td>7.35</td>
<td>screed 2 + floor layout 1</td>
<td>0.05</td>
<td>9.80</td>
</tr>
<tr>
<td>first screed layer (screed 1)</td>
<td>0.08</td>
<td>8.65</td>
<td>screed 2 + floor layout 2</td>
<td>0.04</td>
<td>9.75</td>
</tr>
</tbody>
</table>
Figure 5.7: Typical response of different slab configurations due to normal walking

5.7.3 Sensitivity analysis

A sensitivity check was carried out for each of the two types of loading excitation to find the range of repeatability of the readings. Here, the accelerometer was placed at a fixed point on the slab and each excitation was repeated five times. The readings obtained should ideally be identical for each case and any differences would give an idea of the range of variability expected within the excitation and measuring system. Table 5.11 shows this range for the three loading cases. As expected, the shaker loading has a small range when compared to walking. Also, vertical walking has a smaller range than horizontal walking as it could be reproduced more accurately during testing.

<table>
<thead>
<tr>
<th>Excitation type</th>
<th>Maximum range</th>
</tr>
</thead>
<tbody>
<tr>
<td>steady-state shaker single sine-wave loading</td>
<td>±5%</td>
</tr>
<tr>
<td>horizontal walking</td>
<td>±31%</td>
</tr>
<tr>
<td>vertical walking</td>
<td>±20%</td>
</tr>
</tbody>
</table>
Figure 5.8: Typical response of different slab configurations due to walking on the spot (vertical walking)

5.8 Quality assurance tests

The experimental preparation and setup phases were described in chapter 4, which adhered to quality assurance guidelines laid out in DTA and ISO recommendations [DTA (1993), ISO7626/5 (1994)], and modified for civil engineering testing by Pavic et al. (1997). Here, checks are carried out on the final output results, which also follow the recommendations noted above.

Immediate repeatability check: For each test, two initial FRFs were acquired, using the selected number of averages, one immediately followed by another. For field tests on civil engineering structures, some differences will be expected between these measurements, mainly due to external noise and the low levels of excitation. In laboratory conditions, however, the external influences are minimal and one would expect the readings to be very close. For the model slab, given the relatively high excitation level and minimal noise ratio, the two FRFs were seen to be identical for all slab configurations.

Homogeneity check: In order to check whether the structure behaves linearly, excitation can be applied at two different force levels and FRFs calculated. For a linear system, two such FRFs
should be identical. For the model slab, excitation was applied at full and half amplitude forcing levels and the ensuing FRFs are shown in figure 5.9. Even though very small non-linearities can be observed, particularly at the fundamental mode, the main characteristics of the FRFs agree very closely with each other. For hammer impact excitation, hard and soft hammer blows were used and again the resulting FRFs were in good agreement.

![Figure 5.9: Homogeneity check on initial results](image)

**Reciprocity check:** This is also another linearity check where the excitation was applied at one point on the slab and response measured at another. The excitation and response points were swapped and the measurement repeated. Maxwell’s theorem states that the pairs of FRFs should be identical [Pavic et al. (1997)] and this was observed for the experimental slab.

**Coherence function check:** As described in section 4.6, coherence checks were carried out for each test. Due to laboratory conditions and high relative excitation levels, coherence readings at peak FRFs were in most cases as high as 98% or above.

**End of test repeatability check:** At the end of each testing phase, an FRF was measured using the exact setup as for the immediate repeatability check. This FRF was then compared to one taken at the start of testing and very close agreement was observed in each case. Any significant differences would have indicated problems with noise, changing environmental conditions, or with characteristics of the structure changing slowly through the test (e.g. due to temperature variations or time-dependent changes in material properties).

The above checks were performed on the experimental results of this chapter. As expected, the results were seen to be of high quality due to laboratory conditions. The above checks would become very useful in field testing conditions, particularly in environments with high external noise and weather fluctuations. Further checks were also performed, such as shaker efficiency, accelerometer sensitivity, and load-cell accuracy; these are described in section 3.7.
Chapter 6

Analyses of results

6.1 Introduction

In this chapter, the results from the model slab tests are analysed with regards to the slab dynamic properties. Analytical models are derived to estimate the dynamic parameters after each alteration in the experiment. The test results are then compared with the estimated values and any implications are discussed. Particular attention is given to the effects on the slab dynamic characteristics after changes in prestressing, addition of false floors, addition of partitions, and the inclusion of high damping screed layers. Analyses of tests with TMDs and the effects of structure-human interaction are dealt with in chapters 7 and 8 respectively.

6.2 Effect of prestress

In this section, the effect of prestressing on the dynamic behaviour of the model slab is reported. The tensioning of the tendons was carried out in two stages of 100kN and 75kN (i.e. 57% and 100% of the design force). The resulting natural frequencies of the bare beam are presented and compared with estimated values, using two different approaches. A theoretical analysis is given to represent the changes. The derived damping ratios and their significance are also discussed.

6.2.1 Natural frequency

There is at present some dispute about the effect of prestressing force on natural frequencies of slabs and bridges. While some researchers regard prestressing as a direct influence on the natural frequency of the system [Saïdi et al. (1994)], others see the influence as an indirect effect related to level of cracking [Deák (1996), Jain and Goel (1996)]. Both these approaches are examined below.
with respect to the model slab results. It is shown that incorporating the \textit{prestressing force} as a direct factor in beam dynamics, leads to a poor agreement with experimental results. The results are best represented by consideration of cracking, using the effective cross-section. A theoretical discussion is presented to support these claims.

\textbf{Slab with axially applied force}

Here, the direct effect of prestressing force on natural frequency is investigated from the viewpoint of beam dynamics. It will be shown that in theoretical analyses, prestressing force has a minimal or negative influence on the natural frequency of beams.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{beam_vibration.png}
\caption{Effect of axial compression on beam vibration}
\end{figure}

Consider a vibrating homogeneous beam subjected to a compressive force, $F$, as shown in figure 6.1. The differential equation of deflection, under static loading, is derived by taking moments about a point at $x$:

$$EI \frac{d^2 u}{dx^2} = M - Fu$$ \hspace{1cm} (6.1)

where $M$ denotes the bending moment produced by the loading of intensity $w$ (i.e. $M = \frac{w x^2}{2}$). By double differentiation of equation 6.1, with respect to $x$, the following is obtained:

$$\frac{d^2}{dx^2} \left( EI \frac{d^2 u}{dx^2} \right) = w - F \frac{d^2 u}{dx^2}$$ \hspace{1cm} (6.2)

Assuming the beam is uniform and in free vibration, the inertial force per unit length is substituted for $w$ to give:

$$EI \frac{\partial^4 u}{\partial x^4} + F \frac{\partial^2 u}{\partial x^2} = -\rho A \frac{\partial^2 u}{\partial t^2}$$ \hspace{1cm} (6.3)

which is the equation of free transverse vibration. Assume a harmonic solution of the form:

$$u = \Phi(x) \cos(\omega t - \phi)$$ \hspace{1cm} (6.4)

where $\Phi(x)$ is the assumed mode shape function. Substituting equation 6.4 into equation 6.3 gives:

$$EI \frac{d^4 \Phi(x)}{dx^4} + F \frac{d^2 \Phi(x)}{dx^2} = \omega^2 \rho A \Phi(x)$$ \hspace{1cm} (6.5)
For a simply supported beam, the mode shape, $\Phi(x)$, will be satisfied by:

$$\Phi_n(x) = \sin \frac{n\pi x}{L} \quad (n=1,2,3, \ldots) \quad (6.6)$$

Substituting this expression into equation 6.5 gives the angular frequency of vibration:

$$\omega_n = \frac{n^2\pi^2}{L^2} \sqrt{\frac{E I}{\rho A}} \sqrt{1 - \frac{F L^2}{n^2 E I \pi^2}} \quad (6.7)$$

It can be deduced that the derived frequency expression yields smaller values than that for a simply supported beam without axial compression (i.e. when $F=0$), so that an increase in compressive force decreases natural frequencies. This is contrary to the experimentally derived values of natural frequency with increasing prestress (table 5.1, page 94). Thus, an opposite trend to theory is observed. Similar results were also found by Saiidi et al. (1994) in both field and laboratory conditions. They attributed the differences to a change in the rigidity, $EI$. Other reported tests have also found prestressed beams to possess higher natural frequencies than equivalent reinforced ones [James et al. (1964), Caverson (1992), Zaman and Boswell (1996)].

Dall’Asta and Dezi (1996) and Deák (1996) argued that equation 6.7 is not a true representation for a post-tensioned beam as the axial force is applied externally in the theory (figure 6.1). Such a force maintains its original line of action during the vibration of a member, thus being converted into an eccentric force with respect to the beam axis. This will reduce the natural frequency and may even lead to buckling. Such a situation does not apply to prestressing cables that are themselves anchored to the end faces of the beam, making the axial force an internal force within the system.

Hence, the effect of prestressing cables in concrete may be better represented by energy considerations. The fundamental mode shape of a simply supported uniform beam, with or without the axial force, can be assumed as sinusoidal (equation 6.6). Thus, its fundamental frequency can be obtained by equating the maximum kinetic energy to the maximum potential energy, under free vibration. Considering the beam of figure 6.1, its motion at the first mode can be expressed by combining equations 6.4 and 6.6:

$$u(x, t) = X \sin \left(\frac{\pi x}{L}\right) \cos(\omega t - \phi) \quad (6.8)$$

where $X$ is the amplitude of vibration at midspan. Kinetic energy during the vibration may be expressed as:

$$KE = \int_0^L \frac{1}{2} \rho A \left(\frac{\partial u}{\partial t}\right)^2 \, dx = \frac{1}{4} \rho A \omega^2 X^2 L \sin^2(\omega t - \phi) \quad (6.9)$$

In the case of an externally applied force, potential energy in the beam will be due to flexural deformation minus the work done by the external force due to the movement of the two ends of
the beam [Jain and Goel (1996)]. It is given by:

\[ PE = \int_0^L \frac{1}{2} EI \left( \frac{\partial^2 u}{\partial x^2} \right)^2 \, dx - F \int_0^L \frac{1}{2} \left( \frac{\partial u}{\partial x} \right)^2 \, dx = \left( \frac{\pi^4 EI X^2}{4L^3} - \frac{\pi^2 FX^2}{4L} \right) \cos^2(\omega t - \phi) \]  

(6.10)

Equating maximum kinetic energy with maximum potential energy, for the case of the externally applied axial force, gives:

\[ \omega^2 = \frac{\pi^4 EI}{\rho AL^4} - \frac{\pi^2 F}{\rho AL^2} \]  

(6.11)

Note that equation 6.11 is the square of equation 6.7. Now consider the case of free vibration of a simply supported beam with internal prestress force. Motion at the first mode will still be given by equation 6.8, which again leads to the kinetic energy expression of equation 6.9. However, since there is no longer an externally applied axial force, the second term in equation 6.10 does not exist. For this case, equating maximum kinetic and potential energies, gives the fundamental frequency of a prestressed beam as:

\[ \omega^2 = \frac{\pi^4 EI}{\rho AL^4} \]  

(6.12)

which is the same as equation 2.8 for a simply supported beam without prestress. Hence, representing the prestressing force as an internal force, as opposed to external axial force, shows no effect on the natural frequency. This is again contrary to the experimental results, which show an increase in natural frequency with increased prestress (table 5.1, page 94).

The above analyses show that equations 6.7 and 6.12 do not provide a basis for theoretical prediction of the variation in natural frequency with changes in prestress. This has led to many researchers suggesting phenomena of different origins, such as cracking, where an increase in prestressing force closes the microcracks in the concrete, hence increasing the natural frequency as a result of higher stiffness. This effect has not been modelled adequately to date [Saidi et al. (1994)]. An attempt at such a model is described in the following section.

**Effect of prestress force on cracking**

Table 6.1 shows the average natural frequencies of the slab in the fundamental mode with varying levels of prestress and dead load. With changes in the prestressing force, there was no discernible effect on the mode shapes. As explained in section 2.4.3, the level of cracking is an important factor in determining the stiffness and natural frequency of the floor. To investigate this effect, a theoretical method for predicting natural frequencies, due to level of prestress, was evaluated. The method takes account of the cracked and uncracked sections of the cross-section, at each stage of prestressing. The change in level of cracking will affect section modulus and second moment of area, thus altering the calculated value of natural frequency.
The effective second moment of area, $I_e$, of a cracked section, averaged over length, depends on the loss of effective depth due to cracking and is given by: [ACI (1995), Kong and Evans (1989)].

$$I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_u + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr}$$

(6.13)

where $M_a$ is the maximum gross moment, $I_u$ is the uncracked second moment of area, $M_{cr}$ is the moment needed to start cracking, and $I_{cr}$ is the cracked second moment of area at the most heavily loaded section. Here, $I_{cr}$ is obtained by replacing the actual area of reinforcement with an equivalent concrete area located at the level of the steel, as shown in figure 6.2. By using this transformed section, $I_{cr}$ is calculated from the position of the cracked neutral axis (see appendix C). Equation 6.13 is valid if $M_{cr}$ is smaller than $M_a$, so that the moment needed to initiate cracking is not larger than the maximum possible moment on the slab. This also means that in a cracked section, $I_e$ is always smaller than $I_u$, and no tensile strength is present in the cracked region.

![Figure 6.2: Cracked transformed section](image)

The bending moment at the onset of cracking, $M_{cr}$, can be found by equating the concrete tensile stress, $f_t$, (dependent on the type of concrete section) with the tensile strength, $\sigma_t$, (found experimentally). For a reinforced beam, the maximum tensile stress at the onset of cracking occurs at the furthest distance from the neutral axis. Equating tensile stress and strength at this point gives:

$$f_t = \sigma_t = \frac{M_{cr} y_t}{I_u}$$

(6.14)

where $y_t$ is the depth to the neutral axis. With prestressing force, the added compression in the tension zone reduces $f_t$, so that a higher moment is required to initiate cracking (see figure 6.3). Equating tensile stress and strength at the furthest point from the neutral axis gives:

$$f_t = \frac{M_{cr} y_t}{I_u} - \frac{P_t}{b h} - \frac{P_t e y_t}{I_u}$$

(6.15)

where $P_t$ is the total prestressing force in tendons and $e$ is the tendon eccentricity. Equation 6.15 also shows that if the section is already cracked, increasing the prestressing force has the effect of
reducing the level of cracking, due to the increased compression and an increase in $M_{cr}$. This will lead to stiffening of the system and an increase in its natural frequency. Typical calculations of $M_{cr}$ and $I_c$ in the uncracked, cracked, and partially cracked states are shown in appendix C.

![Diagram of reinforced and prestressed sections](image)

Figure 6.3: Stresses present in uncracked reinforced and prestressed sections

Once the effective second moment of area for each prestressing stage is obtained, it can be used to calculate fundamental frequency using the EBM, static deflection, or Concrete Society methods (see section 2.5.3). Typical calculations of the model slab fundamental frequency are shown in appendix D. Table 6.1 shows the experimental values as compared with predicted values, with and without consideration of cracking.

<table>
<thead>
<tr>
<th>Slab State</th>
<th>$\sigma_{cr}$ (MPa)</th>
<th>$f_0$(exp) (Hz)</th>
<th>Estimates of $f_0$ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EB M</td>
<td>Static def.</td>
<td>Conc. soc.</td>
</tr>
<tr>
<td>Baseline case - bare slab full prestress</td>
<td>5.2</td>
<td>8.0</td>
<td>8.0</td>
</tr>
<tr>
<td>bare slab 57% prestress</td>
<td>3.0</td>
<td>7.6</td>
<td>8.0</td>
</tr>
<tr>
<td>bare slab no prestress</td>
<td>9</td>
<td>7.5</td>
<td>8.0</td>
</tr>
<tr>
<td>Extra live load of 0.25kN/m² full prestress</td>
<td>5.2</td>
<td>7.7</td>
<td>7.7</td>
</tr>
</tbody>
</table>

In their original form, the estimation methods assume the slab to be uncracked and give inaccurate predictions for cracked sections. By revising the methods to take cracking into account, acceptable agreement is observed between estimated and actual results. This indicates that the changes in natural frequency with level of prestress are due largely to changes in the amount of cracking. However, with the partially cracked case, the calculations in appendix C show that the entire section is effective and no cracking exists. This was not reflected in the experimental results. The reason could be the long duration of time between the prestressing stages, and possible prestress losses, leading to changes in the section properties of the slab. The theoretical estimates when fully prestressed and with no prestress, give a very good match with the experimental results when cracking is considered. Hence, it can be stipulated that by using the above mentioned method, the
fundamental frequency of a floor slab may be predicted approximately at a given prestressing level. However, more evidence is needed to qualify the theory.

### 6.2.2 Damping

The damping of the slab is greatly affected by the prestressing force, due to closing up of microcracks. Table 6.2 shows the average damping values obtained for each prestressing stage (from tables 5.1-5.2). With an increase in prestress and fewer cracks within the slab, there is less energy dissipation and hence lower damping. This is discussed more in section 2.6.3.

**Table 6.2: Comparison of experimental damping values with effective second moment of area after cracking**

<table>
<thead>
<tr>
<th>Slab State</th>
<th>( \sigma_{y} ) (MPa)</th>
<th>( \zeta ) (exp) (% critical)</th>
<th>percent change</th>
<th>( I_{e} ) (( \times 10^{-4} ) m(^4))</th>
<th>percent change</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Baseline case - bare slab</td>
<td>5.2</td>
<td>1.1</td>
<td>-</td>
<td>2.2</td>
<td>-</td>
</tr>
<tr>
<td>full prestress</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) bare slab</td>
<td>3.0</td>
<td>1.7</td>
<td>+55% of (i)</td>
<td>2.2</td>
<td>no change</td>
</tr>
<tr>
<td>57% prestress</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iii) bare slab</td>
<td>0</td>
<td>1.9</td>
<td>+73% of (i)</td>
<td>1.9</td>
<td>-14% of (i)</td>
</tr>
<tr>
<td>no prestress</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(iv) Extra live load of 0.25kN/m(^2)</td>
<td>5.2</td>
<td>1.2</td>
<td>+9% of (i)</td>
<td>2.2</td>
<td>no change</td>
</tr>
<tr>
<td>full prestress</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.2 also shows the percentage change in damping as a result of prestress, compared with the change in effective second moment of area calculated previously. An increase in the level of prestress has a severe detrimental effect on the damping of the slab, as compared with a small change in its effective section. This can be attributed to the actual number of cracks involved, rather than their individual depths. Again, the calculations show no change in \( I_{e} \) between the partially prestressed and fully prestressed stages. This is not reflected in the experimental results and is attributed mainly to prestress losses and the time spent between the prestressing stages, which could have the effect of increasing the number of cracks.

With the application of live load, the section modulus is unchanged but an increase in damping is observed, again due to increased bending and appearance of more microcracks in the tension zone.

### 6.3 Effect of false flooring

As explained in section 3.6.4, two false floor layouts were tested, differing in number of panels and their nature of installation. Full results are given in chapter 5 and appendix A. Table 6.3 shows the average slab fundamental frequencies and damping ratios with the two false floor configurations.
Table 6.3: Effect of false flooring on slab vibration properties (fundamental mode)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Fundamental frequency (Hz)</th>
<th>Damping (% critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bare slab (fully tensioned)</td>
<td>8.0</td>
<td>1.10</td>
</tr>
<tr>
<td>Layout 1 total of 7 panels (all rigidly fixed)</td>
<td>7.9</td>
<td>1.17</td>
</tr>
<tr>
<td>Layout 2 total of 14 panels (fixed at centres, loose at corners)</td>
<td>7.6</td>
<td>1.33</td>
</tr>
</tbody>
</table>

It can be seen that in both cases the weight of the panels leads to reductions in natural frequency, depending on the number of panels used. This reduction may be derived theoretically, where the mass of the panels can be added to the overall mass of the slab. From Table 6.3, the predictions agree well with experimentally derived values, indicating that the changes in natural frequency are consistent with the panels simply acting as uniform added mass.

Consideration of changes in damping, shows a small increase with layout 1 as compared with a 66% increase with layout 2 configuration. It thus appears that when rigidly attached, the floor panels add mass to the structure and do not introduce any significant additional stiffness or damping. However, when some edges simply rest on the pedestals, as in layout 2, there is a noticeable increase in damping. This is most likely caused by sliding and/or gapping between the panels and pedestals. It raises the possibility that false floors could be designed to allow this relative motion as a deliberate energy dissipation device. Note that with some false flooring systems, which use heavy panels typically made from timber, all the floor panels rest on the pedestals and rely on their self-weight for stability. In such cases, the overall damping of the slab could be further increased, as compared to layout 2 of the model tests.

6.4 Effect of non-structural partitions

As described in section 3.6, two types of partitions were tested, in the form of cantilever and full-height representations, to simulate those commonly used in offices. Full test results are included in chapter 5 and appendix A. Table 6.4 summarises the effects of the partitions on the model slab vibration parameters. The results suggest a wide difference between the effects of cantilever, as opposed to full-height, partitions. These are described separately in the following sections.
Table 6.4: Effect of partitions on slab vibration properties in the fundamental mode (see figure 3.12, page 60, for partition positions)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Fundamental frequency (Hz)</th>
<th>Damping (% critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>experimental</td>
<td>% change</td>
</tr>
<tr>
<td>bare slab (fully tensioned)</td>
<td>8.0</td>
<td>-</td>
</tr>
<tr>
<td><strong>Layout 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>five cantilever partitions (perpendicular to slab span)</td>
<td>7.7</td>
<td>-3.8</td>
</tr>
<tr>
<td><strong>Layout 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>three cantilever partitions (parallel to slab span)</td>
<td>8.0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Layout 3</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>five perpendicular and three parallel to slab span</td>
<td>7.8</td>
<td>-2.5</td>
</tr>
<tr>
<td><strong>Layout 4</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>one full-height partition (perpendicular to slab span)</td>
<td>20.3</td>
<td>+154</td>
</tr>
<tr>
<td><strong>Layout 5</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>five cantilever and one full-height (all perpendicular to slab span)</td>
<td>20.3</td>
<td>+154</td>
</tr>
</tbody>
</table>

6.4.1 Effect of full-height partitions

Table 6.4 shows a significant change in the slab vibration parameters, after the addition of full-height partitions at the underside of the model slab. Considering the case of one partition at midspan, when rigidly attached at top and bottom, it effectively acts as a line support giving the slab fundamental mode shape of figure 6.4. When compared with the mode shape of the bare slab (figure 5.4, page 96), the added stiffness at the midspan can be seen. This is confirmed by the measurements of fundamental frequency, where an increase of over 150% was observed.

![Fundamental mode shape of slab with a full-height partition at midspan](image)

To assess the behaviour of the partition itself, its horizontal response was measured. Figure 6.5 shows a typical transfer function of the partition at its centre, compared with that of the slab. Note that the fundamental frequencies are identical and the transfer functions are broadly similar in shape, which indicates that the slab and partition act compositely at all frequencies. In addition, figure 6.6 shows the first two measured mode shapes of the full-height partition, indicating a bowing action. Hence, the experiments show that the changes to the slab dynamic behaviour are due to
the partition acting as a flexible, vertical support whose primary mode of deformation is lateral bowing under the vertical load, as shown in figure 6.7.

![Figure 6.5: Comparison between FRF of full-height partition in horizontal direction with that of slab in vertical direction](image)

Figure 6.5: Comparison between FRF of full-height partition in horizontal direction with that of slab in vertical direction

![Figure 6.6: First two mode shapes of full-height partition](image)

Figure 6.6: First two mode shapes of full-height partition

These results illustrate that some non-structural additions on a floor can in fact behave in a structural way and add extra stiffness to the main system, as well as increase the inherent damping. If well designed, this is an inexpensive way of alleviating annoying vibrations on a problematic floor.
Analytical model approximation

To create an analytical model for the effect of a full-height partition, consider the added slab stiffness by representing the partition as a spring with stiffness, $k_p$, placed at a distance, $(a)$, from the support. This leads to the slab behaving as a constrained structure as shown in figure 6.8.

![Figure 6.8: Analytical model approximation of a full-height partition](image)

A common approach for defining the response of a constrained structure is to formulate the problem in terms of generalised co-ordinates and use the mode summation technique. First, the normal modes and mode shapes of the unconstrained structure (without partition/spring) are used to derive the generalised parameters, and then the effect of the spring is taken into account.

Unconstrained beam

Consider a simply supported beam with applied loading of $p(x, t)$, as shown in figure 6.9. The overall deflected shape of the structure, $u(x, t)$, can be approximated as the sum of $n$ mode shapes, $\Phi_i(x)$, multiplied by a time coefficient, $q_i(t)$:

$$u(x, t) = \sum_{i=1}^{n} \Phi_i(x)q_i(t)$$  \hspace{1cm} (6.16)

where $q_i(t)$ is called the generalised co-ordinate, which is effectively the vibration amplitude at a point on the structure (e.g. the midspan of a simply supported beam).

The generalised co-ordinate can be used to analyse the problem by considering the beam as a SDOF system (figure 6.9) with the governing equation:

$$M_i^*\ddot{q}_i + K_i^*q_i = F_i^*$$  \hspace{1cm} (6.17)

In order for the SDOF system to be equivalent to the continuous structure, appropriate values must be chosen for the generalised mass, $M_i^*$, stiffness, $K_i^*$, and force, $F_i^*$. This can be achieved using
energy considerations. For derivation of $M_i^*$, the kinetic energy of the mass of the beam is found over its entire length: (see equation 2.11, page 24)

$$KE = \int_0^L \frac{1}{2} wu'^2 \, dx = \sum_{i=1}^n \frac{1}{2} q_i^2 M_i^* \quad \text{where} \quad M_i^* = w \int_0^L \Phi_i'^2 \, dx \quad (6.18)$$

Similarly, $K_i^*$ is found by considering the strain or potential energy in the structure over its length: (see equation 2.12, page 24)

$$PE = \int_0^L \frac{1}{2} EIu''^2 \, dx = \sum_{i=1}^n \frac{1}{2} q_i^2 K_i^* \quad \text{where} \quad K_i^* = EI \int_0^L \Phi_i''^2 \, dx \quad (6.19)$$

Note that $u'$ denotes differentiation with respect to $t$ and $u''$ denotes differentiation w.r.t $x$. Finally, for the unconstrained beam with uniformly applied load, $p(x,t)$, $F_i^*$ is derived from the work done by the force:

$$WD = \int_0^L pudx = \sum_{i=1}^n q_i F_i^* \quad \text{where} \quad F_i^* = p \int_0^L \Phi_i \, dx \quad (6.20)$$

For equations 6.18-6.20, the exact modes, $\omega_i$, and mode shapes, $\Phi_i$, can be calculated by considering the beam as a continuous system. These can be written as: (see equation 2.8, page 23)

$$\omega_i = \pi^2 \sqrt{\frac{EI}{wL^4}} \quad \text{and} \quad \Phi_i(x) = \sin \left( \frac{i\pi x}{L} \right) \quad (6.21)$$

Constrained beam

At this stage, the effect of the full-height partition can be introduced by considering the force, due to the spring in figure 6.8, as a concentrated point load applied at $(a)$ (see figure 6.10). In such a case, the generalised mass and stiffness of the beam are unaffected but the generalised force, $F_i^*$, must be updated from a distributed load to a concentrated point load, $F_{pi}^*$.

Consideration of the work done at $(a)$, due to the point load, gives:

$$\Delta WD = p(a,t) \Delta u(a,t) = p(a,t) \sum_{i=1}^n \Phi_i(a) \Delta q_i(t)$$

$$\Rightarrow \quad \frac{F_{pi}^*}{q_i} = \frac{\Delta WD}{\Delta q_i} = p(a,t) \Phi_i(a) \quad (6.22)$$
In terms of spring stiffness, $k_p$, equation 6.22 can be expressed as:

$$p(a,t) = -k_p u(a,t) = -k_p \sum_{i=1}^{n} \Phi_i(a) q_i(t)$$

$$\Rightarrow F_{p_i}^* = -k_p \Phi_i(a) \sum_{i=1}^{n} \Phi_i(a) q_i(t)$$  \hspace{1cm} (6.23)

Note here that for the case of the constrained beam, the same modes are used as for the unconstrained structure. This is justified since the spring is treated as an external load and it therefore has no influence on the beam free vibration characteristics. Hence, the response of the beam with the spring is thought of as a linear combination of the modes of the beam without the spring.

Substituting equations 6.18, 6.19 and 6.23 into equation 6.17 and rearranging gives:

$$\ddot{q}_i(t) + \omega_i^2 q_i(t) = \frac{1}{M_i^*} \left[ -k_p \Phi_i(a) \sum_{i=1}^{n} \Phi_i(a) q_i(t) \right]$$  \hspace{1cm} (6.24)

where the terms outside the summation refer to the behaviour of the $i^{th}$ mode of the unconstrained beam, and for the constrained beam, the $i^{th}$ mode is the summation of all the modes of the unconstrained beam. By assuming that the SDOF system vibrates at some frequency, $\Omega$, related to the spring stiffness, and that the response is harmonic, the solution can be written in the form:

$$q_i = \bar{q}_i e^{i\Omega t}$$  \hspace{1cm} (6.25)

Substituting into equation 6.24 and rearranging gives:

$$\ddot{\bar{q}}_i = \frac{1}{M_i^* (\omega_i^2 - \Omega^2)} \left[ -k_p \Phi_i(a) \sum_{i=1}^{n} \Phi_i(a) \bar{q}_i \right]$$  \hspace{1cm} (6.26)

In equation 6.26, with $n$ modes, there will be $n$ values of $\bar{q}_i$ and $n$ equations. The determinant formed by the coefficients of $\bar{q}_i$ will lead to the natural frequencies of the constrained modes. The mode shapes of the constrained structure are then found by substituting the $\bar{q}_i$ into equation 6.16.

Equation 6.26 is simplified by substituting for $\Phi_i$ from equation 6.21:

$$\ddot{\bar{q}}_i = \frac{1}{M_i^* (\omega_i^2 - \Omega^2)} \left[ -k_p \sin \left( \frac{i\pi a}{L} \right) \sum_{i=1}^{n} \sin \left( \frac{i\pi a}{L} \right) \bar{q}_i \right]$$  \hspace{1cm} (6.27)

For the purposes of this discussion, the fundamental mode is the only mode of interest, and hence for a single mode approximation, equation 6.27 reduces to:

$$M_1^* (\omega_1^2 - \Omega^2) = -k_p \sin^2 \left( \frac{\pi a}{L} \right)$$  \hspace{1cm} (6.28)
Chapter 6: Analyses of results

from which the first natural frequency of the constrained beam can be found, where:

\[ \Omega^2 = \omega_1^2 + \frac{k_p}{M_1^*} \sin^2 \left( \frac{\pi a}{L} \right) \]  

(6.29)

In the experimental programme, the partition was placed at the slab midspan, so taking \( a = \frac{L}{2} \), \( M_1^* = 806 \text{kg} \) (see appendix B), \( \omega_1 = (2\pi \times 8.0) \text{rads/s} \) (table 6.4), and \( k_p = 7.52 \times 10^6 \text{N/m} \) (see appendix E), equation 6.29 gives a value for the constrained slab frequency of 17.3Hz. This compares with the experimentally derived value of 20.3Hz. The difference can probably be attributed to the relatively large error range in \( k_p \) (see below and appendix B).

The main assumption in the model is that the base of the partition is fixed to the ground without allowance for any ground motion. In real buildings, the partition would be fixed to the floor above, which would itself be prone to vibrations. Hence, there would be some ground motion at both ends of the partition. In the experimental tests, the difference in mass between the supporting ground and the slab was large, hence the ground could be assumed as completely fixed.

**Application of the model**

The simulated full-height partition, used in these experiments, had a total height of 400mm with an average derived axial stiffness of 7.52 \times 10^6 \text{N/m}. In real floors, the partition height would be expected to range from say 2m to 3.5m. Assuming the change in stiffness is linearly proportional to partition length, (as found experimentally in appendix B to within 15%), this would lead to a reduction in stiffness. Table 6.5 gives typical suggested values for the stiffness of gypsum plasterboard partitions. The figures given are purely based on a few experimental results and should only be taken as a rough guideline. The error ranges given are in accordance with those found experimentally. The material properties of plasterboard are dependent on many factors, including handling history, internal damage, quality of gypsum, and the manufacturing process.

<table>
<thead>
<tr>
<th>Partition thickness (mm)</th>
<th>Partition height (mm)</th>
<th>Predicted stiffness (N/m) ± 15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>2000</td>
<td>( 1.5 \times 10^6 )</td>
</tr>
<tr>
<td></td>
<td>3500</td>
<td>( 8.6 \times 10^5 )</td>
</tr>
<tr>
<td>12</td>
<td>2000</td>
<td>( 2.0 \times 10^6 )</td>
</tr>
<tr>
<td></td>
<td>3500</td>
<td>( 1.1 \times 10^6 )</td>
</tr>
<tr>
<td>18</td>
<td>2000</td>
<td>( 3.0 \times 10^6 )</td>
</tr>
<tr>
<td></td>
<td>3500</td>
<td>( 1.7 \times 10^6 )</td>
</tr>
<tr>
<td>25</td>
<td>2000</td>
<td>( 4.2 \times 10^6 )</td>
</tr>
<tr>
<td></td>
<td>3500</td>
<td>( 2.4 \times 10^6 )</td>
</tr>
</tbody>
</table>
Refering to figure 6.8, for any simply supported slab, equation 6.29 can be further simplified to:

$$f_{0c}^2 = f_0^2 + \frac{k_p}{2\pi^2 \rho bhL} \sin^2 \left( \frac{\pi a}{L} \right)$$  \hspace{1cm} (6.30)

where $f_{0c}$ is the constrained fundamental frequency with the partition, and $f_0$ is the fundamental frequency without the partition. Figure 6.11 gives plots of equation 6.30 for the model slab at quarter and midspan points. Note the effect of partition stiffness on the overall slab frequency.

![Figure 6.11: Plot of equation 6.30 for the model test slab](image)

**Proposed design guideline**

Given a particular floor, a simple design guideline can be utilised to assess the effects, on the floor fundamental frequency, of adding full-height partitions. The suggested design steps are:

**Step 1:** Find the slab fundamental frequency, $f_0$, by experiment or calculation.

**Step 2:** Decide on the target fundamental frequency of the slab, $f_{0c}$, after the addition of full-height partitions. For man-induced vibrations, table 2.5 (page 44) gives recommended values.

**Step 3:** Knowing the position along the span at which the partition is to be placed, $(a)$, use equation 6.30 to arrive at a desirable stiffness, $k_p$, for the partitions.

**Step 4:** For gypsum partitions, table 6.5 gives suggested thicknesses for a given height. With knowledge of their axial stiffness, the use of other types of partition can also be investigated.

- For two-way slabs, the span to be considered is the one perpendicular to the partition
- If using more than one partition, the added stiffness would be higher than in equation 6.30 and $f_{0c}$ will be expected to be increased.
• The steps given above can be used in reverse form, where the type and dimensions of the partitions would be known, leading to a value of stiffness from table 6.5. The increase in slab frequency can then be derived from equation 6.30.

6.4.2 Effect of cantilever partitions

Table 6.4 shows that the direction of the cantilever partitions, with respect to the slab span, affects their contribution to slab vibration. When positioned parallel to the slab span, they were seen to have no effect on the vibration behaviour of the floor. When positioned perpendicular to the span, a noticeable increase in damping was observed. This is most likely due to the dissipation of energy through axial deformations and swaying of the partitions. The biggest increase in damping was observed when the partitions were placed in both directions and joined together.

To investigate the behaviour of individual partitions during vibration, response readings were taken at five points on each partition. In these tests, sine-sweep and single sine-wave excitations were imposed on the slab and the accelerometer was placed on the partitions in both vertical and horizontal directions. Figure 6.12 shows a typical transfer function in the horizontal direction as compared with that of the slab in the vertical direction. Note that the partition exhibits a mode at the fundamental frequency of the slab. The other peaks for the partition are not present in the slab. This indicates that at the slab fundamental frequency, the cantilevers act compositely with the floor, whereas in other modes, they act as separate systems sitting on the floor.

![Graphs showing FRF comparison](image)

Figure 6.12: Comparison between FRF of cantilever partition in horizontal direction (partition 3, layout 1) with that of slab in vertical direction, following sine-sweep loading applied on the slab

Figure 6.13 shows the horizontal acceleration responses of all partitions measured at the free edge (i.e. top of the partitions). Here, it is observed that swaying takes place at all partitions and is
more apparent for partitions which are away from midspan, where the rotation of the slab is highest (i.e. partitions 1 and 5 in figure 3.12).

![Graphs showing acceleration responses at free edge of cantilever partitions](image)

**Figure 6.13:** Acceleration responses at free edge of cantilever partitions (layout 1 of table 6.4), following single sine-wave loading of slab at its fundamental frequency. [see figure 3.12, page 60 for partition positions]

Figure 6.14 shows the acceleration responses of the partitions when arranged as layout 3 (table 6.4), where the individual perpendicular partitions are joined by parallel ones. This has the effect of the plasterboard acting as a single unit and aids the transfer of energy between the partitions. Hence, it is observed that the acceleration responses of individual partitions are significantly increased, when compared with layout 1. This leads to increased swaying of all partitions and, as a result of added energy dissipation, the overall damping of the slab is significantly increased.

The axial deformations of the partitions were investigated by measuring the response at the top of the partitions in the vertical direction. These effects were found to be minimal and were ignored.

**Analytical model representation**

In general, cantilever partitions are very light compared to the slab and so are unlikely to have a significant effect on the slab’s natural frequencies and mode shapes. This was observed by comparing the slab mode shapes with and without the cantilever partitions, which were identical. Also, from table 6.4, the effect of the cantilever partitions on slab fundamental frequency can be
seen to be minimal. However, the experimental results show some swaying of the partitions, which will dissipate energy and hence will add to the overall damping of the system.

In this section, it is assumed acceptable to model the partition in isolation rather than modelling the entire system. This is justifiable because of the very large mass difference, as explained above. Hence, the motion of a cantilever partition can be analysed by considering a fixed-free beam with a sinusoidal rotation applied at the base. As shown in figure 6.15, such a beam can be divided into a cantilever undergoing free vibration, and a rigid beam undergoing base rotation. In the following section, the response of the beam in figure 6.15(c) is derived using generalised coordinates and the mode summation procedure.

The undamped free vibration equation of motion of such a system, figure 6.15(a), is derived by considering the forces acting on an elemental length of the beam, giving:

\[ EI \frac{\partial^4 u_f}{\partial x^4} + \rho A \frac{\partial^2 u_f}{\partial t^2} = 0 \]  \hspace{1cm} (6.31)

where \( u_f \) is the free vibration displacement. Taking the rigid beam of figure 6.15(b), with a rotation at base of \( \theta = \bar{\theta} \sin(\Omega t) \), the displacement of the beam at a point \( x \) will be:

\[ y = \theta x = \bar{\theta} x \sin(\Omega t) \]  \hspace{1cm} (6.32)

Hence, with the inclusion of this base rotation, (figure 6.15(c)), equation 6.31 becomes:

\[ EI u'''' + \rho A [\ddot{y} + \ddot{u}] = 0 \]  \hspace{1cm} (6.33)
where \( u' \) and \( \dot{u} \) imply differentiation with respect to \( x \) and \( t \) respectively. Rearranging equation 6.33 gives:

\[
EIu''' + \rho A \ddot{u} = -\rho A \ddot{y}
\]

(6.34)

This can be solved by approximating the overall response of the beam, \( u(x, t) \), as the sum of \( n \) mode shapes, \( \Phi_i(x) \), multiplied by a time function, \( q_i(t) \) (see equation 6.16). For a cantilever beam, the mode shapes are given by:

\[
\Phi_i(x) = C \{ \cosh(\beta_i x) - \cos(\beta_i x) - \alpha_i [\sinh(\beta_i x) - \sin(\beta_i x)] \}
\]

(6.35)

where \( C \) is an arbitrary amplitude constant and

\[
\beta_i^4 = \frac{\omega_i^2 \rho A}{EI} \quad \text{and} \quad \alpha_i = \frac{\cosh(\beta_i L) + \cos(\beta_i L)}{\sinh(\beta_i L) + \sin(\beta_i L)}
\]

(6.36)

As for full-height partitions, the principle of generalised coordinates can be used, where the beam is approximated to an equivalent SDOF system, (figure 6.15), represented by equation 6.17. The generalised mass, \( M_i^* \), is derived from equation 6.18 and the generalised force is: (see equation 6.20)

\[
F_i^* = \int_0^L -\rho A \ddot{y}(x) \Phi_i(x) dx = \int_0^L -\rho A \ddot{x} \Phi_i(x) dx
\]

(6.37)

Hence, substituting \( M_i^* \) and \( F_i^* \) into equation 6.17 and rearranging, gives the equation for the generalised coordinate \( q_i \):

\[
\ddot{q}_i + \omega_i^2 q_i = \frac{F_i^*}{M_i^*} = \frac{\rho A \Omega^2 \dot{\theta} \sin(\Omega t)}{M_i^*} \int_0^L x \Phi_i(x) dx
\]

(6.38)
which can be written as:

\[ \ddot{q}_i + \omega_i^2 q_i = J_i \sin(\Omega t) \quad \text{where} \quad J_i = \frac{\rho A \Omega^2 \bar{\theta}}{M_i} \int_0^L x \Phi_i(x) \, dx \] (6.39)

Using integration by parts and equation 6.35, the term in the integral is:

\[ \int_0^L x \Phi_i(x) \, dx = C \left[ \left( -L \alpha_i - \frac{1}{\beta_i} \right) \cosh(\beta_i L) \cos(\beta_i L) + \left( L + \frac{\alpha_i}{\beta_i} \right) \sinh(\beta_i L) - \left( L - \frac{\alpha_i}{\beta_i} \right) \sin(\beta_i L) + \frac{\alpha_i}{\beta_i} \right] \] (6.40)

The steady-state solution of equation 6.39 will be harmonic and can be expressed as: \[ q_i = Q_i \sin(\Omega t) \]. Its substitution into equation 6.39 yields:

\[ Q_i = \frac{J_i}{\omega_i^2 - \Omega^2} \quad \text{hence:} \quad q_i = \frac{J_i}{\omega_i^2 - \Omega^2} \sin(\Omega t) \] (6.41)

To express response in terms of acceleration, substitution of equation 6.41 into 6.39 gives:

\[ \ddot{q}_i = J_i \sin(\Omega t) \left( 1 - \frac{\omega_i^2}{\omega_i^2 - \Omega^2} \right) \] (6.42)

so that the total acceleration response of the partition, due to a rotation at the base, is given by:

\[ \ddot{u}(x, t) = \sum_{i=1}^n \ddot{q}_i(t) \Phi_i(x) = \sum_{i=1}^n J_i \Phi_i(x) \sin(\Omega t) \left( 1 - \frac{\omega_i^2}{\omega_i^2 - \Omega^2} \right) \] (6.43)

For the experimental gypsum plasterboard, four-point loading tests gave an average EI value of 1671 Nm², and the density of the partitions was obtained from their mass to be 584 kg/m³. These values had an error range of approximately ±10%. Figure 6.16 shows a typical model response at the free end, assuming the vibration history of the slab as the applied rotation. Note that the frequency of the response is dominated by the frequency of slab vibration. Figure 6.17 shows the response profile of the partition, as derived by the model.

![Model base loading](image1)

![Free end response](image2)

Figure 6.15: Model response at the free end of cantilever with base motion
To further investigate the effect of base motion, the free vibration response of a cantilever beam, with identical material properties, was derived using equation 6.35 as the mode shape. Figure 6.18 shows a comparison of the FFTs of the beam with and without base motion. Notice that the frequency of the slab, $\Omega$, governs the vibration response of the partition.

With respect to the experimental results, as illustrated in figure 6.19, the amplitude of base rotation is dependent on the position of the partition along the slab. This can be found from the measured vertical acceleration response of the slab. Assuming a mode shape function for the slab in the form $\Phi(x) = \sin\left(\frac{nx}{L}\right)$, and that the slab displacement response is given by $u = U \sin \omega t$, the response at any point, $x$, along the slab would be $v = u \sin \left(\frac{n \pi x}{L}\right)$. Hence, the rotation of the slab at $x$ is dependent on the slope and can be expressed as:

$$\theta = \omega' = \frac{\omega \pi}{L} \cos \frac{\pi x}{L} = \frac{U \pi}{L} \cos \frac{\pi x}{L} \sin \omega t$$

so that

$$\bar{\theta} = \frac{U \pi}{L} \cos \frac{\pi x}{L}$$

(6.44)
where $U$ can be found from the measured acceleration responses of the slab at midspan. From equation 6.44, it can be deduced that at slab midspan (i.e. $x = \frac{L}{2}$), the motion is purely vertical with no apparent rotation amplitude (i.e. $\dot{\theta} = 0$). At other points along the slab, the rotation applied at the base of each partition can be derived.

Given the midspan vibration amplitude for the slab of figure 6.13, the base rotation applied to each cantilever partition and the model acceleration response of the partition at its free edge are shown in figure 6.20. For comparison, the experimentally measured responses of figure 6.13 are also plotted for each partition. It can be seen that the model prediction of the responses is generally satisfactory, with best agreement obtained for partitions 4 and 5. Possible causes for discrepancies in the results could be the range of variability of the measured plasterboard stiffness and density values, and the representation of fixed boundary conditions in the experiments. Note that for the partition at midspan, part 3, although the model gives zero response, experiments show some swayng of the partition. This could be because the partition was not exactly at slab midspan, hence experiencing a small amount of base rotation.

From the above results, this model is seen to be capable of predicting, to an acceptable degree, the amount of swaying which can take place as a result of an applied rotation at the base of the cantilever partition. The applied rotation is itself dependent on the slab response and the positioning of the partition with respect to the fundamental mode shape of the slab.

From the experimental and model results, it is evident that the placement of the cantilever partitions along the slab has an influence in their lateral swaying response. The larger the response, the higher the energy dissipation and the greater the likelihood of increased overall slab damping, as seen in table 6.4. The results have also shown that the response of the cantilever partitions is greater at a point on the slab where higher rotational base motion is applied. The analytical model has shown that the frequency of slab vibration has a governing influence on the natural modes of the partitions. Finally, experiments have shown that if the cantilever partitions are connected to each
other, transfer of energy takes place from ones with higher response to ones with lower response, increasing the overall energy absorption of the whole partition system. This increase is seen to be higher than the responses of the partitions when not connected (see figures 6.13 and 6.14).

6.5 Effect of viscoelastic screed layers

As explained in section 3.6.7, two 25mm layers of concrete screed, treated with Concredamp viscoelastic additive, were cast on the model slab. In this section, results from vibration tests on the treated slab are presented. An analytical model is devised to represent the changes following the addition of the layers. First, the material characteristics of the treated concrete are presented.
6.5.1 Screed layer properties

As with the model slab, cube and cylinder samples were cast following the mixing of the Concredamp screed layers. Table 6.6 illustrates the results obtained, which show a reduction in strength depending on the concentration of Concredamp and the amount of water in the mix. Reductions in Young's modulus and density were also observed. During measurements of Young's modulus for the first screed layer, it was found that the specimen cylinder was defective, hence readings for this case were inaccurate and are not included in table 6.6.

The low values of concrete strength and Young's modulus were attributed to problems encountered in the casting process (see section 3.6.7, page 63). Although this was also observed by others [Weiss (1989), O'Rourke & Son Ltd. (1993)], they conducted trials and managed to obtain the correct mix of lower strength but the right Young's modulus. This was not achieved during these tests and the difference in results described here bear no reflection on the usability of Concredamp, but may mainly be due to the casting and pouring procedures used in the laboratory.

In addition, vibration tests were conducted on sample beams to derive their damping characteristics. These tests are explained in section 4.7 and the results are presented in table 6.6, which show an increase in damping following the addition of Concredamp, depending on concentrations used.

<table>
<thead>
<tr>
<th>Test</th>
<th>Average concrete cube strength (N/mm²)</th>
<th>Average Young's modulus (kN/mm²)</th>
<th>Average density (kg/m³)</th>
<th>Average beam damping (% critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) untreated concrete (as per model slab)</td>
<td>45</td>
<td>25.8</td>
<td>2540</td>
<td>4.65</td>
</tr>
<tr>
<td>(ii) screed layer 1 Concredamp conc. of 50l/m³</td>
<td>20</td>
<td>not available</td>
<td>1986</td>
<td>4.70 [+3% of (i)]</td>
</tr>
<tr>
<td>(iii) screed layer 2 Concredamp conc. of 89l/m³</td>
<td>29</td>
<td>17.3</td>
<td>2160</td>
<td>4.92 [+6% of (i)]</td>
</tr>
</tbody>
</table>

6.5.2 Effect on model slab vibration

Results of tests, with the screed layers on the slab, are given in chapter 5 and appendix A. Table 6.7 is a summary of the average natural frequency and damping measurements. Note that the screed has the effect of increasing the cross-sectional area of the slab, and hence may be regarded as a structural addition. This is clearly seen by the increase in natural frequency, which is consistent with predicted values using the EBM (see appendix D).
### Table 6.7: Effect of screed layers on slab vibration properties in the fundamental mode

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Fundamental frequency (Hz)</th>
<th>Damping (% critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) bare slab (fully tensioned)</td>
<td>8.0</td>
<td>1.10</td>
</tr>
<tr>
<td>(ii) bare slab with first screed layer</td>
<td>8.8</td>
<td>1.15</td>
</tr>
<tr>
<td>(iii) bare slab with first and second screed layers</td>
<td>10.1</td>
<td>1.25</td>
</tr>
</tbody>
</table>

The theoretical values in table 6.7 are based on the screed layers being fully structural integrated onto the slab; the section was then transformed to that of the slab. The material properties used for both layers are that of the second layer, given in table 6.6. Experimentally, the screed layers have differing properties and the bond between the screed and the main slab was not fully established, which leads to a reduction in natural frequency, since the screed acts as dead load. The assumption that the screed layers are exactly 25mm thick, could also lead to differences between predicted and experimental frequencies. In reality, the thickness varied along the slab by ±10mm.

Also from table 6.7, it is observed that the damping values of the slab increase as a result of screed addition. However, the increase is less than was expected. This may be due to the properties of the treated concrete mix used in the tests. In the following section, a model is derived to represent the addition of viscoelastic layers on a concrete floor slab.

### 6.5.3 General considerations and theoretical model

#### Possible types of viscoelastic treatment

The behaviour of a viscoelastic material may be explained by considering the properties of elastic and viscous materials separately. In a perfectly elastic material, stress is proportional to strain \((\sigma = E\varepsilon)\) so that the system is conservative and the energy stored is independent of the rate of loading. In a perfectly viscous material, however, stress is a function of the rate of change of strain \((\sigma = k\dot{\varepsilon})\) so the energy stored depends on the rate of loading. In an actual material, the stress is a combination of the elastic and viscous behaviour \((\sigma = E\varepsilon + k\dot{\varepsilon})\). Here, \(k\dot{\varepsilon}\) is a measure of the damping of the system. Normally \(k\dot{\varepsilon}\) is small and it is common to assume a structure to act elastically with some inherent viscous damping, hence the term \(k\dot{\varepsilon}\) is ignored. However, in some materials, the viscous term can be substantial and should not be neglected. These types of material are known as viscoelastic and possess high internal damping. When viscoelastic materials are used for vibration control, they are mainly applied in layers and are subjected to shear or direct strains.
The use of viscoelastic layers to increase damping is mainly confined to mechanical engineering applications, which involve high intensity sources of wide frequency band excitation, such as rocket or jet aircraft noise. Most practical configurations may be considered as variations of two basic arrangements. In the first case, the viscoelastic (dissipative) material is applied in a uniform layer to a sheet or bar of a relatively nondissipative material, such as metal. When the composite is bent, the viscoelastic layer deforms essentially in extension and compression. Thus, it stores energy and supplies damping, due primarily to this mechanism. This type of viscoelastic layer is called a free or unconstrained layer and its damping action is termed extensional damping. It is the type attempted on the model concrete slab, although the viscoelastic behaviour of the screed layers was seen to be minimal due to the reasons explained in the previous section. The second type of treatment can be more effective, with higher composite damping, and is termed constrained layer or shear damping. Here, a viscoelastic layer lies between, and is rigidly connected to, two elastic structures. Bending of this composite causes the viscoelastic layer to deform primarily in shear.

Application of viscoelastic treatment in civil engineering to damp vibrations is not very common, mainly because the amplitudes are small. In cases where it has been utilised, excellent results have been obtained [Nelson (1968), Farah et al. (1977), Ibrahim and Farah (1978)]. For floor slabs, the constrained layer treatment has been adopted on composite construction only, where the viscoelastic material has been applied at the bottom flange of the steel beams. The case of free layer treatment on concrete floors is less common and only one successful application has been reported [Moiseev (1991)]. In the following section, a theoretical analysis, using energy considerations, is presented to assess the effectiveness of such treatment on concrete floor slabs.

**Free layer damping analysis**

Figure 6.21 shows an elementary length of a two layered composite structure with different material properties for each layer. Layer 2 in the figure can be regarded as the screed in the model tests, while layer 1 can represent the model concrete slab. As the tests showed the screed to have relatively little viscoelastic behaviour (see section 6.5.1), the following analysis assumes both layers to undergo elastic deformation with some inherent viscous damping. Hence, the energy stored due to the bending of each layer is assumed to be elastic strain energy only, with pure viscous behaviour ignored.

Assuming full bonding between the two layers of figure 6.21, the neutral axis CC of the composite
Figure 6.21: A two layered composite structure showing the neutral axis (CC) and the centroidal axis of each component (AA for layer 1 and BB for layer 2)

The system is given by:

\[
\bar{y} = \frac{h_1 b_2^4 + \frac{E_2}{E_1} h_2 b \left( h_1 + \frac{h_2}{2} \right)}{h_1 b + \frac{E_2}{E_1} h_2 b} = \frac{E_1 h_1^4}{2 E_1 h_1 + E_2 h_2} \quad (6.45)
\]

Under bending, the strain at a point \( y \) in the composite is expressed as \( \epsilon = \frac{y}{R} \), where \( R \) is the radius of curvature. Similarly, the stress at \( y \) is given by \( \sigma = E \epsilon = \frac{Ey}{R} \). Hence, the stored strain energy at \( y \) due to the bending of an elementary volume \( dV \) is:

\[
S = \int \frac{1}{2} \sigma \epsilon dV = \int \frac{1}{2} \frac{E y^2}{R^2} dV \quad (6.46)
\]

Now consider the energy stored in each individual layer where:

\[
S_1 = \int_{-g}^{h_1-g} \frac{1}{2} \frac{E_1 y^2}{R^2} bLdy \rightarrow S_1 \frac{L}{L} = \frac{1}{2} \frac{E_1}{R^2} \int_{-g}^{h_1-g} y^2 dA = \frac{1}{2} \frac{E_1 I_1}{R^2} \quad (6.47)
\]

and

\[
S_2 = \int_{h_1-g}^{h_1+h_2-g} \frac{1}{2} \frac{E_2 y^2}{R^2} bLdy \rightarrow S_2 \frac{L}{L} = \frac{1}{2} \frac{E_2}{R^2} \int_{h_1-g}^{h_1+h_2-g} y^2 dA = \frac{1}{2} \frac{E_2 I_2}{R^2} \quad (6.48)
\]

where the second moments of area are about the neutral axis CC of the composite section. With reference to figure 6.21, these are:

\[
I_1 = I_{1,CC} = I_{1,AA} + bh_1 a^2 \quad \text{and} \quad I_2 = I_{2,CC} = I_{2,BB} + bh_2 c^2 \quad (6.49)
\]

Having obtained the energy stored in each layer, the overall damping of the composite can be taken as a weighted average of the damping ratios of the individual elements [Ungar (1963)], such that:

\[
\zeta = \frac{\sum (\zeta_i S_i)}{\sum S_i} \quad (6.50)
\]

where \( S_i \) is the energy stored by the \( i^{th} \) component and \( \zeta_i \) is its damping ratio. Substitution of equations 6.47 and 6.48 into 6.50 will give the overall damping of the two layered composite as:

\[
\zeta = \frac{\zeta_1 E_1 I_1 + \zeta_2 E_2 I_2}{E_1 I_1 + E_2 I_2} \quad (6.51)
\]
In the context of the experimental Concredampl screed layers, equation 6.51 can be regarded as a representation of the two screed layers on the slab. From table 6.6, assuming both layers to have properties equal to those of screed layer 2, figure 6.22 shows the original slab with and without the screed layers and a summary of the values involved in the analysis. In the figure, $\zeta_2$ is extrapolated from the given values of tables 6.6 and 6.7.

![Figure 6.22: Summary of slab and screed layer properties](image)

Substituting the relevant parameters into equation 6.51, gives an overall damping with both screed layers of 1.12% critical. This compares with an experimentally derived value of 1.25% (table 6.7). The relatively large discrepancy would be expected, as this analysis does not account for the effects of cracking due to the addition of the screed layers. The extra mass of the layers would increase the microcracks in the main slab, hence increasing its overall damping (see section 6.2.2). Other reasons for the difference in the values could be due to imperfect bonding of screed layer and slab, and the variations in the thicknesses of the layers. In addition, difficulties encountered in the casting process of the Concredampl layers could affect the results (see section 6.5.1).

Ideally, for the dissipative layer to be fully effective it must possess viscoelastic behaviour such that $\zeta_2$ would be much larger than $\zeta_1$, hence increasing the overall damping of the composite. A far better method, than the free layer treatment, is the use of a constrained viscoelastic layer between the slab and another elastic layer. In this way, use can be made of energy loss due to shear. The use of such systems has had particular attention in mechanical engineering applications [Kerwin (1959), Mead and Markus (1969), Rao (1978)]. However, no reports are available on the implementation of a constrained viscoelastic layer to damp vibrations of concrete slabs. This can lead to the possibility of research on such an application, and is described further in section 10.4.
6.6 Model slab accelerations

In this section, the derived acceleration measurements of chapter 5 are analysed, with respect to current perceptibility guidelines, and the different floor configurations are classified. The response of the model floor was measured following three types of loading, namely: continuous sinusoidal vibration at the fundamental frequency, heeldrop, and walking excitations. For each test, the acceleration readings are plotted in the CSA (1989) and [ISO2631/2 (1989), BS6472 (1984), ANSI S3.29 (1983)] vibration perceptibility graphs (see section 2.2.2).

6.6.1 Continuous vibration tests

These tests are explained in section 5.7.1, where the slab was excited at its fundamental frequency with a loading of more than 20 cycles. It can hence be regarded as a continuous vibration in the perceptibility guidelines. The derived acceleration amplitudes are given in table 5.8 (page 98), for each tested slab configuration. For most cases, the excitation was imposed at two differing amplitudes, varying from ±5N to ±30N (see table A.35, appendix A). To obtain a representative acceleration response, a loading of ±15N is assumed for the perceptibility guides of this section. In this way, a consistent set of acceleration readings may be derived for each slab configuration.

Figures 6.23-6.29 show the results obtained by plotting the slab accelerations in the two guidelines at each stage. The values of fundamental frequency are average readings taken from tables 5.1-5.6, (page 94). In the CSA (1989) guide, readings above the continuous vibration iso-perceptibility line are deemed to cause unsatisfactory vibration. In the ISO/BSI/ANSI curves, the acceptability of the floor is dependent on type of use, and hence reference must be made to table 2.2 (page 15), for classification of the slab.

Note that the slab was originally designed as a slender structure, exhibiting annoying vibrations. Figures 6.23-6.27 show that when fully tensioned, the slab is classified as unsatisfactory by both guides. The effect of prestressing can be seen in figure 6.23, where the acceleration response increases by 385% due to an increase of prestressing force of 75%. This illustrates the detrimental effect of post-tensioning on the vibration serviceability of concrete floors.

After each change to the slab configuration, the addition of non-structural components is seen to reduce the acceleration amplitudes by varying amounts. Figure 6.24 shows that false floor layout 2, with some loose panels, reduces response by 73% as opposed to 35% for layout 1 with rigid panels. Note that layout 2 has double the mass of layout 1. By far the most significant improvement to
response is seen following the addition of full-height partitions (figure 6.25), where in both guides the slab becomes acceptable for residential use. Significant improvement is also observed with a stationary occupant on the slab (figure 6.26). Some combinations of components are also given in figures 6.28 and 6.29, which again show improvements to slab acceptability.

The overall impression from figures 6.23-6.29 is that apart from the case of full-height partitions, the model floor falls in the unsatisfactory range in both guides, for the particular loading case. However, the level of unacceptability varies between the two guides. The CSA (1989) guide is essentially relevant to quiet or residential occupancies, while the ISO/BSI/ANSI guide applies to occupancies ranging from critical working areas (base curve) to workshops (8×base curve) and as such, the classification of the floor lies within a large range.

In general, the effect of continuous vibration should be dealt with at the source. The perceptibility guides become more useful when considering occupant activities on a floor, such as walking or jumping, which are intermittent or transient vibrations. Hence, the acceleration readings, given for continuous vibration at the slab fundamental frequency, will be much higher than those encountered within the structure due to occupant activities. Since the loading amplitude of ±15N has a significant effect on the response, these results should be viewed more with respect to the changes caused by non-structural components, than with the response readings.

Figure 6.23: Assessment of the effect of prestressing force in perceptibility guides
Figure 6.24: Assessment of the effect of false flooring in perceptibility guides

Figure 6.25: Assessment of the effect of partitions in perceptibility guides

Figure 6.26: Assessment of the effect of stationary occupant in perceptibility guides
Figure 6.27: Assessment of the effect of screed layers in perceptibility guides

Figure 6.28: Assessment of perceptibility due to combination of components on first screed layer
6.6.2 Walking vibration tests

As explained in section 5.7.2, these tests were conducted as horizontal walking and on-the-spot walking. The resulting RMS acceleration amplitudes are given in tables 5.9 and 5.10 (page 99), and plotted in the perceptibility guides in figures 6.30 and 6.31. Note that in general, slightly higher responses are recorded as a result of walking on-the-spot. This would be expected as a larger vertical force component is being input onto the slab. In the CSA scales, the slab damping iso-perceptibility lines can be used, as the walking was generally less than 10 paces. Taking an average damping for the model slab of $\zeta = 1.5\%$, it is observed that the slab is classified as unsatisfactory for two of the four conditions in these scales.

With reference to table 2.2 (page 15), the ISO/BSI/ANSI curves give a range of occupancy for the floor before its vibration performance can be classified. Assuming walking as an intermittent vibration source, these scales are seen to be more severe than the CSA scales. This is because the CSA scales are based on heeldrop loading, which imposes a larger force compared to walking. In general, the ISO/BSI/ANSI curves are seen to be a more suitable assessment method for walking as they take account of the loading, and are applicable to different occupancy types.
Figure 6.30: Classification of perceptibility scales from horizontal walking

Figure 6.31: Classification of perceptibility scales from walking on the spot
6.6.3 Heeldrop loading tests

This is a transient type of vibration on which the CSA (1989) curves are based. The floor response results following heeldrop loadings are given in table 5.7 (page 97). Figure 6.32 shows these measurements plotted in the perceptibility guides. Again, the CSA guide classifies the slab as unsatisfactory for residential or quiet occupancies. From table 2.2 (page 15), for transient loading, the ISO/BSI/ANSI curves classify the slab as satisfactory for office and workshop environments and also for some residential occupancies during daytimes. Hence, the biggest difference in the classification of the two scales, is observed in the heeldrop tests. For the ISO/BSI/ANSI curves, table 2.2 shows that a large range exists for the acceptability due to transient loading, particularly for residential occupancies. The choice of a suitable base curve weighting factor depends on the location and the amount of outside activity, and as such can be very arbitrary.

![Figure 6.32: Classification of perceptibility scales from heeldrop loading](image-url)
Chapter 7

A simple TMD for concrete floors

7.1 Introduction

As explained in section 2.6.5, tuned mass dampers, TMDs, have mainly been used in buildings and bridges [McNamara (1977), Bachmann (1992a)]. For maximum effect, the use of TMDs would be recommended in situations where the primary structure has high displacement amplitude and low damping and where annoying vibrations occur due to the forcing frequency being in resonance with one distinct mode of the structure only. Many floor slabs, however, exhibit closely spaced modes of vibration with relatively low displacement. In addition, since the TMD consists of a mass that should be placed near the point of maximum displacement (i.e. middle of simply supported floor), this raises problems with its location on a floor. Hence, the application of TMDs on floor slabs has not been widespread and problematic floors have generally been treated by stiffening the structure. Some applications of TMDs on floors have been reported [Setareh and Hanson (1992b), Webster and Vaicaitis (1992)] with reductions in vibration amplitude of up to 78%. However, almost all of these cases have involved cantilever type floors, such as balconies, with high displacement at the free end. In these cases, the TMD was placed in a cabinet at the free edge.

In the research programme described herein, experiments have been performed in an attempt to apply a TMD system on a simply supported slab at its mid-span. The idea was originally proposed by Allen and Pernica (1984) for composite slabs and is modified and extended here for post-tensioned concrete floors. The results show acceptable levels of vibration reduction, as described in section 7.3. Two possible mathematical models for such a system are also proposed, which take the form of a numerical time-stepping method and a closed-form analytical solution. Predictions of the experimental results, using these models, were seen to be acceptable to varying degrees. Following correlations between the experimental and model results, a possible design guideline is presented, which is applicable to any problematic floor.
7.2 Description of the proposed TMD system

The vibration absorbers used were in the form of flat plywood sheets simply supported from the underside of the slab, with weights on top. The weights were placed at the mid-span of the plywood, where maximum displacement occurred. The TMD configurations used were:

- one sheet (1.2m×1m×10mm) hereafter referred to as 'widely'
- two sheets of widely on top of each other with nothing in-between
- two sheets of widely on top of each other with polythene in-between
- two sheets of widely on top of each other with hard rubber in-between
- one sheet (1.2m×0.3m×10mm) hereafter referred to as 'narrowly'
- two sheets of narrowly on top of each other with nothing in-between
- two sheets of narrowly on top of each other with polythene in-between
- two sheets of narrowly on top of each other with hard rubber in-between

The purpose of having different material in-between sheets of ply was to assess the changes in damping that may occur as a result of altering the friction between the layers.

7.2.1 Effect of TMDs on floor vibration

A vibration absorber is an oscillator of much smaller mass, \( m_2 \), than that of the floor structure, \( m_1 \), but with the same natural frequency. Figure 7.1 shows the resulting natural vibration of an idealised floor system following an initial velocity on the slab, equivalent to a single footstep impulse. Figure 7.1(a) shows the undamped natural vibration of the idealised floor system without a vibration absorber. When a vibration absorber is attached to the floor and tuned to its troublesome frequency, the energy of natural vibration of the floor is transferred back and forth between the floor and the absorber. Figure 7.1(b) shows this transfer of energy for an undamped system with a tuned TMD, which shows that a beating vibration of the slab is produced due to this energy transfer.

All real TMDs exhibit some amount of damping. If the damping in the absorber, \( \zeta_2 \), is the same as the damping in the floor, \( \zeta_1 \), no more energy is dissipated than would be without the absorber and although a better performance is observed, the maximum vibration amplitudes during beating are not decreased, (figures 7.1(c)and(d)). If the damping in the absorber is greater than that in
the floor, more energy is dissipated between beats and the absorber becomes much more effective in damping the vibration of the floor, as shown in figure 7.1(e). If damping in the absorber is very large, however, it resists displacement of the absorber mass and therefore prevents the transfer of vibration energy into the absorber, i.e. the absorber again becomes ineffective (figure 7.1(f)).

![Effect of TMD on natural vibration of slab](image)

**Figure 7.1**: Effect of TMD on natural vibration of slab

### 7.2.2 Design of the proposed plywood TMDs

In general, floor slabs with fundamental frequencies of between 3-12Hz are prone to annoying vibrations due to normal human activities. For fundamental frequencies greater than this, natural vibration from footsteps is generally not annoying, because it decays very rapidly. Below this range, TMDs become less effective, because they cannot control the gradual build-up of vibration due to resonance between footstep frequency and natural frequency. The factors involved in the design of the plywood TMD system are described below.

#### Estimation of floor properties

The properties of the floor, in terms of theoretical lumped mass and stiffness parameters, are first calculated from known values of the concrete material properties and slab dimensions. To find the generalised lumped mass, $M_1^*$, and stiffness, $K_1^*$, of the slab, the principle of virtual displacements
can be used where a generalised co-ordinate describes the system. Hence, by idealising the slab as
a simply supported beam with an assumed fundamental mode shape, \( \Phi = \sin(\pi x/L) \), generalised
slab mass and stiffness can be written as:

\[
M_1^* = \int_0^L \rho A \Phi^2 dx = \rho A \frac{L}{2}
\]

(7.1)

\[
K_1^* = \int_0^L EI \Phi'' dx = \frac{EI \pi^4}{2L^5}
\]

(7.2)

and the estimate of the slab natural frequency is simply \( \omega_0 = \sqrt{\frac{K_1^*}{M_1^*}} \). For the model slab, \( M_1^* \) and
\( K_1^* \) are calculated in appendix B.

**Estimation of TMD properties**

These properties can be found from the value of mass ratio, \( \mu = \frac{M_2^*}{M_1^*} \), where \( M_2^* \) is the generalised
mass of the absorber. The mass ratio can typically vary within a range of 0.01 to 0.1 depending
on the mass of the absorber and the required overall slab damping ratio. In general, a heavier
absorber can be more effective in the reduction of floor vibrations, but there exists a limit at which
any increase in TMD mass has little effect on the vibration reduction. In the experiments described,
adding weights on the plywood could alter \( M_2^* \), hence the value of \( \mu \) could be changed. By assuming
a starting arbitrary \( \mu \), the generalised mass of the absorber is derived by \( M_2^* = \mu M_1^* \). Since it is
required for the absorber to have a natural frequency equal to that of the slab (i.e. \( \frac{K_1^*}{M_1^*} = \frac{K_2^*}{M_2^*} \)), the
value of generalised TMD stiffness, \( K_2^* \), can also be found.

**Damping in slab and absorber**

In designing sophisticated TMDs for buildings and bridges, an optimum damping ratio for the
absorber is derived, which depends on the chosen mass ratio. Adequate viscous or Coulomb damping
material will then be added within the TMD system to provide the required value of damping. For
the simple plywood TMD system described here, it would be difficult to provide known values of
damping in the TMD. It must however be possible to achieve this using viscous damping material
in-between layers of plywood, but this was not tried here. In these tests, using plywood sheets of
different size and number altered the damping of the TMD system. In cases where two layers of
plywood were used, friction between planks was assumed to increase damping in the absorber. The
level of friction was altered by placing polythene or hard rubber between the sheets.
Design of planks

After estimation of the absorber weight, the chosen plank stiffness should ensure that the natural frequency of the absorber is equal to the troublesome floor frequency. By assuming the plywood as simply supported with the weights at mid-span, an estimate of mid-span deflection can be made using standard formulae. When related to natural frequency, (equation 2.6, page 23) gives:

\[ \Delta_s = \left( \frac{15.76}{f_0} \right)^2 \]  (7.3)

where \( \Delta_s \) is the mid-span deflection in mm. If the absorber mass is uniformly distributed, the relationship becomes:

\[ \Delta_s = \left( \frac{17.75}{f_0} \right)^2 \]  (7.4)

Hence, having already obtained a value for \( f_0 \), coarse tuning of the absorber can be carried out by placing weights on the plywood, in order to achieve a deflection within the range of equations 7.3 and 7.4. During the experimental procedure, it was found that placing weights at mid-span of the plywood sheets gave better results if the weights were evenly spread by equal amounts across the width of the TMD.

Tuning of the TMD

Once coarse tuning of the TMD was accomplished, according to equations 7.3 and 7.4, fine tuning could be carried out by adjusting weights and checking the absorber frequency in one of two ways. The first is by observing the FRF of the slab when subjected to a sine-sweep. By comparing the FRFs at different weights, the best TMD mass can be obtained. The second method is to shake the slab at its natural frequency so that if the TMD is properly tuned, resonance vibrations may be observed. It would in fact be better to shake the TMD and look for resonance vibration of the slab, but the experimental apparatus at hand did not make this option a possibility.

7.3 Experimental results

An extensive experimental programme was carried out on the model post-tensioned concrete floor slab, in order to test the viability of the proposed TMD system. As explained in section 7.2, two different sizes of plywood were used with different configurations of single and double layered TMDs. Table 7.1 gives an outline of the different TMD arrangements, and the masses required at mid-span to tune them. The loading used was in the form of heel-drop and sine-sweep excitations.
The responses were taken on several points on the slab and TMD, at each configuration and loading condition. The given values represent averages of these points at each test.

<table>
<thead>
<tr>
<th>TMD configuration</th>
<th>Mass of plywood (kg)</th>
<th>Mass at midspan (kg)</th>
<th>Generalised TMD mass (kg)</th>
<th>Mass ratio (generalised)</th>
<th>TMD centre deflection (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 1</td>
<td>one layer of widely (1.2m x 1m x 10mm)</td>
<td>10</td>
<td>20</td>
<td>25</td>
<td>0.031</td>
</tr>
<tr>
<td>Test 2</td>
<td>two layer widely (nothing in-between)</td>
<td>19.5</td>
<td>bad tuning</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Test 3</td>
<td>two layer widely (polythene in-between)</td>
<td>19.5</td>
<td>25</td>
<td>34.75</td>
<td>0.043</td>
</tr>
<tr>
<td>Test 4</td>
<td>two layer widely (rubber in-between)</td>
<td>19.5</td>
<td>23</td>
<td>32.75</td>
<td>0.040</td>
</tr>
<tr>
<td>Test 5</td>
<td>one layer of narrowly (1.2m x 0.3m x 10mm)</td>
<td>3.7</td>
<td>10</td>
<td>11.85</td>
<td>0.015</td>
</tr>
<tr>
<td>Test 6</td>
<td>two layer narrowly (nothing in-between)</td>
<td>7.4</td>
<td>20</td>
<td>23.7</td>
<td>0.029</td>
</tr>
<tr>
<td>Test 7</td>
<td>two layer narrowly (polythene in-between)</td>
<td>7.4</td>
<td>20</td>
<td>23.7</td>
<td>0.029</td>
</tr>
<tr>
<td>Test 8</td>
<td>two layer narrowly (rubber in-between)</td>
<td>7.4</td>
<td>15</td>
<td>18.7</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Note that in cases where two layers of plywood were used with nothing in-between, the TMD could not be tuned satisfactorily. This maybe due to the high amount of friction that exists between these layers, which does not allow sufficient energy dissipation in the TMD. This is because the vibration forces on the TMD are smaller than the limiting friction force between the two layers. By putting polythene or rubber between the sheets, a reduction in friction force was achieved, which allowed energy dissipation. Here, the experimental results are discussed.

**Test 1 - One layer of Wideply:** Figure 7.2 shows FRF plots of the bare slab without TMD, and the application of a TMD in the form of wideply. These plots were obtained from sine-sweep loading tests on the slab, as explained in section 4.5. It can be seen from figures 7.2(a) and 7.2(b) that the transfer function magnitude decreases by nearly 70% as a result of the TMD addition. Figure 7.2(c) shows that the response of the TMD itself is seven times greater than that of the slab. Figure 7.2(d) illustrates the slab response with an out-of-tune TMD. A 10% change in TMD mass leads to approximately 20% rise in response.

The responses of the slab and TMD to heel-drop loading are shown in figure 7.3. It can be seen from figure 7.3(a) that the inclusion of the TMD noticeably increases the damping capacity of the slab. In comparing the responses of the TMD and the slab, figure 7.3(b), note that the vibration amplitude of the TMD is around six times that of the floor.

**Test 2 - Two layers of wideply (nothing in-between):** As mentioned in table 7.1, it was not possible to arrive at a satisfactorily tuned TMD for this case. It was observed, however, that with higher weights the amplitude of response was greatly reduced.
Test 3 - Two layers of wideply (polythene in-between): FRF plots of this case are shown in figure 7.4. Figure 7.4(c) shows that more damping is taking place within the TMD system leading to a faster decay of slab response, as also observed from the heel-drop plots of figure 7.5. In assessing the sensitivity to accurate tuning, figure 7.4(d), it is seen that a 20% change in TMD mass leads to approximately 25% increase in slab response.

Test 4 - Two layers of wideply (hard rubber in-between): In this case, the slab response was seen to decrease six fold at the fundamental frequency, and the damping decay of the slab increased in the same way as with the polythene. The TMD response was around four times as much as that of the slab.
Figure 7.4: FRF plots of TMD as two layers of widely with "polythene" in-between (Test 3)

Figure 7.5: Heeldrop responses with two layers of widely (polythene in-between) TMD (Test 3)

**Test 5 - One layer of Narrowply:** With shortening the width of the plywood, the value of mass ratio is reduced, since less weight is needed for TMD tuning. Figure 7.6 shows that the slab response is reduced as before, but that the TMD response is much higher than with widely. The sensitivity analysis of figure 7.6(d) shows that a 10% change in tuning weight leads to a 20% increase in slab response. Heel-drop plots of figure 7.7 also show a high TMD response and relatively low damping.

**Test 6 - Two layers of narrowply (nothing in-between):** As with widely, the TMD could not be satisfactorily tuned for this case. However, with 20kg of weight, it was possible to reduce the vibrations by around 70%.
**Test 7 - Two layers of narrowly (polythene in-between):** The results for this case are shown in figures 7.8 and 7.9, where the tuning of the TMD was more satisfactory. This is illustrated by the sensitivity analysis of figure 7.8(d). Also, the width of the tuned peak is very broad suggesting that the polythene has aided slippage between the plywood layers, increasing energy dissipation. This leads to an increase in damping at the TMD, as shown in figure 7.9(b).

**Test 8 - Two layers of narrowly (hard rubber in-between):** Less weight was required to tune the TMD for this case, as shown in table 7.1. Comparisons with other cases are made in the following section.
7.3.1 Comparisons of TMD configurations

Case A - One or two layers? Figure 7.10(a) shows a heel-drop plot of slab response when it has one layer of TMD superimposed on a graph of a two layer configuration with polythene in-between. It is deduced that for both widely and narrowly, the slab response decays faster with a two layer TMD. This is evident since a two-layered TMD has been shown to have higher inherent damping, as seen in figure 7.10(b).
**Case B - Wideply or Narrowply?** Wideply and narrowply TMDs are compared in figure 7.11, where for the slab response with wideply there is smaller initial peak and faster decay of vibrations. The difference in response of the plywood in each case is very significant. Figure 7.11(b) shows that the amplitude of vibration for the wideply case is more than half of that for the lighter TMD, and the transfer period of energy is also much shorter. This energy transfer is dependent on the mass ratio, where with a lower $\mu$, it takes longer for the transfer to occur.

**Case C - Nothing or polythene between two layers of ply?** This comparison is made in figure 7.12. It can be seen that with polythene, the decay is marginally increased. This is attributed to more movement at the contact surface, causing more energy dissipation. The polythene in effect aids the damping of the two-layered TMD.

**Case D - Nothing or rubber between two layers of ply?** With rubber in-between, figure 7.13(a) shows a marginally faster decay of vibrations on the slab level. On the TMD response of figure 7.13(b), there is no significant difference.
Figure 7.12: Comparison of having "nothing" or "polythene" between two layers of TMD (Case C)

Figure 7.13: Comparison of "nothing" or "hard rubber" between layers (Case D)

**Case E - Polythene or rubber between two layers of ply?** Figure 7.14 shows the slab and TMD response due to these two conditions. No significant difference is observed and any small changes may be due to the difference in masses needed to tune the TMD, see table 7.1.

Figure 7.14: Comparison of "polythene" or "hard rubber" between layers (Case E)
Summary of experimental results

- two layers of ply give faster decay
- widely shows faster decay of slab vibrations as opposed to narrowly
- narrowly has twice the response at TMD level compared to widely
- with polythene between, slab decay is marginally faster than with nothing in-between
- with rubber in-between, as opposed to nothing, marginally faster rate of decay is observed at both slab and TMD
- polythene and rubber have similar effect on decay of slab and TMD

7.4 Structure-TMD models

Here, two theoretical models are presented in order to simulate the behaviour of the slab and the TMD. These are in the form of a closed form solution of a 2DOF damped lumped mass system, and a numerical time-stepping analysis of the same problem. Diagrammatic representation of the structure-TMD system, used in the models, is shown in figure 7.15. Comparisons are made with the experimental results to validate each model. The time-stepping model is seen to best represent the experimental results. Hence, it is updated to other slab configurations and used to propose a design guideline for the plywood TMD system.

Figure 7.15: 2DOF lumped mass-spring-damper model of slab-TMD system

7.4.1 Closed-form solution

The free vibration equation of motion of the system in figure 7.15 is given in matrix form by:

\[ \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \end{bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 \end{bmatrix} \begin{bmatrix} \dot{u}_1 \\ \dot{u}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]  

(7.5)
where subscripts 1 and 2 represent the slab and TMD respectively. Now assume solutions of the general form:

\[ u_1 = X_1 e^{st} \quad \text{and} \quad u_2 = X_2 e^{st} \]  \hspace{1cm} (7.6)

where \( s \) is the complex frequency. Substitution of equations 7.6 and their derivatives into equation 7.5, results in the following eigenvalue problem:

\[
\begin{bmatrix}
    m_1s^2 + (c_1 + c_2)s + (k_1 + k_2) & -c_2s - k_2 \\
    -c_2s - k_2 & m_2s^2 + c_2s + k_2
\end{bmatrix}
\begin{bmatrix}
    X_1 \\
    X_2
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    0
\end{bmatrix}
\]  \hspace{1cm} (7.7)

For a non-trivial solution, the determinant of the matrix in equation 7.7 must be zero, leading to a fourth degree polynomial:

\[
m_1m_2s^4 + [m_1c_2 + m_2(c_1 + c_2)]s^3 + [m_1k_2 + c_1c_2 + m_2(k_1 + k_2)]s^2
+ (c_1k_2 + c_2k_1)s + k_1k_2 = 0
\]  \hspace{1cm} (7.8)

For the system to vibrate freely, damping must be small and all nonzero roots of equation 7.8 will be complex. They occur in conjugate pairs that may be expressed as:

\[ s_{11}, s_{12} = -\zeta_1\omega_1 \pm i\omega_1 \quad \text{and} \quad s_{21}, s_{22} = -\zeta_2\omega_2 \pm i\omega_2 \]  \hspace{1cm} (7.9)

By substituting the roots of equation 7.9 into equation 7.7, the corresponding amplitude ratios can be obtained:

\[ r_{ij} = \frac{c_2s_{ij} + k_2}{m_1s_{ij}^2 + (c_1 + c_2)s_{ij} + (k_1 + k_2)} = \frac{m_2s_{ij}^2 + c_2s_{ij} + k_2}{c_2s_{ij} + k_2} \]  \hspace{1cm} (7.10)

where \( i = 1 \) or 2 and \( j = 1 \) or 2. The resulting ratios \( r_{11}, r_{12} \) and \( r_{21}, r_{22} \) are complex conjugate pairs. The complete solution can then be written as:

\[ u_1 = r_{11}X_{11}e^{s_{11}t} + r_{12}X_{12}e^{s_{12}t} + r_{21}X_{21}e^{s_{21}t} + r_{22}X_{22}e^{s_{22}t} \]

\[ u_2 = X_{11}e^{s_{11}t} + X_{12}e^{s_{12}t} + X_{21}e^{s_{21}t} + X_{22}e^{s_{22}t} \]  \hspace{1cm} (7.11)

in which the coefficients \( X_{11}, X_{12} \) and \( X_{21}, X_{22} \) are complex conjugate pairs determined from initial conditions. The first two terms of \( u_2 \) in equation 7.11 can be converted into equivalent trigonometric expressions by writing:

\[ X_{11}e^{s_{11}t} + X_{12}e^{s_{12}t} = e^{-\zeta_1\omega_1 t}(C_1 \cos \omega_1 t + C_2 \sin \omega_1 t) \]  \hspace{1cm} (7.12)

where \( C_1 = X_{11} + X_{12} \) and \( C_2 = i(X_{11} - X_{12}) \) are real constants. Similarly for \( u_1 \), the amplitude ratios can be expressed in terms of their real and imaginary parts:

\[ r_{11} = a + ib \quad , \quad r_{12} = a - ib \quad , \quad r_{21} = c + id \quad , \quad r_{22} = c - id \]  \hspace{1cm} (7.13)
giving the first two terms of $u_1$ as:

$$r_{11}X_{11}e^{s_{11}t} + r_{12}X_{12}e^{s_{12}t} = e^{-\zeta_1\omega_1 t} [(C_1 a - C_2 b) \cos \omega_1 t + (C_1 b + C_2 a) \sin \omega_1 t]$$  \hspace{1cm} (7.14)

This conversion process can be repeated throughout equation 7.11 giving the total solution to the problem as:

$$u_1 = e^{-\zeta_1\omega_1 t} [(C_1 a - C_2 b) \cos \omega_1 t + (C_1 b + C_2 a) \sin \omega_1 t]$$

$$+ e^{-\zeta_2\omega_2 t} [(C_3 c - C_4 d) \cos \omega_2 t + (C_3 d + C_4 c) \sin \omega_2 t]$$

$$u_2 = e^{-\zeta_1\omega_1 t} [C_1 \cos \omega_1 t + C_2 \sin \omega_1 t] + e^{-\zeta_2\omega_2 t} [C_3 \cos \omega_2 t + C_4 \sin \omega_2 t]$$  \hspace{1cm} (7.15)

where $C_1 \rightarrow C_4$ are determined from initial conditions. For the case of a floor subjected to footsteps, a heel-drop excitation is a suitable representation of loading. Here, the heel-drop is applied as an initial velocity, $V$, on the model slab. Hence, the initial conditions to equation 7.15 are:

$$u_1 = u_2 = \dot{u}_2 = 0 \quad \text{and} \quad \dot{u}_1 = V \quad \text{at} \quad t = 0$$  \hspace{1cm} (7.16)

With application of these conditions, the solutions for $C_1 \rightarrow C_4$ can be obtained:

$$C_1 = \frac{C_2 b + C_4 d}{a - c} \quad C_2 = \frac{-\alpha_4 C_4}{\alpha_3}$$

$$C_3 = -C_1 \quad C_4 = \frac{\alpha_3 V}{\alpha_3 \alpha_2 - \alpha_1 \alpha_4}$$  \hspace{1cm} (7.17)

where $\alpha_1 \rightarrow \alpha_4$ are constants depending on the natural frequencies and damping ratios:

$$\alpha_1 = \frac{\omega_1^2 - \zeta_1 \omega_1 ab - \omega_2 bd + \zeta_2 \omega_2 bc}{a - c} + \omega_1 a + \zeta_1 \omega_1 b$$

$$\alpha_2 = \frac{\omega_1 bd - \zeta_1 \omega_1 ad - \omega_2 d^2 + \zeta_2 \omega_2 cd}{a - c} + \omega_2 c + \zeta_2 \omega_2 d$$

$$\alpha_3 = \frac{b (\zeta_2 \omega_2 - \zeta_1 \omega_1)}{a - c} + \omega_1$$

$$\alpha_4 = \frac{d (\zeta_2 \omega_2 - \zeta_1 \omega_1)}{a - c} + \omega_2$$  \hspace{1cm} (7.18)

A MATLAB programme was written to evaluate this solution. Figure 7.16 shows plots of this model using experimental values of $m_1, k_1, m_2, k_2$ and $\zeta_1$ with different values of $\zeta_2$ and a fixed value of $V$. Notice that with increased TMD damping, the slab response decays faster until a limit of optimum TMD damping is reached. This is the limit of useability of this model, since the model is only valid for low damping values and the response ceases to be vibratory with high damping.

### 7.4.2 Numerical model

Having shown that cases of high TMD damping cannot be clearly defined by a closed form solution, a numerical time-stepping method was hence implemented. The model is still a damped 2DOF
lumped mass system of figure 7.15 but this time the equations of motion are solved using the linear acceleration method [Clough and Penzien (1993)]. The equations of motion can be written in incremental form as:

\[
\begin{bmatrix}
m_1 & 0 \\
0 & m_2
\end{bmatrix}
\begin{bmatrix}
\frac{\Delta \ddot{u}_1}{\Delta t} \\
\frac{\Delta \ddot{u}_2}{\Delta t}
\end{bmatrix}
+ \begin{bmatrix}
c_1 + c_2 & -c_2 \\
-c_2 & c_2
\end{bmatrix}
\begin{bmatrix}
\Delta \ddot{u}_1 \\
\Delta \ddot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
k_1 + k_2 & -k_2 \\
-k_2 & k_2
\end{bmatrix}
\begin{bmatrix}
\Delta u_1 \\
\Delta u_2
\end{bmatrix} = \begin{bmatrix} 0 \\
0 \end{bmatrix} \tag{7.19}
\]

where \(i = 1, 2\) to denote slab or TMD. The method assumes acceleration to vary linearly over a time-step, giving the resultant velocity and displacement expressions shown in figure 7.17. By rearranging these and substituting into equation 7.19, the response of the system is written as:

\[
\begin{bmatrix}
m_1 \left( \frac{6 \ddot{u}_1}{\Delta t^2} + \frac{3(c_1+c_2)}{\Delta t} + (k_1 + k_2) \right) \\
m_2 \left( \frac{6 \ddot{u}_2}{\Delta t^2} + \frac{3c_2}{\Delta t} + k_2 \right)
\end{bmatrix}
\begin{bmatrix}
\Delta u_1 \\
\Delta u_2
\end{bmatrix} =
\begin{bmatrix}
m_1 \left( \frac{6 \ddot{u}_1}{\Delta t} + 3\ddot{u}_1 \right) + c_1 \left( 3\ddot{u}_1 + \frac{\ddot{u}_1 \Delta t}{2} \right) + c_2 \left( 3\ddot{u}_1 + \frac{\ddot{u}_1 \Delta t}{2} - 3\ddot{u}_2 - \frac{\ddot{u}_2 \Delta t}{2} \right) \\
m_2 \left( \frac{6 \ddot{u}_2}{\Delta t} + 3\ddot{u}_2 \right) + c_2 \left( -3\ddot{u}_1 - \frac{\ddot{u}_1 \Delta t}{2} + 3\ddot{u}_2 + \frac{\ddot{u}_2 \Delta t}{2} \right)
\end{bmatrix} \tag{7.20}
\]

Equation 7.20 can be solved in stepwise form using the initial conditions given earlier in equation 7.16. A MATLAB programme was written to carry out the solution using a time-step of two milliseconds. Figure 7.18 shows plots of this model for an arbitrary value of \(V\), and different values of \(\zeta_2\). In this case, it is observed that with high TMD damping, the model is still valid (see figure 7.18(c1)). Figure 7.19 illustrates a comparison between the numerical model and the analytical one for same values of all variables. It can clearly be seen that at low values of \(\zeta_2\), both models give very similar responses (see figure 7.19(a)). With higher values of \(\zeta_2\), the predictions of response become different.
Figure 7.17: Expressions used in the linear acceleration method

Typical comparisons of the numerical model with the experimental results are shown in figure 7.20, where a satisfactory agreement is observed, even at high values of $\mu$ and $\zeta_2$. This solution method was adopted for the remainder of the analysis to obtain generalised values of $V$ and $\zeta_2$.

Figure 7.18: Numerical model plots of slab and TMD responses with different values of TMD damping ($\zeta_2$)
Figure 7.19: Comparisons between analytical and numerical models

Figure 7.20: Comparisons between experimental and numerical model results
Derivation of Initial Velocity

To update $V$ in the model, a plot of the model was superimposed on top of an experimental plot and the value of $V$ was iterated until its initial peak best fitted the particular experimental case. A MATLAB programme was written to derive the best value of initial model response amplitude at each test. The range of the derived values for $V$ was different due to variations in heeldrop impacts. In this way, an average value could be derived from many experimental plots to represent a typical initial velocity due to heeldrop. This is discussed in section 7.4.3.

Derivation of TMD damping

Here, once the best model initial velocity was derived, an extensive MATLAB programme was written, which would iterate through a range of values for $\zeta_2$ and by comparing with the experimental data, would suggest the best numerical value to fit the data. The programme first assumed an arbitrary value for $\zeta_2$ and plotted the resultant numerical graph on top of the equivalent experimental one. By correlating the peaks of the two graphs, $\zeta_2$ was incremented within a given range and a value was obtained, which best fitted the particular experimental case. This procedure was carried out for every TMD configuration at every point tested, giving a large number of values for $\zeta_2$, which could be averaged to obtain a general figure.

7.4.3 Generalisation of the model

Table 7.2 summarises the averages of the derived values for $\zeta_2$ and $V$, which best fit the experimental data at each TMD configuration. It also includes approximate reductions in experimental slab vibrations. With all other parameters in the system remaining constant, the two variables of TMD damping and mass ratio are the only factors which can affect the performance of the plywood TMD.

In the following sections, a typical value for the initial velocity, $V$, is first deduced and used in a generalisation study of the two key parameters of mass ratio and TMD damping. The model is applied to different slab configurations and best starting values of $\zeta_2$ and $\mu$ are derived, which could be assumed to be effective for any slab. In carrying out these optimisations, suggested acceleration amplitude limits, given by Allen (1990a) and Bachmann (1992b), are used. For ease of presentation, the responses are plotted for the rest of this section as peak responses only, see figure 7.21.
Table 7.2: Summary of derived model values

<table>
<thead>
<tr>
<th>TMD configuration</th>
<th>$\zeta_0$</th>
<th>Average $V$ m/s</th>
<th>Mass ratio ($\mu$)</th>
<th>Approx. experiment vibration reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>one layer widely</td>
<td>0.023</td>
<td>0.013</td>
<td>0.031</td>
<td>70%</td>
</tr>
<tr>
<td>two layer widely (polythene in-between)</td>
<td>0.034</td>
<td>0.013</td>
<td>0.043</td>
<td>80%</td>
</tr>
<tr>
<td>two layer widely (rubber in-between)</td>
<td>0.034</td>
<td>0.013</td>
<td>0.040</td>
<td>80%</td>
</tr>
<tr>
<td>one layer narrowly</td>
<td>0.014</td>
<td>0.020</td>
<td>0.015</td>
<td>60%</td>
</tr>
<tr>
<td>two layer narrowly (nothing in-between)</td>
<td>0.021</td>
<td>0.011</td>
<td>0.029</td>
<td>75%</td>
</tr>
<tr>
<td>two layer narrowly (polythene in-between)</td>
<td>0.020</td>
<td>0.011</td>
<td>0.029</td>
<td>70%</td>
</tr>
<tr>
<td>two layer narrowly (rubber in-between)</td>
<td>0.020</td>
<td>0.009</td>
<td>0.023</td>
<td>70%</td>
</tr>
</tbody>
</table>

Figure 7.21: Representation of responses as their peaks

Choice of initial velocity

From table 7.2, it can be seen that the range of values for $V$ is fairly close. This indicates that when averaging many cases of heeldrop loading, a good estimate of the heel-impact can be derived, particularly if the heeldrop is performed by the same person. Hence it is possible to consider a fixed value of $V$ for all the TMD configurations, and simplify the model accordingly to a typical heeldrop impulse. The value recommended for these tests is taken as 0.015m/s. Note that from table 7.2, this value is a conservative one leading to an initial acceleration amplitude of 0.7m/s² ($\approx 7\%g$), see figure 7.21. Since heeldrop is already regarded as a severe loading, the resultant model accelerations are assumed to be higher than would occur normally.

Generalisation of mass ratio

Table 7.2 clearly shows that the mass of the TMD directly dictates the amount of reduction in the slab vibration response. The higher the TMD mass, the greater the reduction in slab response. However, it is not always possible to use a large or heavy TMD, as constraints may exist in the placement of the plywood. In addition, the size of the planks and the tuning weights may also
dictate the size of the TMD system for a particular floor. Hence in any design, account must be
taken of a possible range of mass ratios, which can be used for a particular slab.

Figure 7.22 shows an idealised model slab response without a TMD system, subjected to a typical
heel impact. Note that the acceleration amplitude is slightly above acceptable limits of 0.5-1m/s²(5-
10%g) for active occupancies, and well above the limits for offices (see table 2.3, page 15). In the
following discussion, a single limit of acceptance of 4%g is used to account for an average structure.

![Figure 7.22: Single heeldrop on model slab without TMD](image)

Taking a typical value for TMD damping ratio of 3% critical, figure 7.23 shows the model slab
response with TMD systems of varying mass ratios. It is evident that the initial peak amplitude is
unaffected by the addition of the TMD system. However, response is seen to decrease noticeably
with higher mass ratios, until an optimum value is reached. Here, the optimum $\mu$ for the model
slab is taken as 0.04, since at higher values a significant vibration reduction is not observed, and in
some cases the secondary peaks may cause annoyance, as they can exceed 4%g. Hence, from figure
7.23 for a given TMD damping, any mass ratio within the range 0.01-0.04 can be very effective.

**Other slab configurations**

In order to support the values of mass ratio given above, two typical flat slab configurations, taken
from field tests, were input into the model and their vibration behaviour after addition of TMDs
was observed. These two slabs are described briefly below.

**Slab 1** Taken from Caverson (1992), this is a loose representation of the Vantage West car park
comprising a cast-in-place post-tensioned flat slab with unbonded tendons. A typical panel
has dimensions of $8.4 \times 7.2$ m with a slab thickness of 225mm. Slab damping ratio is 4.6% critical with fundamental frequency of 8.5Hz. Assuming the concrete properties to be similar
to the model experiment, these values were input into the numerical model and found to be
acceptable and below 4%. For the sake of this example, the slab was made more flexible and its damping reduced to 2% critical to make it more problematic. The model was then implemented with an arbitrary TMD damping of 3% critical, which is fairly conservative. Figure 7.24 shows the resultant responses without TMD and with TMD of varying mass ratios. In this case, a suitable range of \( \mu \), for fast reduction of vibration below 4%, is between 0.01-0.025. The limit of amplitude taken here is conservative for a car park.

Slab 2 Taken from Bachmann (1992a), this is a loose representation of a problematic gymnasium floor, supported on reinforced concrete beams. A typical panel has dimensions of 7 × 6m with a slab thickness of 250mm. Slab damping ratio is 2.4% critical with fundamental frequency of 7.5Hz. The responses at different mass ratios are shown in figure 7.25, using model slab concrete properties and a TMD damping of 2.5% critical. The range of mass ratios to reduce vibrations to acceptable levels is seen to be 0.01-0.04.
Summary

Higher mass ratios for TMDs lead to quicker decay of vibration amplitudes. There exists an optimum ratio where any increase in $\mu$ does not greatly affect the response. A typical range of mass ratios for reducing vibrations to acceptable limits is 0.01-0.04, where a balance exists between having a lower $\mu$ with lighter TMD and a higher $\mu$ with heavier TMD but faster decay.

Generalisation of TMD damping

For the experimental slab, the response is plotted at an average mass ratio of 0.02 against varying values of $\zeta_2$, figure 7.26. As expected, the higher the TMD damping, the faster the decay of slab vibrations. To reduce the vibration below 4%$g$ at the second peak, a TMD damping of 0.015 or above is required. Notice that this is the value of the slab damping ratio, so that for the TMD to be fully effective, it should have greater damping capacity than the slab. From table 7.2, it can be seen that all TMD configurations would satisfy this requirement, although with two layers of widely having polythene or rubber in-between, the highest value of $\zeta_2$ is achieved.

Other slab configurations

The range of TMD damping was observed against the two field slabs described above, using an average mass ratio of 0.02, and the following results were obtained.

Slab 1 Figure 7.27 shows variations of $\zeta_2$ for the Vantage West slab. An acceptable TMD damping for reducing vibration below 4%$g$ is 3% critical or higher.

Slab 2 Figure 7.28 gives the variations of $\zeta_2$ for the gymnasium floor with minimum acceptable value being 1.5% critical for vibration reduction below 4%$g$ at the second peak.
Figure 7.26: Model slab response with variations in TMD damping

Figure 7.27: Vantage West slab response with variations in TMD damping

Figure 7.28: slab 2 response with variations in TMD damping
Summary
For a given mass ratio, the higher the TMD damping, the more effective the system and the faster the vibration decay. If $\zeta_2$ becomes too large, the displacement of the TMD is inhibited, hence less energy can be dissipated. A minimum value for reducing vibrations due to a heel impact is around 2% critical for a typical slab. This value could be reduced with an increase in mass ratio if possible.

7.4.4 Extension of the model to walking vibrations
Until now, heeldrop loading was used as the impact on a typical slab giving a design criterion for a TMD system. This is acceptable as a heeldrop impact represents a very severe loading, so that the design for remedying the resulting vibrations can be considered as conservative. Here, consideration is given to intermittent vibrations, such as walking. Depending on the walking frequency, the loading is simulated as several heel impacts occurring after each other. This is shown in figure 7.29, where a comparison is made between simulating walking as successive heeldrops and an actual signal taken from Bachmann et al. (1995). It can be seen that limitations exist in representing walking in this way. Firstly, the impacts are very severe, particularly at higher harmonics, and in effect closer to rhythmic jumping, leading to greater slab response amplitude. Secondly, there is no horizontal walking momentum and in effect the person is considered as walking on spot. Hence, both these limitations make the model rather conservative and therefore it may be feasible to increase the acceptable acceleration limits for a given floor subjected to such loading. Here, these limits are increased from an average of $4\%g$ to $6\%g$.

![Figure 7.29: Simulated and actual walking time-histories and frequency spectra for walking at a frequency of 2Hz](image)
Figure 7.30(a) shows the experimental model slab subjected to four simulated heeldrop footsteps at 2Hz. Similarly, figure 7.30(b) shows the model slab, with TMD, subjected to the same walking load. It can be seen that without the TMD, the responses to successive footsteps overlap, leading to a build-up of acceleration amplitudes up to around 20%g after four steps. In comparison, with a suitable TMD system designed for the floor, there is only one peak that may cause annoyance. By assuming an acceleration limit of 6%g, with a TMD the slab vibration becomes acceptable after 0.3 seconds, as compared with over 3 seconds without a TMD system.

(a1) No TMD (one step)  
(a2) two steps  
(a3) three steps  
(a4) four steps  
(b1) With TMD (one step)  
(b2) two steps  
(b3) three steps  
(b4) four steps

Figure 7.30: Walking simulations with and without TMD at 2Hz

Similar results are obtained with walking vibrations of different frequencies, as shown in figure 7.31, where only the peaks are plotted for clarity. Hence, it is seen that the proposed TMD system is very effective for walking vibrations as well as transient. Notice here that the TMD is more effective for certain walking frequencies than others. In the case of 3.6Hz footstep frequency, the vibration reduction is less significant. This is because 3.6Hz may not be a multiple of the slab fundamental frequency. However, this problem can be overcome by using multiple TMD systems, as explained in section 10.4. The effectiveness of the designed TMD system, in the model slab tests, is shown in figures 5.7 and 5.8 (pages 100-101) for horizontal and vertical walking respectively.
Figure 7.31: Walking simulations with and without TMD at other pacing frequencies

7.5 Sensitivity analyses

Having obtained a suitable model for a slab-TMD system, a sensitivity analysis was carried out to verify its ranges of usability. Here, the sensitivity to variations in each of the main variables was investigated separately, as follows:

**Slab dimensions:** Alterations to these parameters, particularly slab length, \( L \), and depth, \( d \), will affect the value of the model natural frequency. This is very significant as the tuning of the TMD is based on an accurate estimate of slab natural frequency. Figure 7.32 shows that with small changes in \( d \), there is a significant change in slab frequency so that the model simulation becomes out of phase and the decay amplitudes are affected (N.B: as expected \( f \propto d^2 \)). Similarly, figure 7.33 illustrates the importance of choosing the correct boundary conditions so that the model slab length is the distance undergoing displacement. This is particularly important in design, where one might want to estimate the equivalent simply supported span, taking into account effects of support stiffness and continuity. It would be advisable for the model to have values of slab dimensions within ±10% of real values.

**Mass ratio (\( \mu \)):** Figure 7.34 shows how a change in mass ratio can affect the model simulation. It can be seen that the amplitude and decay rate of vibration are dependent on the mass
ratio. As explained in section 7.2.2, the larger the TMD mass, the faster the rate of decay.

**TMD damping ratio ($\zeta_2$):** Average values of $\zeta_2$ for each TMD stage are given in table 7.2. The sensitivity of the model to these values is illustrated in figure 7.35 for three values within a large range. Notice how this parameter affects amplitude of decay of the slab and must be included in any TMD design. It does however affect slab natural frequency.

**Summary**

In addition to the above variables, sensitivity of the model to variations in Young’s modulus, slab width, slab damping ratio, and initial applied velocity were also checked. In general, the proposed model can be acceptably used with given parameters within 10% of actual values, hence the model is not severely sensitive. It is however important to note that variations in the key parameters of mass ratio and TMD damping do not have any effect on the initial peak acceleration amplitudes of the slab. The TMD’s effectiveness is only observed once the energy has been transferred from the primary to the secondary mass in the model.
7.6 Application of the model

Here, a design guideline is proposed which could be applied to a given slab. The guideline in its present form is at its early stages and perhaps somewhat too complex for use by designers. However, it is envisaged that with more application and practical results, various data can be accumulated, leading to suggested values for parameters and hence simplifying the guideline for designers. This is particularly relevant in steps 3 and 4 below.

7.6.1 Proposed design guideline

For a given floor with specified usage, indicating acceleration limit and loading type, it is possible to follow a few steps and arrive at a suitable plywood TMD system, which can be used to reduce the vibrations of the floor. These steps can be followed as below for heeldrop or walking excitations:

step 1 Adapt the model such that the natural frequency of the primary mass becomes equal to the measured fundamental frequency of the given slab. This is very important, since the boundary
conditions of real slabs can rarely be modelled and their effect on natural frequency cannot easily be predicted. For example, in cases where a slab is supported on edge-beams, the connection between the slab and the beams is such that the boundary condition is between a simply supported and a fixed state. Hence, it is necessary to adapt the model parameters (particularly slab length) to arrive at an equivalent span, at which the model fundamental frequency matches that of the measured slab value.

**step 2** Establish the type of floor, in terms of its occupancy and usage, and decide on a value of limiting vibration amplitude. Here, the suggested values in table 2.3 (page 15) can be used. Furthermore, by looking at the floor span and type of activity, arrive at some possible loading patterns, including the worst possible case (e.g. several consecutive steps). From these, derive the highest response peak amplitude for the slab, either by assuming successive heeldrops (section 7.4.4), or using one of the methods of section 2.7.1.

**step 3** Keeping the TMD damping constant at 3% critical, investigate the slab response due to the worst loading case of step 2, with a range of mass ratios, and arrive at a suitable value for $\mu$.

**step 4** Keeping mass ratio constant at the value derived in step 3, investigate the slab response due to a range of TMD damping ratios and arrive at a suitable value for $\zeta_2$.

**step 5** Having arrived at optimum values for $\mu$ and $\zeta_2$, determine the overall performance of the slab with the TMD system installed, due to worst and normal cases of loading.

**step 6** With the derived mass ratio and TMD damping, use table 7.2 to suggest the best plywood TMD configuration and by calculating the mass of the TMD, arrive at a suitable size for the plywood and the mass needed to tune the TMD.

A detailed design example, using the above steps, is given in appendix F for a typical floor slab.
Chapter 8

Human-structure interaction

8.1 Introduction

In section 2.6.6, a discussion was presented on the forces exerted by people on a floor and the research conducted on the human-structure interaction. It was observed that most design codes treat the occupants of a floor as extra load, uniformly distributed across the slab, while biomechanical studies suggest that the human body has significant energy absorption capabilities and is best modelled as a spring-mass-damper system. Since people not only impose dynamic loads but also interact with the structure, an awareness of this interaction is required to understand firstly, how the structure will respond, and secondly, how people may perceive the vibrations.

Ellis et al. (1994) found that when the mass of the people is reasonably large in comparison with the mass of the structure, such as in sports grandstands, the natural frequency of the system changes to reflect the crowd fundamental frequency, and a significant amount of energy absorption takes place. Similarly, some work has also been conducted on the best model to represent a single person [Ji and Ellis (1994b), Foschi et al. (1995), Folz and Foschi (1991)]. These studies all concluded that a SDOF mass-spring-damper is sufficient in representing a human body. Other representations have also been proposed, such as modelling the body as a continuous system [Ji (1995)], as two SDOF systems [ISO2631/1 (1985)], or as a 15DOF system [Nigam and Malik (1987)].

In general, the subject of human-structure interaction is poorly covered in literature and only a few notable findings can be recorded. Indeed, none of the perceptibility design guidelines, presented in section 2.2.2, takes the occupants of the structure into account in any way. It is likely, however, that an individual's annoyance due to floor vibrations is a function of the amount of energy being transmitted to the body, which must be dependent on the interaction between the body and the vibrating structure. This chapter presents the results of experiments on the human-structure interaction, which give a natural frequency for the human body of 10.43Hz and a damping capacity
of 50% critical. An analytical model is presented, which assumes the human body as a SDOF system, and the results are compared with the existing design codes, which assume the occupants as extra mass. The models are seen to be in good agreement with the experimental results.

### 8.2 Description of experiments

The main purpose of these experiments was to assess the effect of a human on the vertical vibration behaviour of the slab. Therefore, the majority of the tests were conducted with a single person (or two people) standing on the floor. Crowd effects and the significance of the occupants’ position and posture were not part of this investigation.

Tests with the human body were conducted on the following slab configurations:

- Bare slab (fully tensioned)
- Slab with one standing man
- Slab with dead weight equivalent to mass of the man placed at same position as the man (hereafter referred to as equivalent weight)
- Slab with two standing men
- Slab with equivalent weight of the two men

Figure 3.18 (page 64) shows the position of the standing man on the model slab. In tests where two men were used, they were symmetrically positioned on the slab. In each case, a sine-sweep test was carried out to obtain the FRF and also a shaker cut-off test was conducted to obtain time-domain decay response. The majority of the analysis is focused on the fundamental frequency of the model floor, as it is in the range of possible annoyance to occupants. However, some analysis is also presented on the effect of a human on the higher harmonics of floor vibration.

### 8.3 Experimental results

Having obtained FRF and time-domain data for the various configurations described above, the results were analysed to derive natural frequency and damping values of the slab in each case. Detailed results of these tests are included in appendix A. Table 8.1 gives selected relevant readings at some slab configurations. These are discussed in the following sections. In each plot, the response of the bare slab is also illustrated for comparison purposes.
Table 8.1: Selected human-structure interaction results (taken from appendix A)

<table>
<thead>
<tr>
<th>Test configuration</th>
<th>Average fund. frequency (Hz)</th>
<th>Average damping (% of critical)</th>
</tr>
</thead>
<tbody>
<tr>
<td>bare slab (fully tensioned)</td>
<td>8.02</td>
<td>1.10</td>
</tr>
<tr>
<td>bare slab + one man</td>
<td>7.76</td>
<td>3.84</td>
</tr>
<tr>
<td>bare slab + equiv. mass of one man</td>
<td>7.68</td>
<td>1.45</td>
</tr>
<tr>
<td>slab + both screed layers</td>
<td>10.15</td>
<td>1.25</td>
</tr>
<tr>
<td>slab + screed layers + one man</td>
<td>9.96</td>
<td>3.11</td>
</tr>
<tr>
<td>slab + screed layers + equiv. mass of one man</td>
<td>9.93</td>
<td>1.28</td>
</tr>
<tr>
<td>slab + screed layers + two men</td>
<td>9.96</td>
<td>3.46</td>
</tr>
<tr>
<td>slab + screed layers + equiv. mass of two men</td>
<td>9.81</td>
<td>1.28</td>
</tr>
</tbody>
</table>

8.3.1 Effects on natural frequency

From table 8.1, it can be seen that the fundamental frequency of the slab reduced with the presence of a standing man, and with his equivalent weight. The latter result would be expected, as an increase in the mass of a system would lead to a decrease in its natural frequency. The reduction in frequency with the standing man, however, is in contrast with the results obtained by Ellis and Ji (1997). They conducted similar tests on a laboratory beam of fundamental frequency 18.68Hz, and found this to increase when a person stood on the slab. The reasons for the discrepancies between the two findings are explained later in section 8.4.1, and are basically due to the difference in the fundamental frequencies of the two slabs.

8.3.2 Damping effects

Effect of one man on bare slab

Figure 8.1 shows FRF magnitude plots of the bare slab with the standing man and with his equivalent weight. It can be seen that by considering the human as extra mass, the transfer function amplitude is only marginally reduced, whereas this decrease is around 75% when considering the actual person. The width of the fundamental peak is also much broader with the man, indicating a substantial increase in damping. From the damping measurements, this increase is seen to be over three-fold, as compared to 32% when considering the man as a mass (table 8.1). The time-domain plots of figure 8.2 substantiate these results and it can clearly be seen that the standing man absorbs a significant amount of the slab vibration energy.
Effect of two men on the slab

Figures 8.3 and 8.4 show frequency and time-domain results when two men were standing on the slab. From table 8.1, it is seen that with two men, the increase in damping is of the same order as with one standing man, leading to the observation that the effect of the extra person is marginal. A comparison between one and two people standing on the slab is shown in figure 8.5, from which it can be reasonably assumed that in laboratory experiments, one person is sufficient in representing the increase in damping due to the occupants. This assumption will become incorrect when considering a crowd, as the mass of the people is significantly increased. In such cases, consideration must also be given to the percentage of the crowd that is stationary. A person who is not stationary has minimal energy absorption capabilities [Ji and Ellis (1993)].
8.3.3 Effect of man at higher frequencies

Figure 8.6 illustrates FRF plots of the effect of the standing man and his equivalent mass on the amplitude of the second natural frequency of the slab. Similarly, Figure 8.7 shows this effect at the third natural frequency. It can clearly be seen that at higher frequencies, the energy absorption
effect of the body is completely lost. It is evident that at these frequencies resonance of the body does not occur, and it becomes feasible to assume the body as a mass. Hence, the energy absorption effect of the body must be confined to its range of fundamental frequency, which is reported to be between 4-16Hz when standing [Herterich and Schnauber (1992)].

![Figure 8.6: FRF plots of man and equivalent weight at second slab frequency](image)

![Figure 8.7: FRF plots of man and equivalent weight at third slab frequency](image)

8.3.4 Summary of experimental results

- The fundamental frequency of the model slab decreased with the addition of a stationary standing man, and also with his equivalent weight.

- A stationary man significantly increased the model slab damping ratio by as much as 250%, compared to 32% when considering the man as a mass.

- Two men stationary on the slab lead to an increase in damping which was only marginally more than one man stationary on the slab.

- The energy absorption capacity of the body was confined within a fixed frequency range and was lost at higher frequencies.
8.4 Human-structure models

In this section, a numerical model is devised, which assumes the body and the slab as two separate SDOF systems. First, the undamped response of the 2DOF system is analysed, with respect to the experimental results, to derive values for the fundamental frequency and stiffness of the human body. These are then used in a time-stepping damped 2DOF model to derive a value for the damping inherent in the body. It is shown that a SDOF model is adequate in representing the stationary body, and that its interaction with a floor is best represented by a 2DOF model.

8.4.1 Undamped 2DOF model

Consider the undamped 2DOF system of figure 8.8, with $M_1^*$ and $K_1^*$ representing the generalised slab mass and stiffness (see appendix B), and $M_2^*$, $K_2^*$ being the generalised body mass and stiffness respectively. Of the above parameters, the only unknown is $K_2^*$, which can be derived using the experimentally measured natural frequencies of table 8.1, when considering the combined 2DOF human-structure system.

![Figure 8.8: 2DOF undamped lumped mass-spring model of slab-human system](image)

For the system of figure 8.8, the equation of motion is given by:

\[
\begin{bmatrix}
M_1^* & 0 \\
0 & M_2^*
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_1 \\
\ddot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
K_1^* + K_2^* & -K_2^* \\
-K_2^* & K_2^*
\end{bmatrix}
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\] (8.1)

Assuming harmonic motion of the system with frequency, $\omega$, the determinant of the co-efficients in equation 8.1 must be equal to zero, leading to:

\[
M_1^* M_2^* \omega^4 - [K_2^* M_1^* + M_2^* (K_1^* + K_2^*)] \omega^2 + K_1^* K_2^* = 0
\] (8.2)

This equation can be used to analyse the effect of the body when considered as a mass only, and also when considering its interaction with the supporting structure. These cases are dealt with separately below.
Consideration of equivalent mass

Assuming the human body to be represented only as a mass, the value of $K_2^*$ in figure 8.8 will tend to $\infty$. Rearranging equation 8.2, given $\omega_1 = \sqrt{\frac{K_1^*}{M_1^*}}$ and $\omega_2 = \sqrt{\frac{K_2^*}{M_2^*}}$, leads to:

$$K_2^* = \frac{\omega_1^2 M_1^* M_2^* (\omega_1^2 - \omega_2^2)}{M_1^* (\omega_1^2 - \omega_2^2) - M_2^* \omega_2^2}$$

(8.3)

If $K_2^*$ tends to $\infty$, the denominator of equation 8.3 must tend to zero, so that the natural frequency of the combined system will be:

$$\omega_2^2 = \frac{\omega_1^2 M_1^*}{M_1^* + M_2^*}$$

(8.4)

This means that, as expected, the addition of extra mass leads to a reduction in the fundamental frequency of the primary system, depending on $M_2^*$. In the case of the experimental results, table 8.1 gives $\omega_1 = (2\pi \times 8.02)$ rads/s, and appendix B gives $M_2^* = 806$ kg. Also, as the focus is on addition of mass only, $M_2^*$ is taken to be the total mass of the body (i.e., 75 kg). By substituting the values into equation 8.4, the natural frequency of the system is obtained as 7.65 Hz. This compares with the experimentally derived value of 7.68 Hz (table 8.1), which shows good agreement.

Consideration of human-structure interaction

With $K_2^*$ in figure 8.8 being positive and finite, the interaction between the two masses, $M_1^*$ and $M_2^*$, must be considered. By expressing $\mu = \frac{M_2^*}{M_1^*}$, equation 8.2 can be written as:

$$\omega^4 - \left[\omega_1^2 + \omega_2^2 (1 + \mu)\right] \omega^2 + \omega_1^2 \omega_2^2 = 0$$

(8.5)

for which the solution is:

$$\omega^2 = \frac{1}{2} \left[\omega_1^2 + \omega_2^2 (1 + \mu) \mp \sqrt{[\omega_1^2 + \omega_2^2 (1 + \mu)]^2 - 4 \omega_1^2 \omega_2^2}\right]$$

(8.6)

Introducing a new parameter, $\Omega = \frac{\omega_2}{\omega_1}$, and substituting into equation 8.6, gives:

$$\left(\frac{\omega}{\omega_1}\right)^2 = \frac{1}{2} \left[1 + \Omega^2 (1 + \mu) \mp \sqrt{[1 + \Omega^2 (1 + \mu)]^2 - 4 \Omega^2}\right]$$

(8.7)

Figure 8.9 shows a plot of equations 8.7 for different values of mass ratio, $\mu$. The following observations can be gathered from the figure:

i) At any value of $\mu$, the fundamental frequency of the slab is in between the two natural frequencies of the combined 2DOF system.

ii) As the number of people increases (i.e., larger $\mu$), the difference between the two natural frequencies of the combined 2DOF system increases.
iii) A clear dividing line can be seen at the point where $\Omega=1.0$. For $\Omega > 1.0$ (i.e. structure frequency is lower than body frequency), the fundamental frequency will gradually reduce as $\mu$ increases, and a higher frequency mode appears. For $\Omega < 1.0$ (i.e. high frequency structure), the original frequency will increase as $\mu$ increases, with a new lower frequency mode appearing. This explains the difference between the results described here and those found by Ellis and Ji (1997) (see section 8.3.1). In these tests, the slab frequency was lower than the body frequency, hence a reduction in fundamental frequency was observed. The beam tested by Ellis & Ji had a fundamental frequency higher than the body frequency, which increased with the standing man. Ellis & Ji also noted that the 'new' modes, as a result of the occupants, will be mainly due to the occupants vibrating and will not necessarily be seen from the structural response. This was found to be true for the case of the experimental slab.

iv) Figure 8.9 can also be used for structure-TMD systems where for maximum effect $\Omega=1.0$, and for a given $\mu$, the natural frequencies of the system can be obtained. However, for a structure-TMD system, the mass ratio is generally very small and $\omega \approx \omega_1$.

v) From the results of the experimental slab, it is possible to derive a value for $\omega_2$, given all other parameters in equation 8.7, as described below.
Derivation of the generalised mass of the body

The derivation of an accurate value for $M^*_2$ is at present not fully understood and very few findings are reported. The most notable investigation was perhaps by Ji (1995) who represented the body as a continuous system and derived its modal masses. For the purpose of this discussion, a simple method is used to derive $M^*_2$ by assuming the body as a rod undergoing axial vibrations (figure 8.10), and considering the energies involved.

![Figure 8.10: Simplification of the body as an axially vibrating rod](image)

Equating kinetic energies of the rod and its generalised SDOF representation, leads to:

$$\int_0^L \frac{1}{2} m u^2 \, dx = \frac{1}{2} M^*_2 \dot{z}^2$$

which gives

$$M^*_2 = \int_0^L m \frac{x^2}{L^2} \, dx = \frac{mL}{3}$$

(8.8)

In the case of the experimental slab, the standing man had a mass of 75kg, giving $M^*_2 = 25$kg.

Derivation of body natural frequency and stiffness

Refering to equation 8.7 and figure 8.9, a value for the fundamental frequency of the human body is derived. Taking $M^*_2 = 25$kg and $M^*_1 = 806$kg (appendix B), gives a value for $\mu = \frac{M^*_2}{M^*_1} = 0.03$. Similarly, from the experiments, table 8.1 gives a combined frequency of 7.76Hz, which with a slab frequency of 8.02Hz, gives $\frac{\omega_2}{\omega_1} = 0.968$. By substituting these into equation 8.7, a value of $\Omega = \frac{\omega_2}{\omega_1} = 1.30$ is obtained. Hence, these tests give a fundamental frequency for the body of $f_2 = 10.43$Hz. This corresponds well with 9.96Hz given in [Ellis and Ji (1997)] and is within the 2DOF values of ISO2631/1 (1985), being 5.03 and 12.49Hz [Folz and Foschi (1991)]. It is also within the range of values given by Ji (1995), for different modal masses, which are between 9.96 and 11.87Hz. When considering a group of people, the frequency differences between individuals can be neglected and the frequency of the human bodies are treated as a constant coefficient [Ji and Ellis (1994b)]. Furthermore, equation 8.7 also suggests a new mode to appear at 10.77Hz. This would be mainly due to the person vibrating and was not observed in the measured results, which were taken on
the slab. Note that both the individual body and slab fundamental frequencies are inbetween the frequencies of the combined human-structure system.

Using the fundamental frequency of the body, its stiffness is found from \( \omega_2 = \sqrt{\frac{K_2}{M_2}} \), which is \( K_2 = 1.07 \times 10^5 \text{N/m} \). This is greater than the 2DOF values of ISO2631/1 (1985), being \( 6.2 \times 10^4 \) and \( 8.0 \times 10^4 \text{N/m} \), but is smaller than the derived values by Ji (1995) for a continuous system, being in the range \( 1.88 \times 10^5 \) to \( 8.55 \times 10^5 \text{N/m} \) for different modal masses.

### 8.4.2 Damped 2DOF and SDOF models

In this section, the derived values of the body frequency and stiffness are used in a damped 2DOF model, (figure 8.11), and a typical value for the damping of the human body is obtained. The approach is very similar to the time-stepping model presented for the structure-TMD system, explained in section 7.4.2. The main difference between the human and TMD models is the value of mass ratio, \( \mu \), and the damping inherent in the secondary mass, \( \zeta_2 \). The human body is heavier than a TMD system, particularly if a crowd is involved, hence increasing \( \mu \). Similarly, the damping of a human body can be as high as 47.5% critical [Ji and Ellis (1994b)]. This significant increase in damping of the secondary mass leads to much faster decay of slab vibration, as shown by the experimental results. The variables in the model are updated to best represent the experimental data and comparisons are made. Also, a SDOF damped model is presented for the slab without the standing man. These models are described in more detail below.

![Diagram of 2DOF lumped mass-spring-damper model of slab-human system](image)

Figure 8.11: 2DOF lumped mass-spring-damper model of slab-human system

**Description of the models**

The 2DOF model is a time-stepping solution of a lumped mass-spring-damper system, as described in section 7.4.2. The generalised values of mass and stiffness, for the slab and the body, are taken
as those given in the previous section. The initial conditions are taken as zero displacement for both man and slab but an initial velocity, $V$, is assumed applied to the slab (as in equation 7.16, page 152). The subsequent slab response decay is then analysed. Figure 8.12 is a plot of this model for an arbitrary $V$ and $\zeta_2$, using the experimental values for all other slab variables.

In addition to the 2DOF model, a SDOF model was also devised, which would represent the slab without the standing man. This model is used to investigate the possibility of assuming the man as an extra mass on the slab, without any inherent damping and stiffness. Figure 8.13 shows a typical plot of the model using the experimental slab properties.

![Figure 8.12: Typical plot of 2DOF slab response model with standing man](image)

![Figure 8.13: Typical plot of SDOF slab response model with equivalent mass of man](image)

**Derivation of model human damping**

Having arrived at the SDOF and 2DOF models and updated $V$ to best fit the experimental results, it was possible to compare the experimental and model plots and hence arrive at a value for human body damping ($\zeta_2$ in model). An extensive MATLAB programme was written which would iterate for $\zeta_2$ through a range of values. This iteration routine is similar to that described in section 7.4.2.
After carrying out the operation for all experimental plots, the value of body damping was derived to be mostly within the range 45% to 55% critical. Factors such as posture, tensioning of the muscles, or bending of the knees would contribute to the observed variability in \( \zeta_2 \). The derived range, however, is in good agreement with Griffin (1990), [as reported by Ji and Ellis (1994b)], who gives a value of 47.5% at a sitting position. For the remainder of this report, an average value of 50% critical is taken to represent the damping ratio for the human body.

Useability of the models

Figure 8.14 shows a typical plot of the 2DOF model compared with the experimental response of the slab with a standing man. It can be seen that the model satisfactorily represents the experimental data. This fit was also observed in all other cases of the slab with the standing man, including the case of two standing men. Similarly, figure 8.15 shows a typical plot of the SDOF model with mass of slab adjusted up to account for the increase of mass due to the standing man. In both cases, good agreement is achieved. However, when model results of slab response with man and his equivalent weight are plotted, figure 8.16, a large difference is observed, which illustrates that the standing man has very high energy absorption capabilities and should not be regarded as extra mass.

![Graph](image)

**Figure 8.14:** Typical plot of final model for slab response with standing man

8.4.3 Sensitivity analyses

Having derived suitable models for the interaction of the human body with the structure, here a sensitivity analysis is carried out to investigate the effect of changes in body mass and stiffness. Figure 8.17 shows plots of slab response with a stationary person of average weight, as compared with 40% heavier and lighter occupants. It can be seen that the rate of decay of slab vibrations is
marginally faster with the heavier occupant. This is to be expected, as the heavier the occupant, the greater the effect of the secondary dashpot in the model (figure 8.11). However, the model is not very sensitive to a change in occupant mass. Here, an increase in body mass of 40% leads to an increase in damping of the slab of approximately 10%, and also to a reduction of slab natural frequency of nearly 1%.

Perhaps a better indication of the sensitivity of the model is when changes to the fundamental frequency of the body are considered (i.e. variations in body stiffness). As described previously, these experiments yielded a natural frequency for the body of 10.43Hz, which lies within the suggested range of 4-16Hz. Figure 8.18 illustrates the model slab response with the body frequency at 10.43Hz, and also at the upper and lower boundaries of the recommended range. It can be seen that the fastest decay of vibrations occurs at the calculated frequency of 10.43Hz. Damping of the slab decreases at other values of body frequency.
8.5 Derivation of human body response

Having shown that the human body has a significant energy absorption capacity, in this section the body response to a given floor vibration is evaluated. Figure 8.19 shows the model response ratio of the secondary mass (human body) to the primary mass (floor slab), with damping of the body taken as 50% critical. It is observed that at resonance, the response of the body is 46% higher than the acceleration of the floor. Hence, the avoidance of such a case can be regarded as a design target for the human perception of vibration. The implications in design of this difference in the responses are discussed in the next section.

To investigate the response of the human body to a given vibratory motion of the floor, consider the primary structure of figure 8.11, $M_1^*$, subjected to a harmonic excitation, $P e^{i\omega t}$. The motion of such a system will take the form of equation 8.1, with damping and force incorporated: (also see equation 7.5, page 150)

$$
\begin{bmatrix}
M_1^* & 0 \\
0 & M_2^*
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_1 \\
\ddot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
c_1 + c_2 & -c_2 \\
-c_2 & c_2
\end{bmatrix}
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
-K_1^* + K_2^* & -K_2^* \\
-K_2^* & K_2^*
\end{bmatrix}
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2
\end{bmatrix}
= \begin{bmatrix}
P \\
0
\end{bmatrix} e^{i\omega t}
$$

(8.9)
Assume the solution to be in the form:

\[
\begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix} = \begin{bmatrix}
    X_1 \\
    X_2
\end{bmatrix} e^{i\omega t} \tag{8.10}
\]

and for simplicity write:

\[
\begin{bmatrix}
    K_1^* + K_2^* & -K_2^* \\
    -K_2^* & K_2^*
\end{bmatrix} = \begin{bmatrix}
    K_{11} & K_{12} \\
    K_{21} & K_{22}
\end{bmatrix} = K \quad \begin{bmatrix}
    c_1 + c_2 & -c_2 \\
    -c_2 & c_2
\end{bmatrix} = \begin{bmatrix}
    C_{11} & C_{12} \\
    C_{21} & C_{22}
\end{bmatrix} = C
\]

\[
\begin{bmatrix}
    M_1^* & 0 \\
    0 & M_2^*
\end{bmatrix} = \begin{bmatrix}
    M_{11} & M_{12} \\
    M_{21} & M_{22}
\end{bmatrix} = M \quad \begin{bmatrix}
    P \\
    0
\end{bmatrix} = P \quad \begin{bmatrix}
    X_1 \\
    X_2
\end{bmatrix} = X \quad \begin{bmatrix}
    u_1 \\
    u_2
\end{bmatrix} = U \tag{8.11}
\]

Substitution of equations 8.10 and 8.11 into 8.9 gives:

\[
\begin{bmatrix}
    K - \omega^2 M + i\omega C
\end{bmatrix} X = P \tag{8.12}
\]

for which the total response is derived using equation 8.10 and is given by:

\[
U = BP e^{i\omega t} \quad \text{where} \quad B = \left[ K - \omega^2 M + i\omega C \right]^{-1} \tag{8.13}
\]

Hence:

\[
B = \begin{bmatrix}
    B_{11} & B_{12} \\
    B_{21} & B_{22}
\end{bmatrix} = \frac{1}{D} \begin{bmatrix}
    K_{22} - \omega^2 M_{22} + i\omega C_{22} & -K_{12} - i\omega C_{12} \\
    -K_{21} - i\omega C_{21} & K_{11} - \omega^2 M_{11} + i\omega C_{11}
\end{bmatrix} \tag{8.14}
\]

where \( D \) is the determinant of \( K - \omega^2 M - i\omega C \) given by:

\[
D = (K_{11} - \omega^2 M_{11} + i\omega C_{11}) \left( K_{22} - \omega^2 M_{22} + i\omega C_{22} \right) - (K_{12} + i\omega C_{12})^2 \tag{8.15}
\]

Using standard relationships for complex numbers, the coefficients in \( B \) can be expressed in terms of real amplitudes, giving:

\[
B = \begin{bmatrix}
    \sqrt{\omega^2 + R^2} & \sqrt{\omega^2 + f^2} \\
    \sqrt{\omega^2 + h^2} & \sqrt{\omega^2 + h^2}
\end{bmatrix} \tag{8.16}
\]
where

\[ a = K_{22} - \omega^2 M_{22} \quad b = \omega C_{22} \quad c = K_{12} \]
\[ d = \omega C_{12} \quad e = K_{11} - \omega^2 M_{11} \quad f = \omega C_{11} \]
\[ g = (K_{11} - \omega^2 M_{11}) (K_{22} - \omega^2 M_{22}) - K_{12}^2 - \omega^2 (C_{11} C_{22} - C_{12}^2) \]
\[ h = \omega \left[ C_{11} (K_{22} - \omega^2 M_{22}) + C_{22} (K_{11} - \omega^2 M_{11}) - 2C_{12} K_{12} \right] \]

(8.17)

Substitution of equation 8.16 into equation 8.13, gives the overall response of the two masses as:

\[ u_1 = \frac{\sqrt{a^2 + b^2}}{\sqrt{g^2 + h^2}} Pe^{i\omega t} \quad \text{and} \quad u_2 = \frac{\sqrt{c^2 + d^2}}{\sqrt{g^2 + h^2}} Pe^{i\omega t} \]

(8.18)

Hence, the response ratio of the secondary mass (human body) to the primary mass (floor slab) is given by:

\[ \left( \frac{u_2}{u_1} \right)^2 = \frac{c^2 + d^2}{a^2 + b^2} = \frac{K_{12}^2 + \omega^2 C_{12}^2}{(K_{22} - \omega^2 M_{22})^2 + \omega^2 C_{22}^2} \]

(8.19)

In terms of the floor and body properties, equations 8.11 can be used to express the above as:

\[ \left( \frac{u_2}{u_1} \right)^2 = \frac{K_{2}^2 + \omega^2 C_{2}^2}{(K_{2}^2 - \omega^2 M_{2}^2)^2 + \omega^2 C_{2}^2} \]

(8.20)

Writing \( K_2^* = M_2^* \omega_2^2 \) and \( \zeta_2 = 2\zeta_2 M_2^* \omega_2 \), reduces equation 8.20 to:

\[ \frac{u_2}{u_1} = \sqrt{\frac{\omega_2^2 \left( \omega_2^2 + 4\omega_2^2 \zeta_2^2 \right)}{(\omega_2^2 - \omega^2)^2 + 4\omega_2^2 \omega_2^2 \zeta_2^2}} = \sqrt{\frac{1 + 4 \left( \frac{\omega_2}{\omega} \right)^2 \zeta_2^2}{1 - \left( \frac{\omega_2}{\omega} \right)^2}} \sqrt{4 \left( \frac{\omega_2}{\omega} \right)^2 \zeta_2^2} \]

(8.21)

Figure 8.20 is a plot of equation 8.21 for different values of \( \zeta_2 \). It can be seen that for each \( \zeta_2 \), the response ratio reaches a maximum at the condition of resonance, when the body frequency equals the loading frequency. For avoidance of human annoyance to vibration, the response ratio should be small. As expected, figure 8.20 shows that notable reductions in the ratio are achieved with higher values of human body damping. In each case, the maximum response ratio can be derived by substituting \( \omega = \omega_2 \) in equation 8.21 to get:

\[ \left( \frac{u_2}{u_1} \right)_{max} = \sqrt{\frac{1 + 4 \zeta_2^2}{4\zeta_2^2}} \]

(8.22)

From the experimental results, putting the derived value of \( \zeta_2=50\% \) critical into equation 8.22, gives a response ratio of 1.41 between the body and the floor. This means that the body response is 41\% larger than the structural response. It compares with 46\% obtained from figure 8.19, hence good agreement is observed. Using the response ratio of 1.46 from figure 8.19, equation 8.22 gives an optimal human damping ratio of 47\% critical, which is within the range obtained from the experiments (i.e. 45-55\% critical) given in section 8.4.2.
8.6 Design implications

In order to understand how structures respond to dynamic loads generated by people, it is necessary to understand how people and structures interact. It has been shown that the human-structure interaction is similar in nature to a structure-TMD system, with the difference that the ratio of critical damping of the body has been measured as 50%, as compared to generally below 10% for a TMD. This suggests that the human body is a very good energy absorber. However, a TMD is usually a specially designed device, fixed onto a structure, and is always available to perform its function. The downside with people is that although the human body is a natural mass-spring-damper system, it will not always be present on a structure. In addition, when a human body changes from its stationary state to a moving state, its effect as an energy absorber is totally lost, because the body does not vibrate with the slab and acts solely as a load [Ellis and Ji (1997)]. This limits the use of people as energy absorbers.

The experiments have shown that at frequencies low enough to cause annoyance to occupants, the human body absorbs much of the floor energy. This effect is lost at higher frequencies. Therefore, with its significant energy absorption capacity and usable range of natural frequencies, the human body is very effective in reducing floor vibrations. In cases where a crowd is present, it is unlikely that all the occupants are not stationary, hence the stationary people will absorb the energy imposed by the moving ones. As the number of occupants and their movements change, so will the nature of the 2DOF system representing their interaction with the structure. It follows that this possible
source of energy dissipation should be accounted for in design codes, rather than assuming the occupants as extra mass. This leads to two implications: (1) evaluating structural response where people are involved, and (2) evaluating human response to vibration.

It has been shown in section 8.5 that the response ratio of the secondary mass (human body) to the primary mass (model slab) is of the order of 1.5, with damping of the body at 50%. In other words, when the integer multiple of the load frequency equals the fundamental frequency of the human body, the body experiences accelerations up to 50% higher than the measured floor accelerations. The avoidance of such a case can be regarded as a design target, which leads to perhaps the most important difference between the 2DOF models of TMD-structure and human-structure systems. In the design of a TMD, the frequency of the TMD should be close to the frequency of the structure for maximum response reduction, and thus maximum response in the TMD. The design for reducing human sensitivity, however, should focus on avoiding resonance with the structure frequency, hence aiming for a low response of the secondary mass (i.e. the body). So the TMD is designed on the basis of the slab but the slab should be designed on the basis of the human body.

The importance of considering the response of the human body is explained by a simple example. For vibrational serviceability problems, such as people walking across a floor, there must always be someone in a stationary position to feel the vibration. This person will therefore absorb energy and reduce the vibrations. If the stationary person were to be replaced by a transducer, it would monitor a larger vibration from the same source (i.e. there is no added energy absorption). Also, if people's perception of vibration were to be investigated, a transducer on the floor would only measure floor motion, which would be different to a person's motion, as proved in section 8.5.

It can therefore be argued that the accelerations quoted in subjective ratings of some criteria are that of the structure, and the accelerations within the human body are up to 50% higher than the measured floor accelerations. This is implicitly taken into account in the perceptibility criteria, by the subjective ratings of occupants, but not explicitly noted in any of the guidelines. It is hence important that an appropriate model of human-structure interaction is adopted in the guidelines for different situations and activities, and that the effect of the occupants be considered in the design of the structure. Furthermore, in the design codes where minimum frequency limits are quoted, the presence of a crowd will lead to interactions with the supporting structure, affecting its fundamental frequency, which also needs to be considered in the design.
Chapter 9

Field test results

9.1 Introduction

This chapter presents the results of vibration tests performed on a full-scale floor in order to obtain its dynamic properties. There are many reported floor vibration tests in the literature [Pernica and Allen (1982), Rainer and Swallow (1986), Maguire and Severn (1987), Osborne and Ellis (1990), Caverson (1992), Pavic et al. (1994), Zaman (1996)]. These all give fundamental parameters of natural frequency and damping, but less evidence exists of tests conducted to investigate the acceleration response of floors in detail. In the tests described here, the usual dynamic parameters were first measured and then the floor was excited at its fundamental frequency, as well as controlled walking tests, to derive its acceleration response. These were used in different acceptability guides to classify the slab dynamic behaviour. This was part of a series of tests commissioned by the Steel Construction Institute (SCI) in order to establish a database of floor acceleration measurements due to human activities.

The tested slab was the first floor of the SCI headquarters in Silwood Park, Ascot. The test was of a commercial nature and was conducted on 24th October 1998, when the building was unoccupied. A plan of the floor area was provided, from which locations of expected high response were deduced. Impact tests were conducted on these points to find the most critical, but representative, location for more comprehensive tests. Forced vibration experiments were then carried out on the chosen location to measure more accurately the floor natural frequencies, damping, stiffness, and mode shapes. Based on the derived fundamental frequencies, walking tests were conducted across the floor with eleven different paces and walking styles, and the responses measured accordingly. Where possible, the results are compared with those of the model slab tests and differences are discussed. A detailed description of test procedures and results can be found in Williams and Falati (1998).
9.2 Description of the floor

The first floor of the SCI headquarters building is currently being used as an open-plan office area, divided by both full-height and cantilever partitions. Other non-structural components include false flooring, false ceiling, mechanical ductwork, and a significant amount of office furniture. Figure 9.1 gives an indication of the layout of the floor, and its non-structural components. Figure 9.2 shows a detailed plan as supplied by the SCI. The dotted lines indicate the approximate positions of the false flooring panels. The floor itself consists of a 130mm deep concrete slab cast on top of a 1.2mm gauge Holorib deck, supported by the main and intermediate beams. Above the slab, the 20mm false flooring panels provide 150mm of void for electrical cables and mechanical ductwork. The panels merely rest on their pedestals in a similar formation as layout 2 of the model slab tests (see section 3.6.4), and are covered by a layer of carpet tiles. The full-height partitions are made of glass and span from top of floor panels to the underside of the false ceiling, a height of 2600mm.

The vibration tests were conducted on the floor panels (after lifting carpet tiles) and on the bare concrete slab in a selected grid, as shown in figure 9.3. At time of testing, the floor was free of personnel, other than those performing the tests. The following sections give a detailed account of these tests.

Figure 9.1: A view of the first floor in the SCI building
Figure 9.2: Detailed plan of the floor showing main and intermediate beams
9.3 Impact tests

Two types of impact test were undertaken on the floor to find the critical location. These were instrumented hammer and heeldrop tests. The hammer test is deemed to be a more accurate measurement technique, as the force exerted on the floor is known.

9.3.1 Instrumented hammer excitation

These tests were conducted on seven locations across the floor area to find the panel with the lowest fundamental frequency and highest response. The points tested were B2, B4, C3, D2, D4, G3 and J3 (figure 9.3). The equipment used is explained in section 3.3.1 and the test procedure is described in section 4.5.1. Following initial trials, the position of hammer impacts was chosen to be stationary on the bare concrete, near the centre of a selected panel, as shown in figure 9.3. The response was measured by placing the accelerometer at each point and applying ten hammer blows to obtain an accurate average reading for that point. During each test, the coherence function was constantly monitored for effects of external excitations and the tests were repeated if the coherence at the modes fell below 80%. A typical transfer function and its coherence is shown in figure 9.4.
Note that the coherence for this slab is less than the model floor tests. This is because the hammer excitation does not always input a large enough force to excite a real size floor adequately.

The frequencies obtained at all seven points are given in table 9.1 in increasing order. The lowest frequency mode appeared at around 8.4Hz for many of the tested points, and was hence deemed to be the fundamental natural frequency of the particular floor area. Tests at the centres of adjacent panels gave equal or higher values of fundamental frequency. This could have been anticipated from examination of figure 9.3, as position C3 is the centre of one of the largest spanning sections of the floor area. It was this position, and the entire floor panel in which it lies, which was selected for subsequent detailed tests. The heeldrop tests confirmed this choice as described below.

![Graph showing amplitude and coherence against frequency]

Figure 9.4: Hammer test transfer function and coherence at C3

<table>
<thead>
<tr>
<th>Test position</th>
<th>Transfer function peak frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B2</td>
<td>8.63 9.63 11.0 11.38</td>
</tr>
<tr>
<td>B4</td>
<td>8.44 9.25 9.88 11.25 12.0</td>
</tr>
<tr>
<td>C3</td>
<td>8.38 9.50 10.25 10.75 11.25</td>
</tr>
<tr>
<td>D2</td>
<td>8.38 9.63 11.25 11.0 11.88</td>
</tr>
<tr>
<td>D4</td>
<td>8.38 9.0 9.75 10.38 11.38</td>
</tr>
<tr>
<td>G3</td>
<td>8.38 9.25 10.15 11.50</td>
</tr>
<tr>
<td>J3</td>
<td>8.50 9.25 10.75 11.50 11.88</td>
</tr>
</tbody>
</table>
9.3.2 Heeldrop excitation

These tests were conducted at grid points $A3, B2, B3, C3, D2$ and $G3$ as shown in figure 9.3. At each point, a minimum of three heeldrops was undertaken to give an average of response readings. It was found that the position of application of a heeldrop, in respect to the accelerometer, affected the readings. Similar readings were obtained when: (i) the heeldrop and accelerometer were on the concrete slab, (ii) heeldrop on bare false floor panel with accelerometer on adjacent panel, and (iii) heeldrop on carpeted false floor panel with accelerometer on adjacent panel. However, when the heeldrop and accelerometer were on the same false floor panel, much higher readings were recorded, presumably due to localised high-frequency "rattling" of the panel on its supports. These tests were therefore discounted. All results reported here are for a heeldrop on a bare panel with the accelerometer on the adjacent panel.

After recording, the response data in the time-domain were converted to the frequency domain in the form of a power spectrum. For clarity, all frequencies above 25Hz were filtered out. The response and corresponding power spectrum for position $C3$ is shown in figure 9.5. The frequencies obtained from the spectra of all tested points are given in table 9.2 in increasing order. These can be compared with values of table 9.1, which show good agreement.

Table 9.2 also shows the values of peak acceleration response obtained at each point. These are averages of all the tests at a particular point, which sometimes exhibited a substantial range. Note that as expected, the peak accelerations for this floor are significantly lower than those for the model slab tests (table 5.7, page 97). This was due to the heavy mass of the field floor compared to the amplitude of applied load (i.e. lower mass ratio between applier and floor). For this floor, the largest responses were measured at points $C3$ and $G3$, which are centres of adjacent panels. Having a smaller fundamental frequency, the panel containing $C3$ was considered the more critical panel and was tested further, as described below.

<table>
<thead>
<tr>
<th>Test position</th>
<th>Peak accel. (m/s²)</th>
<th>Power spectra peak frequencies (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A3$</td>
<td>0.17</td>
<td>8.44 10.10</td>
</tr>
<tr>
<td>$B2$</td>
<td>0.27</td>
<td>8.23 9.61 10.47</td>
</tr>
<tr>
<td>$B3$</td>
<td>0.28</td>
<td>8.23 9.64</td>
</tr>
<tr>
<td>$C3$</td>
<td>0.31</td>
<td>8.28 9.53 10.51</td>
</tr>
<tr>
<td>$D2$</td>
<td>0.25</td>
<td>8.22 9.55 10.55</td>
</tr>
<tr>
<td>$G3$</td>
<td>0.35</td>
<td>5.82</td>
</tr>
</tbody>
</table>
9.4 Forced vibration tests

Following initial impact tests, which identified the shaded panel in figure 9.3 as a critical area, a comprehensive shaker test setup was used to measure the dynamic characteristics of the floor at the critical modes of vibration. The equipment used is described in section 3.3.2 and the test procedure is explained in section 4.5.2. The shaker was positioned on the bare concrete 0.5m from C3, near the centre of the test area. The floor panel was then divided into a 5×5 grid as shown in figure 9.3. Sine-sweep and single sine-wave loading tests were undertaken, as described below.

9.4.1 Sine-sweep loading

Figure 9.6 shows a typical transfer function following a sine-sweep loading, and figure 9.7 shows SDOF curves fitted at the first two modes. The values of frequency, damping, and stiffness are the parameters which define these curves and hence provide the best estimate of the equivalent parameters defining the modes of the structure (see section 4.6.4). From these tests, at the fundamental mode, the average damping ratio was 5.4% at a frequency of 8.3Hz, with the stiffness at C3 being 8.9×10^6N/m. At the second mode, damping was 4.2% with 9.7Hz and stiffness of 1.4×10^7N/m at C3. These values of damping are around a factor of four greater than the model slab readings.
Apart from obvious structural differences, the higher damping values are due to the presence of furniture, false flooring, partitions and mechanical ductwork. The derived values are also in good agreement with the ratios suggested by Murray (1981) and particularly with those mentioned in AISC (1997) (see section 2.6.4). For higher modes, the closeness of the peaks made the SDOF curve fitting less accurate. The peak frequencies (from the transfer function) for modes 3 and 4 were 10.6Hz and 11.4Hz respectively.

The mode shapes for the chosen panel were derived, and the continuity of the floor slab was assessed by taking measurements of response at adjacent panels. These measurements were taken along the centreline of the panels (gridline 3 in figure 9.3). The first four perceived modes of vibration for the tested floor panel and cross-sections of the mode shapes are shown in figures 9.8-9.11.

Figure 9.6: Transfer function and coherence of sine-sweep test at C3
Figure 9.7: SDOF model fitted to modes 1 and 2 at B4

Figure 9.8: Mode 1 modeshape and cross-section at gridline 3
Figure 9.9: Mode 2 modeshape and cross-section at gridline 3

Figure 9.10: Mode 3 modeshape and cross-section at gridline 3
Figure 9.11: Mode 4 modeshape and cross-section at gridline 3

Figure 9.12: Measured and best fit decay model for modes 1 and 2 at C3
9.4.2 Single sine-wave loading

A total of thirteen single sine-wave tests were carried out with 8.5Hz and 9.6Hz loadings at points A4, C2, C3 and D2 in figure 9.3. The shaker was suddenly stopped and the resulting decay of oscillation was measured, which would give a further independent assessment of damping for the particular mode. Figure 9.12 shows typical decay responses at the first two frequencies, with the damping calculated by fitting a SDOF curve on a specified section of the decay. The best fit curve, based on a linear system, is defined by the parameters of damping and frequency (see section 4.6.4). With these tests, average damping was 4.7% at 8.3Hz and 4.1% at 9.5Hz. Note that the decay of vibrations is not a pure exponential, as the damping will not be purely viscous (see section 2.6.2). In comparison with a typical model slab decay response (figure 4.15, page 86), more non-linearity exists in the field slab. Although the curve fitting is not perfect, the overall derived values for damping and natural frequency agree satisfactorily with the results of the previous section.

Tests with standing man

Single sine-wave tests were also conducted at points C2, C3 and D2 with three people standing by the accelerometer during the tests. Figure 9.13 shows a comparison of the response decays, with and without the men, at the second mode. The presence of the men has little effect on the overall response measured by the accelerometer. However, the average readings of all tests showed that at the first mode, the damping increased to 5.7% with a frequency of 8.1Hz, and at the second mode, damping became 4.8% with a frequency of 9.5Hz. Hence, the presence of the men had the effect of marginally increasing damping at the region where they were standing.

![Figure 9.13: Comparison between experimental results of bare slab and with three men standing by the sensor at D2 (sine-wave loading of 9.5Hz)](image)

To investigate this effect further, the 2DOF and SDOF models of chapter 8 were implemented on the field results with $\zeta_2$ taken as 50% (as for one man). Figure 9.14 shows that the models represent
the presence of the men satisfactorily. In this case, the mass of the men compared to the mass of the slab is very small and hence the effect is not as significant as for the model slab tests. This is observed in figure 9.15, which shows a comparison between the 2DOF and SDOF models. When compared to figure 8.16 (page 181), these results show that the mass ratio is a critical factor in the structure-human interaction and if the mass of the people is small compared to the mass of the slab, a SDOF model would suffice. However, if the mass ratio is large, the representation is best shown by a 2DOF model (as in chapter 8).

Figure 9.14: Comparison between model and experimental results

![Figure 9.14](image1)

Figure 9.15: Comparison between 2DOF human-structure model and SDOF slab model

![Figure 9.15](image2)

### 9.5 Walking tests

With the accelerometer positioned at $C3$, walking tests were undertaken to monitor the vibration levels generated on the floor when in normal use. A portable computer was used to generate a beat at a pre-selected frequency, so that the pace of the walker could be controlled. The direction of walking was in a straight line across the panel from position $A3\rightarrow E3$ and was started at an adjacent panel so that the walker had already established his forward momentum on approaching
A3. Only the lower modes of vibration are likely to be excited and pacing at an integer fraction of these frequencies can cause resonance, representing the worst loading cases.

The types of test were: normal walking, running, walking on the spot, and jumping on the spot. Each test was repeated with a different walker to account for variability of loading. Table 9.3 gives the type of test, along with walking frequency, response frequency, root mean square acceleration, and peak acceleration (both averaged over tests with different walkers). When compared with the results of the model slab (section 5.7.2, page 98), the induced acceleration responses for the field slab are significantly lower, due to differences in boundary conditions and mass ratios. Below, each test is examined in more detail.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Description of walk</th>
<th>Typical pace frequency (Hz)</th>
<th>Typical response freq. (Hz)</th>
<th>Average peak accel. (m/s²)</th>
<th>Average RMS accel. (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 Hz (controlled)</td>
<td>2.03</td>
<td>8.03, 10.06</td>
<td>0.071</td>
<td>0.014</td>
</tr>
<tr>
<td>2</td>
<td>2.5 Hz (controlled)</td>
<td>2.25</td>
<td>8.47, 9.09, 10.75, 11.31</td>
<td>0.094</td>
<td>0.015</td>
</tr>
<tr>
<td>3</td>
<td>3 Hz (controlled)</td>
<td>3.16</td>
<td>9.56</td>
<td>0.126</td>
<td>0.021</td>
</tr>
<tr>
<td>4</td>
<td>normal (uncontrolled)</td>
<td>2.16</td>
<td>8.63, 10.88</td>
<td>0.066</td>
<td>0.012</td>
</tr>
<tr>
<td>5</td>
<td>Two men at 2 Hz (controlled)</td>
<td>2.06</td>
<td>8.13, 9.44, 10.25, 10.84, 11.41</td>
<td>0.108</td>
<td>0.022</td>
</tr>
<tr>
<td>6</td>
<td>Two men at 2.5 Hz (controlled)</td>
<td>2.41</td>
<td>8.09, 9.53, 10.28, 11.56, 12.16</td>
<td>0.124</td>
<td>0.020</td>
</tr>
<tr>
<td>7</td>
<td>Running at 3 Hz (controlled)</td>
<td>3.28</td>
<td>9.69</td>
<td>0.115</td>
<td>0.022</td>
</tr>
<tr>
<td>8</td>
<td>normal running (uncontrolled)</td>
<td>2.63</td>
<td>8.0, 9.69</td>
<td>0.127</td>
<td>0.020</td>
</tr>
<tr>
<td>9</td>
<td>Walking on the spot at 2 Hz (controlled)</td>
<td>1.97</td>
<td>8.0, 9.97</td>
<td>0.097</td>
<td>0.021</td>
</tr>
<tr>
<td>10</td>
<td>Jumping on the spot at 2 Hz (controlled)</td>
<td>2.0</td>
<td>8.06</td>
<td>0.215</td>
<td>0.043</td>
</tr>
<tr>
<td>11</td>
<td>Jumping on the spot at 2.5 Hz (controlled)</td>
<td>2.5</td>
<td>7.53, 10.03</td>
<td>0.339</td>
<td>0.087</td>
</tr>
</tbody>
</table>

**Test 1** was conducted at a pacing rate of approximately \( \frac{1}{4} \) of the fundamental frequency of the floor. Thus, the fourth harmonic of the walking pace would be in resonance with the floor. A typical acceleration time-history and power spectrum are shown in figure 9.16. Note the build up of vibrations as the walker approaches and passes the centre of the floor. The footfalls can also be noticed from the increasing peak at every fourth cycle. The relatively high damping of the floor eliminates the possibility of a build-up of floor vibration due to successive footfalls. The spectrum shows that this pace of walking excites the first and third modes of the floor.

**Test 2** was an attempt to excite the second floor frequency with the fourth harmonic of walking, hence the walker traversed at a faster pace. From table 9.3, it is observed that as well as first and second modes, some higher modes were also excited at this pace.
Test 3 was an even faster walk, which verged on the border with running, in an attempt to excite the second and third modes with the third harmonic of walking force. Table 9.3 shows that the response was dominated by the second mode at 9.56Hz. In this instance, the pace of walking was so high that the damping between footfalls was less than previous cases, leading to accumulation of vibration response and higher acceleration readings.

Test 4 originated after testing all obvious possible controlled walking paces. The walker was now asked to traverse across the panel at a normal pace without the use of the timing signal. This test was repeated with three different walkers and a typical result is shown in figure 9.17, for time and frequency domains. The walking frequencies and induced accelerations varied in a wide range from 0.007m/s² to 0.017m/s² RMS. The mode of vibration being excited in this case depended strongly on the pace of walking of each individual involved.

Tests 5 and 6 were similar to tests 1 and 2 but two walkers were used in each case. A timing signal was used and the walkers attempted to co-ordinate their walking pace. The results of figure 9.18 indicate that at the same walking frequency, the same modes are excited as with one walker (figure 9.16), but the accelerations are increased by a sizeable margin.

Test 7 involved a person running across the panel at 3Hz and test 8 was running without the use of the timing signal. Results of figure 9.19, for uncontrolled running, show that consecutive footsteps are fast enough not to allow damping of the vibrations between footfalls. In this case, first and second modes were excited and in comparison to uncontrolled walking (figure 9.17), much higher acceleration readings were measured.

Test 9 was performed by a person walking on the spot, adjacent to the accelerometer, at 1/3 of floor fundamental frequency. The essential difference between this test and normal walking is that the walker does not have a forward momentum and hence the energy is concentrated only in the vertical direction. Therefore, a relatively large acceleration response can be observed in figure 9.20, as compared with the similar case of normal walking (figure 9.16). Notice however, that roughly the same modes of vibration were excited in both cases.

Tests 10 and 11 were performed in an attempt to produce the worst possible loading case, by which the person was jumping on the spot adjacent to C3 at a controlled frequency of 1/3 and 1/4 of floor fundamental frequency. As expected, these cases produced the highest responses. This type of activity can be regarded as a gymnasium or dance floor loading and from figure 9.21, it has significantly high harmonic component amplitudes and can excite one or more of the lower modes of the floor. For these cases, also note the substantial peaks at the lower harmonics of the loading. Similar results were also found in BRE 426 (1997).
Chapter 9: Field test results

Figure 9.16: Typical response to controlled walking at 2Hz measured at C3 (Test 1)

Figure 9.17: Typical response to uncontrolled walking measured at C3 (Test 4)
Figure 9.18: Typical response to controlled walking of two men at 2Hz measured at C3 (Test 5)

Figure 9.19: Typical response to uncontrolled running measured at C3 (Test 8)
Figure 9.20: Typical response to walking on the spot at 2Hz measured at C3 (Test 9)

Figure 9.21: Typical response to jumping on the spot at 2Hz measured at C3 (Test 10)
9.6 Classification of the floor

In this section, the walking and heeldrop response readings are used and the floor is assessed for vibration acceptability, using the CSA and ISO/BSI/ANSI perceptibility guidelines of section 2.2.2.

Figures 9.22-9.26 show the obtained acceleration responses plotted in the two scales. For forces imposed due to walking, both guidelines classify the floor as acceptable in office or residential environments (figure 9.22). When walking was controlled to create the worst case loading, (test 3), the ensuing accelerations are seen to be at the limit of acceptability of the ISO/BSI/ANSI scales for offices, when the load was regarded as intermittent (i.e. 4×base curve, see table 2.3, page 15). In comparison with equivalent model slab results, (figure 6.30, page 136), smaller responses are observed with the field slab. The effect of more than one person walking is shown in figure 9.23, where an increase in acceleration is observed to levels which are on, or slightly above, ISO/BSI/ANSI scales. These vibrations are below the acceptibility levels of CSA scales. The floor is still regarded as acceptable with two men walking in unison. For cases involving a group, the likelihood of walking-in-step is low, but in the event, higher accelerations would be expected.

The measurements for running at 3Hz and walking at 3Hz are similar (figure 9.24), since at this pace the walker is close to breaking into running. The accelerations are again higher than normal walking and on the limit of acceptability of ISO/BSI/ANSI scales. For cases of jumping, much higher response readings were observed (figure 9.25), to an extent that the guidelines classify the ensuing vibrations as unacceptable. If the jumps are of transient nature, the responses fall within the acceptibility limits of both guidelines. Hence, the use of this floor for sports activities (such as gymnasium, etc.) would need further investigation for serviceability vibrations.

Figure 9.26 shows the response of the floor due to heeldrop loading. As with the results of heeldrop tests on the model slab, (figure 6.32, page 137), the responses are much higher than the cases of walking or even jumping. Both scales classify the floor as on the border of acceptibility following heeldrop tests. Hence, the use of heeldrop as a simplified loading function for the classification of floors is questionable.

In summary, both scales regard this floor as acceptable for walking vibrations, although the acceptibility limits vary. Since the CSA scales are based on heeldrop loading results, the levels of acceptibility are seen to be higher than the ISO/BSI/ANSI scales. When subject to heeldrop loading, which is transient, the classification of the floor becomes very similar between the two guidelines. This pattern of results was also observed in the model slab tests (see section 6.6).
Figure 9.22: Vibration acceptability due to walking

Figure 9.23: Vibration acceptability due to more than one person walking

Figure 9.24: Vibration acceptability due to running
Figure 9.25: Vibration acceptability due to jumping

Figure 9.26: Vibration acceptability due to heeldrop loading
Chapter 10

Conclusions

10.1 Introduction

This research programme has investigated the effects of various architectural and non-structural components on the overall dynamic behaviour of floor slabs. In particular, the research was focused on post-tensioned floors, with the effect due to: prestressing force, false flooring, full-height and cantilever partitions, viscoelastic screed layers, TMD systems, and human-structure interaction. Such additions on a floor have the effect of changing its dynamic behaviour, particularly the key parameters of natural frequency and damping. Although natural frequency can be estimated with varying accuracy, the damping mechanisms on a floor are generally less understood. The common approach in design is to pre-suppose a damping value based on past experience of similar structures. As it is widely believed that non-structural components on a floor make a significant contribution to its damping capacity, research into the effects of such components is merited.

This dissertation has described tests on a 50% scaled one-way spanning post-tensioned concrete slab, subjected to continuous and transient loading (as well as walking). The response and dynamic properties of the test floor were measured after each change to its configuration and the results were complemented by analytical modelling. The test procedure and laboratory results were investigated on a full-scale field slab and were seen to agree well with real floors. In this chapter, the findings are explained due to each non-structural component, and recommendations are given as to their use. Suggestions for further investigation are explained in section 10.4. First, some comments are made as to the best test procedure technique.
10.2 Experimental procedures

10.2.1 Loading apparatus

The impact hammer test equipment was supplied as a standard kit and did not require significant modification. The shaker equipment, however, was designed and built specifically for the research programme, as explained in section 3.3.2. The following list summarises the advantages and disadvantages of each method:

- In the hammer test, the frequency and amplitude of the forcing vibrations are inversely proportional such that for a larger frequency range, the tester must be content with a low forcing amplitude. Moreover, the actual range of applied frequencies is restricted, which is determined by different hammer tips (see figure 3.3, page 50). As a result, one cannot look at the behaviour of the floor in the proximity of its natural frequencies, but has to be content with a large range around one or more of the natural frequencies.

- In the shaker test, the frequency and amplitude of the imposed force are independent, so that one can be altered without affecting the other. In this way, by increasing the amplitude at a frequency range close to one of the floor natural frequencies, better resonance at that frequency can be achieved, with sharper and clearer peaks of the transfer function.

- In the shaker test, the input energy can be concentrated at the region of resonance, so that the response of the floor becomes large. This is a distinct advantage in the floor damping measurements, because once the shaker is switched off, a clear exponential decay of response is observed. In the hammer test, the applied impulse energy to the floor is not sufficient to give a very high response.

- For the same grid point on a floor, the shaker test gives better coherence values than the hammer test. This is illustrated for the field slab by comparing figures 9.4 (page 191) and 9.6 (page 194). The reasons for better shaker coherence are that more energy is concentrated in the required frequency range, and that the continuous forcing vibration can drown out the external noise. In the hammer test, the short transient vibration is prone to external noise, which may occur at the instant of impact.

- For field tests, the hammer apparatus is less bulky and involves fewer components, hence requires less time to be ready on site. The comparison between the two test setups can be seen in figure 4.4 (page 75).
A detailed study of the hammer and shaker apparatus was conducted by Falati (1996), who concluded that both methods were perfectly sound in the testing of floors for vibration. However, he suggested that if all aspects were considered, the advantages of the shaker test out-scored those of the hammer. With the research programme being focused on damping capacity measurements, the ability of the shaker to input extra energy at given frequencies was seen to be a major advantage. Hence, the shaker equipment was utilised in most of the tests.

10.2.2 Measurement apparatus

During the testing programme, measurements were made of the loading applied, and the acceleration response of the slab. The data were analysed and recorded by the spectrum analyser. Throughout the experiments, the loadcell and accelerometers were checked for errors and recalibrated, where possible. At all times, the QA procedures of section 5.8 were adhered to, which included: immediate repeatability checks, homogeneity checks, reciprocity checks, coherence function checks, and end of test repeatability checks. The use of these checks in any site investigation is strongly recommended.

10.3 Model slab vibration behaviour

10.3.1 Effect of prestressing force

The slab was tested in the states of: no prestress, 57% prestress, and full prestress. The following conclusions were derived from these tests and their analytical representation:

- Modelling prestressing as an external axial force is not a true representation of its effects. It can be shown by energy considerations that the line of action of the prestressing force moves integral with the bending of the slab and has no effect on the slab fundamental frequency (see section 6.2.1).

- Slab natural frequency is independent of prestressing force but indirectly affected by it, due to a change in section modulus and amount of cracking in the slab.

- The theoretical methods of natural frequency estimation, in their present form, can give inaccurate predictions for post-tensioned floors, partly because they do not take the level of prestress into account.
• It is possible to approximately predict the changes in natural frequency, due to a prestress force, by considering the changes in the effective second moment of area, $I_e$, of the slab. If these changes in $I_e$ are included in the theoretical methods of frequency estimation, closer predictions to experimental results can be obtained.

• Damping of the slab greatly reduces with an increase in prestress due to closing up of micro-cracks and, hence, an increase in effective second moment of area, $I_e$. However, the relationship between $\zeta$ and $I_e$ is not clear.

• The addition of live load decreases the natural frequencies and increases damping depending on the level of prestress.

10.3.2 Effect of false flooring

Tests to assess the contribution of false floor panels to slab dynamic behaviour, gave the following conclusions and recommendations:

• False floor panels reduce the overall slab fundamental frequency due to the added mass. This reduction is dependent upon the number of false floor panels used.

• When rigidly attached to the pedestals, the floor panels simply added mass to the structure and did not introduce any significant additional damping. In these tests, the increase in slab damping was 6%.

• When some panel edges rested on the pedestals, there was a marked increase in damping. This was most likely caused by sliding and/or gapping between the panels and pedestals. Here, the increase in slab damping was 66%.

The results show that contrary to popular belief, merely the presence of false floors does not enhance the dynamic behaviour of the system. It depends on factors such as the type of flooring used and the nature of their installation. This raises the possibility that false floors could be designed to allow increased motion between panel and pedestal as a deliberate energy dissipation device. Of the false flooring systems available in market today, the results suggest that the use of panels which rest on the pedestals (e.g. heavy timber panels), may lead to an increased damping capacity of a floor, compared to panels which are rigidly fixed to the floor.
10.3.3 Effect of full-height partitions

The results of tests have shown that a significant effect on the slab vibration parameters is gained by the addition of full-height partitions. When rigidly attached between two floors, the partition effectively acts as a line support, resulting in a substantial increase in slab stiffness. In the model slab tests, addition of a single full-height partition at midspan, led to an increase in floor fundamental frequency from 8Hz to 20Hz (i.e. +154%). Also, an increase in fundamental mode damping of 77% was observed. By measuring the behaviour of the partition itself, it was found to undergo axial bending deformations at the same frequency as the slab vibration. Hence, the results illustrate that partitions, which are regarded as non-structural components, can in fact behave in a structural way and add extra stiffness to the main system. If well designed, this is a very inexpensive way of alleviating annoying vibrations on a problematic floor.

As described in section 6.4.1, an analytical model was established, which represented the full-height partition as an extra spring, acting on the underside of the slab and rigidly fixed at the bottom (see figure 6.8, page 113). From this model, a simple design proposal was derived for simply supported beams in the form:

\[
\frac{\omega_n^2}{\omega_0^2} = 1 + \frac{k_p}{2\pi^2\rho bhL} \sin^2 \left( \frac{\pi a}{L} \right)
\]  

(10.1)

where \(k_p\) is the stiffness of the partition material, \((a)\) is the distance of the partition from the nearest support, \(\omega_0\) is the beam frequency before addition of the partition, and \(\omega_n\) is the frequency after the partition has been installed. This formula can be used in two ways. Either the effect on the fundamental frequency following the addition of pre-planned partitions can be derived, or a target frequency can be decided (see table 2.5, page 44), which would determine the type and stiffness of the partition walls needed. Typical stiffness values for gypsum plasterboard partitions are given in table 6.5, which were derived experimentally (see appendix E).

10.3.4 Effect of cantilever partitions

It has been found that cantilever partitions can be designed to have maximum beneficial effect on slab vibration properties. They were seen to be most effective when positioned perpendicular to the slab span and at a point where the fundamental mode shape of the slab has a high slope. This is because when the slab is undergoing vibration, a rotation is applied at the base of the partitions, leading to sway and hence dissipation of energy. The larger the rotation, the bigger the sway. Therefore, a partition at midspan will have less energy dissipated compared to one at quarter span (see section 64.2).
The experimental results were supported by an analytical model of a cantilever beam subjected to a sinusoidal base rotation (see figure 6.15, page 121), which was derived from the vertical vibration response of the slab. The model results show that the frequency of the base rotation has a governing influence on the natural modes of the partitions. The model and experimental results were seen to be in good agreement for the tested partitions (see figure 6.20, page 125).

Finally, experiments have shown that if the cantilever partitions are connected to each other, transfer of energy takes place from ones with higher response to ones with lower response, increasing the overall energy absorption of the whole partition system.

10.3.5 Effect of viscoelastic screed layers

As explained in section 6.5.1, problems were encountered in the mixing and casting stages of the screed layers. It was observed that using viscoelastic material, as an admixture to concrete, requires a certain amount of experience in order to obtain the correct mix. Hence, the problems encountered with casting of the screed layers were more likely due to the inexperience of the casting team. As a result, the cube strengths and Youngs moduli of the treated concrete were much lower than expected.

The effect of the screed layers on natural frequency of the slab was satisfactorily predicted by regarding the layers as structural, rather than non-structural, additions. The predictions were seen to be marginally higher than the experimental results and the differences were attributed to the bonding between the screed and the main slab. The damping capacity of the slab was also seen to increase with the addition of the layers. This increase was 5% with a layer of 50l/m$^3$ Concreddamp concentration, and 14% with a screed of 85l/m$^3$ Concreddamp concentration. These increases were less than expected and attributed to the casting problems explained above.

To show that using such treated screed layers is a viable option in reducing slab response, an analysis was presented of a two layered composite structure (see section 6.5.2). In the analysis, full bonding of the two layers was assumed and the overall damping of the composite structure was derived as the weighted average of the damping ratios of individual components such that:

$$\zeta = \frac{\sum \zeta_i S_i}{\sum S_i}$$  \hspace{1cm} (10.2)

where $S_i$ is the energy stored by the $i^{th}$ component and $\zeta_i$ is its damping ratio. For a free viscoelastic layer, by considering the stored strain energy in each layer, a prediction of the overall slab damping due to the addition of the two screed layers could be achieved (equation 6.51, page 129).
Ideally, the damping of the viscoelastic layer would be expected to be much higher than the slab. Although this was not achieved in the experimental tests, the overall damping of the composite would be expected to increase with higher values of screed layer damping ratio.

10.3.6 Addition of tuned vibration absorbers

As explained in chapter 7, a simple tuned mass damper (TMD) system was proposed to reduce vibrations of long-span floor slabs. The system consisted of layers of plywood sheets, simply supported on angle sections at the underside of the slab. The tuning of the system was carried out by placing weights on the plywood. Different sizes and configurations of plywood were attempted and it was possible to reduce vibrations of the model experimental slab by as much as 80%. Of the eight different layouts attempted, the one with two layers of plywood and polythene in-between gave best results (see section 7.3.1). Some of the findings from the experiments were:

- two layers of plywood TMD gave faster slab vibration decay than one layer
- wider plywood sheets showed faster decay of slab as opposed to narrow plywood
- narrowply had twice the response at TMD level compared to widely
- with polythene or rubber between two ply layers, slab decay was marginally faster than with nothing in-between (due to aiding of friction)

To investigate the structure-TMD interaction further, a time-stepping numerical model for the TMD system was devised and optimised to best represent the experimental results (section 7.4.2). The model was then extended to other slab and TMD configurations. A possible design method was proposed for the TMD system, which could be applied to many slab configurations (see section 7.6.1). The model was also seen to be very effective in reducing annoying slab vibrations due to intermittent loading, such as jumping and walking. A full worked example was illustrated for a typical slab (see appendix F).

10.3.7 Effect of human-structure interaction

Tests with a stationary standing man and his equivalent weight on the slab, led to the following findings:

- The fundamental frequency of the model slab decreased with the addition of a stationary standing man, and also with the addition of his equivalent mass.
• A stationary man significantly increased the model slab damping ratio by as much as 250%, as compared to 32% when considering the man as a mass (see table 8.1, page 170).

• Two stationary people had an approximately similar effect on slab vibration reduction as one stationary person (section 8.3.2). Hence, one person is adequate in laboratory experiments of this nature, when considering floors with few occupants.

• The energy absorption capacity of the body was confined within a fixed frequency range and was lost at higher frequencies (section 8.3.2). This is the range of natural frequencies of the component parts of the body.

Two analytical models were derived (section 8.4). The first model was an undamped 2DOF representation of the slab and the man, used to derive the body fundamental frequency (section 8.4.1). The second model represented the body as a SDOF system, with inherent stiffness and damping, on top of the damped SDOF model for the slab (section 8.4.2). The conclusions obtained from the models were:

• By considering the generalised slab mass and stiffness, and the generalised mass of the standing man, the frequency of the human body was derived as 10.43Hz, with a stiffness of $1.07 \times 10^5 \text{N/m}$ (section 8.4.1).

• The models show that the addition of occupants on a structure leads to changes in its fundamental frequency, depending on the mass ratio and the original fundamental frequency of the structure. If the structure frequency is lower than the body frequency, the presence of occupants will lead to a reduction in the fundamental frequency of the structure, with a new higher mode appearing to represent the motion of the occupants. For high frequency structures, the opposite is the case and the presence of occupants will increase the fundamental frequency with a new lower mode appearing for the occupants (see figure 8.9, page 176). In all cases, the individual structure and body frequencies lie in between the two frequencies of the human-structure system.

• With the addition of people on a structure, the new mode representing the vibration of the body can be calculated. However, it was not possible to observe this mode from the measurements of response on the structure.

• The overall damping of the human body was found to be in the range 45-55% critical.

• At resonance, the human body may experience accelerations up to 50% higher than the measured floor accelerations. This is implicitly accounted for in the perceptibility criteria of section 2.2.2 (by subjective ratings), but not explicitly noted in any of the guidelines.
• The current design codes, which assume the occupants as live load, should be altered to take account of the significant vibration reduction attributable to the human body, and also of the changes to the fundamental frequency of a structure, which may be caused by the occupants.

10.3.8 Field testing

A vibration test is reported in chapter 9 on a full scale office floor. Modal analysis techniques of chapter 4 were used to derive the dynamic characteristics of the floor in terms of its natural frequencies, damping, and stiffness parameters. The types of loading were hammer, heeldrop, shaker, and walking tests. During the tests, the observations of section 10.2.1 were confirmed in that a much better coherence and control of the test could be achieved with the use of the shaker, as opposed to the hammer. With the floor being fully furnished, its damping was higher than that of the model slab, but within the recommended ranges given in section 2.6.4.

Tests were also carried out with three men standing on the slab near the accelerometer, and the results confirmed the human-interaction model of chapter 8 (see section 9.4.2). Walking tests were conducted on the floor with various paces and types of walk to resonate the required natural frequencies (section 9.5). The highest responses were observed as a result of jumping on the floor.

10.3.9 Summary

Table 10.1 shows a simple summary of the effects of the various non-structural components on the vibration characteristics of the model experimental slab. It also shows the effects due to the addition of the TMD system and stationary occupant.

10.4 Scope for further research

The results obtained in these tests have given scope for further research into the field of concrete slab vibration behaviour. Possibilities for investigations into the effects of individual aspects are discussed below.

Prestressing force: Following these tests on a one-way spanning slab, the effects of prestressing on the dynamic behaviour of two-way slabs can be investigated. The prestressing force in each of the two orthogonal directions can be altered, changing aspect ratios, and the subsequent effects can be studied in terms of the ratios of prestress force in the two directions. Using
Table 10.1: Summary of the effects of various additions on the model slab
(↑ = increased, ↓ = decreased, ↔ = no change)

<table>
<thead>
<tr>
<th>Alteration to slab</th>
<th>Effect on slab dynamic behaviour</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>fund. freq.</td>
</tr>
<tr>
<td>Increase of prestress force</td>
<td>↑</td>
</tr>
<tr>
<td>Inclusion of false flooring</td>
<td>↓</td>
</tr>
<tr>
<td>Inclusion of full-height partitions</td>
<td>↑</td>
</tr>
<tr>
<td>Inclusion of cantilever partitions</td>
<td>↓</td>
</tr>
<tr>
<td>Inclusion of screed layers</td>
<td>↑</td>
</tr>
<tr>
<td>Addition of TMD system</td>
<td>↔</td>
</tr>
<tr>
<td>With stationary standing man</td>
<td>↓</td>
</tr>
</tbody>
</table>

This data, and the effects on natural frequency, can give rise to the derivation of a method for predicting the fundamental frequency of two-way spanning post-tensioned floors. Present methods do not give accurate predictions. The Concrete Society (1994) method, specifically designed for two-way slabs, considers each orthogonal direction separately, leading to poor estimates of frequency [Williams and Waldron (1994)]. In addition, more detailed research can be conducted to investigate the amount of cracking in a concrete section, and its relationship with damping. This may prove to be difficult and could need sophisticated equipment to quantify the amount of cracking.

**False flooring:** The conclusions derived from these laboratory experiments can be tested on real floors, where the type and manner of installation can be thoroughly investigated in relation to slab damping. At present, merely the existence of false floors is seen as evidence of additional damping. The experimental results have shown this assumption to be somewhat incorrect, hence more evidence is needed to confirm the results.

**Full-height partitions:** The analytical model of these partitions can be implemented on site and the accuracy of the prediction of constrained frequency can be gauged. To make the suggested design guideline applicable to types of partition other than plasterboard, a database of different partition stiffnesses can be developed. Furthermore, the model could be extended to more than one partition, and also to assess the effects on two-way spanning slabs.

**Cantilever partitions:** The sway characteristics of these partitions need to be investigated further with regards to their energy dissipation. Although the positioning of cantilever partitions is mainly an architectural decision, the results have shown that their direction and placement, in respect to the slab span, affects their useful contribution to the floor dynamic behaviour.
There is therefore scope for more research, particularly with other partition types, such as mobile partitions.

**Viscoelastic screed layers:** Constrained viscoelastic materials have been used extensively in vehicle and aeronautical industries. The use of such materials on composite floors, applied at the flanges, has also given satisfactory results [Farah et al. (1977)]. Although using the viscoelastic admixture as a free (unconstrained) layer on the main slab gave only a very small increase in damping, its usefulness has been shown by an analytical model and also recorded experimentally by others [Moiseev (1991)]. More research is possible in this field to find the most suitable concentration of material and best layer of screed, to increase damping of a given slab. Also, its effectiveness can be linked analytically to a reduction in slab response, by a suitable model. The use of constrained layers is also possible in concrete floors, where a layer of a suitable material (e.g. rubber) can be sandwiched between two thin screed layers. Bending of the system will cause the composite layer to undergo shear, dissipating energy in the rubber. The advantage of such a system is that properly designed constrained layers can usually provide more damping with less added weight than free layers. However, this brings the added complication of frequency-dependent effects as explained by Ungar (1963).

**Plywood vibration absorbers:** Having proposed a design guideline for the application of plywood TMDs, tests need to be conducted in the field for its validation. The TMDs can be improved with further investigation on the damping capacity of the plywood layers and on the possibility of using viscous dampers between the sheets, with known values of energy absorption. This also raises the potential of using other more uniform and permanent materials in place of the plywood sheets. The downside with any TMD system is that it is only applicable to a given frequency. However, recent research has shown successful results in the use of multiple liquid dampers for buildings [Fujino and Sun (1993)]. Hence, the use of multiple plywood TMDs can be investigated, where a row of absorbers could be installed on a floor, each tuned to a different frequency. These would coincide with any possible annoying frequency and also with second and third natural frequencies of the slab, where applicable.

**Structure-human interaction:** Some of the research needs in this field are: (i) the precise difference in energy absorption between a stationary and a non-stationary man, (ii) effect of the number of occupants as a function of vibration perceptibility and annoyance, (iii) more accurate measurements of the damping of the human body for a range of conditions, (iv) full evaluation of the modal mass of the human body [Ji (1995)], (v) consideration of body vibrations other than vertical (e.g. sway), (iv) possibility of calculation of the human-structure
interaction for a given activity and structure, hence modification of existing design codes to incorporate the number of occupants.

**Field test results:** At present, most vibration tests on full scale floors concentrate on measurements of natural frequency, damping, and heeldrop peak acceleration. Less data is available on the acceleration responses induced by normal human activities on a given floor, particularly walking. Hence, a database of floor types and their respective acceleration responses is needed, from which a better judgement can be made on the merits of various design guidelines.
Appendix A

Full experimental results

This appendix presents the detailed results of all experiments. First, the natural frequency and damping values are given from frequency and time domain analyses. The derived acceleration responses, following single sine wave loading tests, are then presented.

Frequency domain natural frequencies

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18 0.66 0.18 0.66 0.18</td>
<td>0.18 0.66 0.18</td>
</tr>
<tr>
<td>reinforced without post-tension (weekly tests)</td>
<td>39*</td>
<td>7.60 - 30.76 - 65.38</td>
<td>7.54 30.38 64.58</td>
</tr>
<tr>
<td></td>
<td>39*</td>
<td>7.54 - 30.68 - 65.33</td>
<td>7.54 30.38 64.94</td>
</tr>
<tr>
<td></td>
<td>39*</td>
<td>7.51 - 29.69 - 62.05</td>
<td>7.54 30.38 64.58</td>
</tr>
<tr>
<td>100kN post-tension force</td>
<td>39</td>
<td>7.60 7.61 29.38 29.72 63.96</td>
<td>7.63 29.72 64.19</td>
</tr>
<tr>
<td></td>
<td>39*</td>
<td>7.76 - 30.07 - 64.42</td>
<td>7.63 29.72 64.19</td>
</tr>
<tr>
<td>175kN post-tension force (design force)</td>
<td>39</td>
<td>8.00 8.05 30.29 30.14 65.75</td>
<td>8.01 31.03 65.95</td>
</tr>
<tr>
<td></td>
<td>39*</td>
<td>7.99 - 31.47 - 66.00</td>
<td>8.01 31.03 65.95</td>
</tr>
<tr>
<td></td>
<td>39*</td>
<td>7.98 - 31.72 - 66.09</td>
<td>8.01 31.03 65.95</td>
</tr>
<tr>
<td>detensioned before demolishing (with aired layers)</td>
<td>21</td>
<td>9.16 9.17 36.65 36.33 71.26</td>
<td>9.16 35.59 71.26</td>
</tr>
<tr>
<td></td>
<td>[* = Instrumented hammer tests]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A.2: Effect of dead load on Natural frequency (frequency-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18 0.66 0.18 0.66 0.18</td>
<td>0.18 0.66 0.18</td>
</tr>
<tr>
<td>100kN post-tension force with dead load of 0.25kN/m²</td>
<td>39</td>
<td>7.25 7.20 28.71 28.31 61.53</td>
<td>7.22 28.51 61.53</td>
</tr>
<tr>
<td>175kN post-tension force with dead load of 0.25kN/m²</td>
<td>39</td>
<td>7.71 7.69 29.13 29.16 63.75</td>
<td>7.70 29.15 63.75</td>
</tr>
</tbody>
</table>
Table A.3: Effect of false flooring on Natural frequency (frequency-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18</td>
<td>0.06</td>
</tr>
<tr>
<td>floor layout1 (see fig. 3.15)</td>
<td>39</td>
<td>7.88</td>
<td>7.87</td>
</tr>
<tr>
<td>(measured on slab level)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>floor layout1 (measured on false floor level)</td>
<td>10</td>
<td>7.88</td>
<td>7.87</td>
</tr>
<tr>
<td>floor layout2 (see fig. 3.16)</td>
<td>39</td>
<td>7.61</td>
<td>7.63</td>
</tr>
<tr>
<td>(measured on slab level)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>floor layout2 (measured on false floor level)</td>
<td>15</td>
<td>7.56</td>
<td>7.61</td>
</tr>
</tbody>
</table>

Table A.4: Effect of partitions (see figure 3.12 for layout) on Natural frequency (frequency-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18</td>
<td>0.06</td>
</tr>
<tr>
<td>five cantilever partitions perpendicular to slab span (nos. (1)-(5) in fig. 3.12)</td>
<td>39</td>
<td>7.77</td>
<td>7.76</td>
</tr>
<tr>
<td>three cantilever partitions perpendicular to slab span (nos. (2)-(4) in fig. 3.12)</td>
<td>39</td>
<td>7.87</td>
<td>7.87</td>
</tr>
<tr>
<td>five cantilever partitions and three transverse to slab span (nos. (1)-(6) in fig. 3.12)</td>
<td>39</td>
<td>7.80</td>
<td>7.82</td>
</tr>
<tr>
<td>three parallel to slab span (nos. (6)-(8) in fig. 3.12)</td>
<td>39</td>
<td>-</td>
<td>8.00</td>
</tr>
<tr>
<td>one full-height partition perpendicular to slab span</td>
<td>39</td>
<td>20.37</td>
<td>20.31</td>
</tr>
<tr>
<td>five cantilever and one full-height partitions</td>
<td>39</td>
<td>20.35</td>
<td>20.27</td>
</tr>
</tbody>
</table>

Table A.5: Effect of stationary man on Natural frequency (frequency-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18</td>
<td>0.06</td>
</tr>
<tr>
<td>one man standing near slab centre</td>
<td>39</td>
<td>7.93</td>
<td>7.91</td>
</tr>
<tr>
<td>equivalent mass of man near slab centre</td>
<td>39</td>
<td>7.67</td>
<td>7.70</td>
</tr>
</tbody>
</table>

Table A.6: Effect of visco-elastic resin screeds on Natural frequency (frequency-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18</td>
<td>0.06</td>
</tr>
<tr>
<td>25mm thick screed of low Conceldamp concentration (see section 3.6.7)</td>
<td>39</td>
<td>8.75</td>
<td>8.79</td>
</tr>
<tr>
<td>additional 25mm thick screed of high Conceldamp concentration</td>
<td>8</td>
<td>10.09</td>
<td>10.17</td>
</tr>
</tbody>
</table>
### Table A.7: Effect of combinations over slab on Natural frequency (frequency-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td>0.18 0.06</td>
<td>0.18 0.06 0.18</td>
<td>0.18 0.06 0.18</td>
</tr>
<tr>
<td>false floor layout2 and standing man (on slab level)</td>
<td>39</td>
<td>- 7.64 - noisy -</td>
<td>7.64 - -</td>
</tr>
<tr>
<td>false floor layout2 and standing man (on floor level)</td>
<td>15</td>
<td>- 7.69 - noisy -</td>
<td>7.69 - -</td>
</tr>
<tr>
<td>five cantilever partitions and dead load at 0.25kN/m²</td>
<td>39</td>
<td>7.63 7.66 28.94 28.88 63.76</td>
<td>7.63 28.91 63.76</td>
</tr>
</tbody>
</table>

### Table A.8: Effect of combinations over the first screed layer on Natural frequency (frequency-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td>0.18 0.06</td>
<td>0.18 0.06 0.18</td>
<td>0.18 0.06 0.18</td>
</tr>
<tr>
<td>five cantilever partitions</td>
<td>33</td>
<td>- 8.75 - 34.57 -</td>
<td>8.75 34.57 -</td>
</tr>
<tr>
<td>one standing man</td>
<td>21</td>
<td>- 8.85 - 34.94 -</td>
<td>8.85 34.94 -</td>
</tr>
<tr>
<td>five cantilever partitions and standing man</td>
<td>12</td>
<td>- 8.72 - 34.46 -</td>
<td>8.72 34.46 -</td>
</tr>
</tbody>
</table>

### Table A.9: Effect of combinations over second screed layer on Natural frequency (frequency-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td>0.18 0.06</td>
<td>0.18 0.06 0.18</td>
<td>0.18 0.06 0.18</td>
</tr>
<tr>
<td>one standing man</td>
<td>21</td>
<td>- 10.02 - 39.62 -</td>
<td>10.02 39.62 -</td>
</tr>
<tr>
<td>two standing man</td>
<td>21</td>
<td>- 10.00 - 39.48 -</td>
<td>10.00 39.48 -</td>
</tr>
<tr>
<td>equiv. mass of two standing men</td>
<td>24</td>
<td>- 9.87 - 38.13 -</td>
<td>9.87 38.13 -</td>
</tr>
<tr>
<td>five cantilever partitions</td>
<td>27</td>
<td>- 10.04 - 39.00 -</td>
<td>10.04 39.00 -</td>
</tr>
<tr>
<td>five cantilever partitions and dead load at 0.25kN/m²</td>
<td>9</td>
<td>- 9.77 - 38.31 -</td>
<td>9.77 38.31 -</td>
</tr>
<tr>
<td>five cantilever partitions and one standing man</td>
<td>21</td>
<td>- 9.97 - 38.88 -</td>
<td>9.97 38.88 -</td>
</tr>
<tr>
<td>false floor layout1</td>
<td>21</td>
<td>- 10.09 - 38.98 -</td>
<td>10.09 38.98 -</td>
</tr>
<tr>
<td>false floor layout1 and one standing man</td>
<td>12</td>
<td>- 10.07 - 38.91 -</td>
<td>10.07 38.91 -</td>
</tr>
<tr>
<td>false floor layout1 and equiv. mass of one standing man</td>
<td>12</td>
<td>- 9.80 - 39.25 -</td>
<td>9.80 39.25 -</td>
</tr>
<tr>
<td>false floor layout2 and one standing man</td>
<td>21</td>
<td>- 9.89 - 37.86 -</td>
<td>9.89 37.86 -</td>
</tr>
<tr>
<td>false floor layout2 and one standing man</td>
<td>11</td>
<td>- 9.91 - noisy -</td>
<td>9.91 - -</td>
</tr>
</tbody>
</table>
### Time-domain natural frequencies

#### Table A.10: Effect of prestress on Natural frequency (time-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>±5</td>
<td>±10</td>
</tr>
<tr>
<td>100kN post-tension force</td>
<td>39</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>175kN post-tension force</td>
<td>39</td>
<td>8.04</td>
<td>8.03</td>
</tr>
<tr>
<td>detensioned before demolishing (with screw)</td>
<td>8</td>
<td>8.96</td>
<td>9.01</td>
</tr>
</tbody>
</table>

#### Table A.11: Effect of dead load on Natural frequency (time-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>±3</td>
<td>±6</td>
</tr>
<tr>
<td>100kN prestress force with load of 0.25kN/m²</td>
<td>39</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>175kN prestress force with load of 0.25kN/m²</td>
<td>39</td>
<td>7.60</td>
<td>7.73</td>
</tr>
</tbody>
</table>

#### Table A.12: Effect of false flooring on Natural frequency (time-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average vals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>±10</td>
<td>±15</td>
</tr>
<tr>
<td>floor layout1 (see figure 3.15) (on slab level)</td>
<td>39</td>
<td>7.89</td>
<td>7.91</td>
</tr>
<tr>
<td>floor layout1 (on false floor level)</td>
<td>9</td>
<td>7.90</td>
<td>7.90</td>
</tr>
<tr>
<td>floor layout2 (see figure 3.16) (on slab level)</td>
<td>39</td>
<td>-</td>
<td>7.49</td>
</tr>
<tr>
<td>floor layout2 (on false floor level)</td>
<td>15</td>
<td>-</td>
<td>7.49</td>
</tr>
</tbody>
</table>

#### Table A.13: Effect of partitions (see figure 3.12 for layout) on Natural frequency (time-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average vals</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>±10</td>
<td>±15</td>
</tr>
<tr>
<td>five cantilever partitions perpendicular to slab span (nos. (1)-(8) in fig. 3.12)</td>
<td>39</td>
<td>7.66</td>
<td>7.68</td>
</tr>
<tr>
<td>three cantilever partitions perpendicular to slab span (nos. (2)-(4) in fig. 3.12)</td>
<td>39</td>
<td>7.77</td>
<td>7.83</td>
</tr>
<tr>
<td>five perpendicular and three transverse to slab span (nos. (1)-(8) in fig. 3.12)</td>
<td>39</td>
<td>7.70</td>
<td>7.75</td>
</tr>
</tbody>
</table>
### Table A.14: Effect of stationary man on Natural frequency (time-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td>±10 ±20 ±30</td>
<td>±10 ±20 ±30</td>
<td>±10 ±20 ±30</td>
</tr>
<tr>
<td>one man near slab centre</td>
<td>39</td>
<td>7.81 - 7.57 7.57 - 31.51 65.63</td>
<td>7.65 31.51 65.63</td>
</tr>
<tr>
<td>equivalent mass of man near</td>
<td>39</td>
<td>7.58 7.65 7.74 - 31.41 64.10</td>
<td>7.66 31.41 64.10</td>
</tr>
<tr>
<td>slab centre</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table A.15: Effect of visco-elastic resin screeds on Natural frequency (time-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td>±10 ±15 ±20</td>
<td>±10 ±15 ±20</td>
<td>±10 ±15 ±20</td>
</tr>
<tr>
<td>25mm thick screed of low Concrodamp conc. (see</td>
<td>21</td>
<td>8.71 8.73 - 34.91 noisy</td>
<td>8.72 34.91 noisy</td>
</tr>
<tr>
<td>section 5.6.7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>additional 25mm thick screed of high Concrodamp</td>
<td>27</td>
<td>- 10.17 10.14 39.02 noisy</td>
<td>10.15 39.02 noisy</td>
</tr>
<tr>
<td>conc.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table A.16: Effect of combinations over first screed layer on Natural frequency (time-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td>±15 ±20 ±30</td>
<td>±15 ±20 ±30</td>
<td>±15 ±20 ±30</td>
</tr>
<tr>
<td>five cantilever partitions</td>
<td>21</td>
<td>8.64 8.65 - - 8.64</td>
<td>8.64 - -</td>
</tr>
<tr>
<td>one standing man</td>
<td>12</td>
<td>8.75 8.71 - - 8.73</td>
<td>8.73 - -</td>
</tr>
<tr>
<td>five cantilever partitions and standing</td>
<td>21</td>
<td>8.63 8.64 - - 8.63</td>
<td>8.63 - -</td>
</tr>
<tr>
<td>man</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table A.17: Effect of combinations over second screed layer on Natural frequency (time-domain analysis)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Natural Frequencies (Hz)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td>±15 ±20 ±30</td>
<td>±15 ±20 ±30</td>
<td>±15 ±20 ±30</td>
</tr>
<tr>
<td>one standing man</td>
<td>12</td>
<td>9.94 9.93 - - 9.93</td>
<td>9.93 - -</td>
</tr>
<tr>
<td>two standing men</td>
<td>12</td>
<td>9.90 9.95 - - 9.93</td>
<td>9.93 - -</td>
</tr>
<tr>
<td>equiv. mass of one man</td>
<td>12</td>
<td>9.86 9.88 - - 9.87</td>
<td>9.87 - -</td>
</tr>
<tr>
<td>equiv. mass of two men</td>
<td>12</td>
<td>9.75 9.74 - - 9.75</td>
<td>9.75 - -</td>
</tr>
<tr>
<td>five cantilever partitions</td>
<td>12</td>
<td>9.93 9.95 37.84 - 9.94</td>
<td>37.84 - -</td>
</tr>
<tr>
<td>five cantilever partitions and dead</td>
<td>9</td>
<td>9.74 9.74 - - 9.74</td>
<td>9.74 - -</td>
</tr>
<tr>
<td>load at 0.25kN/m²</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>five cantilever partitions and</td>
<td>9</td>
<td>9.88 9.91 - - 9.90</td>
<td>9.90 - -</td>
</tr>
<tr>
<td>one standing man</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>false floor layout</td>
<td>18</td>
<td>9.97 9.98 - - 9.97</td>
<td>9.97 - -</td>
</tr>
<tr>
<td>false floor layout and one standing</td>
<td>8</td>
<td>9.97 9.95 - - 9.96</td>
<td>9.96 - -</td>
</tr>
<tr>
<td>man</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>false floor layout2 and equiv. mass of</td>
<td>8</td>
<td>9.89 9.89 - - 9.89</td>
<td>9.89 - -</td>
</tr>
<tr>
<td>one man</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>false floor layout2</td>
<td>10</td>
<td>9.78 9.79 - - 9.79</td>
<td>9.79 - -</td>
</tr>
<tr>
<td>false floor layout2 and one standing</td>
<td>8</td>
<td>9.82 9.80 - - 9.81</td>
<td>9.81 - -</td>
</tr>
</tbody>
</table>
Frequency domain damping ratios

Table A.18: Effect of prestress on Damping (half-power method)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 1st 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequency resolution (Hz)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18 0.06</td>
<td>0.18 0.06</td>
</tr>
<tr>
<td>reinforced without post-tension</td>
<td>39*</td>
<td>2.30 - 2.47 - 1.19</td>
<td>1.90 2.42 1.48</td>
</tr>
<tr>
<td></td>
<td>39*</td>
<td>2.05 - 2.22 - 1.16</td>
<td></td>
</tr>
<tr>
<td></td>
<td>39*</td>
<td>1.65 - 2.40 - 1.51</td>
<td></td>
</tr>
<tr>
<td></td>
<td>39*</td>
<td>1.60 - 2.57 - 2.07</td>
<td></td>
</tr>
<tr>
<td>100kN post-tension force</td>
<td>39</td>
<td>1.83 1.83 1.83 1.83 1.83 1.83</td>
<td>1.75 1.80 1.73</td>
</tr>
<tr>
<td>(design force)</td>
<td>39*</td>
<td>1.73 - 2.05 - 1.98</td>
<td></td>
</tr>
<tr>
<td>175kN post-tension force</td>
<td>39</td>
<td>1.05 1.13 2.03 1.76 1.73 1.09 1.59 1.50</td>
<td></td>
</tr>
<tr>
<td>(detensioned before demolishing)</td>
<td>21</td>
<td>1.78 1.91 2.08 1.32 2.55 1.84 1.70 2.55</td>
<td></td>
</tr>
</tbody>
</table>

[* = Instrumented hammer tests]

Table A.19: Effect of dead load on Damping (half-power method)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 1st 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequency resolution (Hz)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18 0.06</td>
<td>0.18 0.06</td>
</tr>
<tr>
<td>100kN post-tensioning force</td>
<td>39</td>
<td>1.91 1.99 1.58 1.81 1.70 1.95 1.78 1.70</td>
<td></td>
</tr>
<tr>
<td>with dead load of 0.25kN/m²</td>
<td>39*</td>
<td>1.09 - 1.49 - 1.62</td>
<td></td>
</tr>
<tr>
<td>175kN post-tensioning force</td>
<td>39</td>
<td>1.44 1.11 1.79 2.42 2.12 1.27 2.11 2.12</td>
<td></td>
</tr>
<tr>
<td>with dead load of 0.25kN/m²</td>
<td>39*</td>
<td>1.09 - 1.49 - 1.62</td>
<td></td>
</tr>
</tbody>
</table>

Table A.20: Effect of false flooring on Damping (half-power method)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td>fund. 1st 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequency resolution (Hz)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18 0.06</td>
<td>0.18 0.06</td>
</tr>
<tr>
<td>floor layout1 (see fig. 3.15)</td>
<td>39</td>
<td>1.24 1.08 1.10 1.08 1.56</td>
<td>1.16 1.09 1.56</td>
</tr>
<tr>
<td>(measured on slab level)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>floor layout1 (measured on false floor level)</td>
<td>10</td>
<td>- 1.05 - 1.49 - 1.05 1.49 -</td>
<td></td>
</tr>
<tr>
<td>floor layout2 (see fig. 3.16)</td>
<td>39</td>
<td>1.69 1.69 1.41 1.37 noisy 1.69 1.39 -</td>
<td></td>
</tr>
<tr>
<td>(measured on slab level)</td>
<td></td>
<td>noisy</td>
<td>noisy</td>
</tr>
<tr>
<td>floor layout2 (measured on false floor level)</td>
<td>15</td>
<td>1.93 1.93 1.39 noisy noisy 1.93 1.39 -</td>
<td></td>
</tr>
</tbody>
</table>
Table A.21: Effect of partitions (see figure 3.12 for layout) on Damping (half-power method)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequency resolution (Hz)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18 0.06 0.18 0.06 0.18</td>
<td></td>
</tr>
<tr>
<td>five cantilever partitions perpendicular to slab span</td>
<td>39</td>
<td>1.44 1.19 2.91 3.52 1.45 1.31 3.22 1.45</td>
<td></td>
</tr>
<tr>
<td>(nos. (1)-(6) in fig. 3.12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>three cantilever partitions perpendicular to slab span</td>
<td>39</td>
<td>1.51 1.22 1.55 1.24 1.42 1.37 1.40 1.42</td>
<td></td>
</tr>
<tr>
<td>(nos. (2)-(4) in fig. 3.12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>five perpendicular and three transverse to slab span</td>
<td>39</td>
<td>1.82 1.48 1.83 1.17 1.17 1.65 1.40 1.17</td>
<td></td>
</tr>
<tr>
<td>(nos. (1)-(8) in fig. 3.12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>three parallel to slab span</td>
<td>39</td>
<td>- 1.10 - 0.81 - 1.10 0.81 -</td>
<td></td>
</tr>
<tr>
<td>(nos. (6)-(8) in fig. 3.12)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>three full-height partitions perpendicular to slab span</td>
<td>39</td>
<td>1.55 1.94 3.10 2.71 2.17 1.75 2.91 2.17</td>
<td></td>
</tr>
<tr>
<td>one full-height partition perpendicular to slab span</td>
<td>39</td>
<td>2.04 1.86 3.95 2.15 1.09 1.95 3.04 1.09</td>
<td></td>
</tr>
<tr>
<td>one full-height partition perpendicular to slab span</td>
<td>39</td>
<td>- 3.71 2.56 2.02 1.95 3.14 2.02</td>
<td></td>
</tr>
<tr>
<td>five cantilever and one full-height partitions</td>
<td>39</td>
<td>2.03 1.88 3.09 3.31 1.09 1.96 3.16 1.69</td>
<td></td>
</tr>
</tbody>
</table>

Table A.22. Effect of stationary man on Damping (half-power method)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequency resolution (Hz)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18 0.06 0.18 0.06 0.18</td>
<td></td>
</tr>
<tr>
<td>one man standing near slab centre</td>
<td>39</td>
<td>4.59 5.00 1.42 1.28 1.31 3.90 1.33 1.31</td>
<td></td>
</tr>
<tr>
<td>equivalent mass of man near slab centre</td>
<td>39</td>
<td>1.95 1.15 1.31 1.16 1.19 1.41 1.24 1.19</td>
<td></td>
</tr>
</tbody>
</table>

Table A.23: Effect of visco-elastic resin screeds on Damping (half-power method)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequency resolution (Hz)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18 0.06 0.18 0.06 0.18</td>
<td></td>
</tr>
<tr>
<td>25mm thick screed of low Concrelump concentration (see section 3.6.7)</td>
<td>39</td>
<td>1.12 1.06 2.27 2.14 2.31 1.14 2.15 2.31</td>
<td></td>
</tr>
<tr>
<td>additional 25mm thick screed of high Concrelump conc.</td>
<td>39</td>
<td>1.33 1.08 2.67 2.59 3.34 1.17 2.42 3.34</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1.33 1.12 1.08 1.99 -</td>
<td></td>
</tr>
</tbody>
</table>

Table A.24: Effect of combinations over bare slab on Damping (half-power method)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund. 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequency resolution (Hz)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.18 0.06 0.18 0.06 0.18</td>
<td></td>
</tr>
<tr>
<td>false floor layout and standing man (on slab level)</td>
<td>39</td>
<td>- 4.46 - noisy -</td>
<td>4.46 -</td>
</tr>
<tr>
<td>false floor layout and standing man (on floor level)</td>
<td>15</td>
<td>- 5.43 - noisy -</td>
<td>5.43 -</td>
</tr>
<tr>
<td>five cantilever partitions and dead load at 0.2 kN/m²</td>
<td>39</td>
<td>1.35 1.59 2.24 1.36 1.51 1.47 1.80 1.51</td>
<td></td>
</tr>
</tbody>
</table>
Table A.25: Effect of combinations over first screed layer on Damping (half-power method)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td>five cantilever partitions</td>
<td>33</td>
<td>-</td>
<td>1.22</td>
</tr>
<tr>
<td>one standing man</td>
<td>21</td>
<td>-</td>
<td>4.02</td>
</tr>
<tr>
<td>five cantilever partitions and standing man</td>
<td>12</td>
<td>-</td>
<td>4.57</td>
</tr>
</tbody>
</table>

Table A.26: Effect of combinations over second screed layer on Damping (half-power method)

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Frequency resolution (Hz)</td>
<td>0.12</td>
</tr>
<tr>
<td>one standing man</td>
<td>21</td>
<td>-</td>
<td>3.09</td>
</tr>
<tr>
<td>two standing men</td>
<td>21</td>
<td>-</td>
<td>3.42</td>
</tr>
<tr>
<td>equiv. mass of one standing man</td>
<td>21</td>
<td>-</td>
<td>1.09</td>
</tr>
<tr>
<td>equiv. mass of two standing men</td>
<td>21</td>
<td>-</td>
<td>1.12</td>
</tr>
<tr>
<td>five cantilever partitions</td>
<td>27</td>
<td>-</td>
<td>1.16</td>
</tr>
<tr>
<td>five cantilever partitions and dead load at 25kN/m²</td>
<td>9</td>
<td>-</td>
<td>1.10</td>
</tr>
<tr>
<td>five cantilever partitions and one standing man</td>
<td>21</td>
<td>-</td>
<td>2.90</td>
</tr>
<tr>
<td>false floor layout1</td>
<td>21</td>
<td>-</td>
<td>1.19</td>
</tr>
<tr>
<td>false floor layout1 and one standing man</td>
<td>12</td>
<td>-</td>
<td>2.81</td>
</tr>
<tr>
<td>false floor layout1 and equiv. mass of one standing man</td>
<td>12</td>
<td>-</td>
<td>1.07</td>
</tr>
<tr>
<td>false floor layout2</td>
<td>21</td>
<td>-</td>
<td>1.08</td>
</tr>
<tr>
<td>false floor layout2 and one standing man</td>
<td>11</td>
<td>-</td>
<td>2.64</td>
</tr>
</tbody>
</table>
## Time-domain damping ratios

**Table A.27: Effect of prestress on Damping (logarithmic decrement method)**

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>±5%</td>
<td>±10%</td>
</tr>
<tr>
<td>100kN post-tension force</td>
<td>35</td>
<td>-</td>
<td>1.70</td>
</tr>
<tr>
<td>175kN post-tension force</td>
<td>39</td>
<td>1.16</td>
<td>1.05</td>
</tr>
<tr>
<td>Detensioned before demolishing</td>
<td>8</td>
<td>-</td>
<td>1.39</td>
</tr>
</tbody>
</table>

**Table A.28: Effect of dead load on Damping (logarithmic decrement method)**

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>±5%</td>
<td>±10%</td>
</tr>
<tr>
<td>100kN post-tensioning force with dead load of 0.25kN/m²</td>
<td>39</td>
<td>-</td>
<td>2.10</td>
</tr>
<tr>
<td>175kN post-tensioning force with dead load of 0.25kN/m²</td>
<td>39</td>
<td>0.90</td>
<td>1.51</td>
</tr>
</tbody>
</table>

**Table A.29: Effect of false flooring on Damping (logarithmic decrement method)**

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>±5%</td>
<td>±10%</td>
</tr>
<tr>
<td>Floor layout 1 (see fig. 3.15) (on slab level)</td>
<td>39</td>
<td>1.28</td>
<td>-</td>
</tr>
<tr>
<td>Floor layout 1 (on false floor level)</td>
<td>9</td>
<td>1.21</td>
<td>-</td>
</tr>
<tr>
<td>Floor layout 2 (see fig. 3.16) (on slab level)</td>
<td>39</td>
<td>1.32</td>
<td>1.98</td>
</tr>
<tr>
<td>Floor layout 2 (on false floor level)</td>
<td>15</td>
<td>-</td>
<td>2.04</td>
</tr>
</tbody>
</table>

**Table A.30: Effect of partitions (see figure 3.12 for layout) on Damping (logarithmic decrement method)**

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>fund.</td>
<td>2nd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>±5%</td>
<td>±10%</td>
</tr>
<tr>
<td>Five cantilever partitions perpendicular to slab span (nos. (1)-(6) in fig. 3.12)</td>
<td>39</td>
<td>-</td>
<td>1.13</td>
</tr>
<tr>
<td>Three cantilever partitions perpendicular to slab span (nos. (2)-(4) in fig. 3.12)</td>
<td>39</td>
<td>1.21</td>
<td>1.49</td>
</tr>
<tr>
<td>Five perpendicular and three transverse to slab span (nos. (1)-(8) in fig. 3.12)</td>
<td>39</td>
<td>1.25</td>
<td>1.71</td>
</tr>
</tbody>
</table>
### Table A.31: Effect of standing man on Damping (logarithmic decrement method)

<table>
<thead>
<tr>
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<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Loading amplitude (N)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>± 10 ± 15 ± 20 ± 30</td>
<td>± 10 ± 15 ± 20</td>
</tr>
<tr>
<td>one man near slab centre</td>
<td>39</td>
<td>2.69 - 4.1 4.95 1.43</td>
<td>1.61 3.88 1.43</td>
</tr>
<tr>
<td>equivalent mass of man near slab centre</td>
<td>39</td>
<td>0.58 1.88 1.99 -</td>
<td>1.23 1.46 1.46</td>
</tr>
</tbody>
</table>

### Table A.32: Effect of visco-elastic resin screeds on Damping (logarithmic decrement method)

<table>
<thead>
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<th>Slab configuration</th>
<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Loading amplitude (N)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>± 10 ± 15 ± 20 ± 30</td>
<td>± 10 ± 15 ± 20</td>
</tr>
<tr>
<td>25mm thick screed of low</td>
<td>21</td>
<td>1.07 1.23 - 2.35</td>
<td>noisy 1.15 2.39</td>
</tr>
<tr>
<td>Concrodamp concentration (see section 3.6.7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>additional 25mm thick screed of</td>
<td>27</td>
<td>- 1.34 1.33 2.23</td>
<td>noisy 1.33 2.52</td>
</tr>
<tr>
<td>high Concrodamp conc.</td>
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</tbody>
</table>

### Table A.33: Effect of combinations over first screed layer on Damping (logarithmic decrement method)

<table>
<thead>
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<th>Slab configuration</th>
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<th>Damping (% critical)</th>
<th>Average values</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Loading amplitude (N)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>± 10 ± 15 ± 20 ± 30</td>
<td>± 10 ± 15 ± 20</td>
</tr>
<tr>
<td>five cantilever partitions</td>
<td>21</td>
<td>1.31 1.44 -</td>
<td>1.38</td>
</tr>
<tr>
<td>one standing man</td>
<td>12</td>
<td>3.36 4.25 -</td>
<td>3.81</td>
</tr>
<tr>
<td>five cantilever partitions and</td>
<td>21</td>
<td>3.52 3.90 -</td>
<td>3.76</td>
</tr>
<tr>
<td>standing man</td>
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<td></td>
</tr>
</tbody>
</table>

### Table A.34: Effect of combinations on second screed layer on Damping (logarithmic decrement method)

<table>
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<th>no. of tests</th>
<th>Damping (% critical)</th>
<th>Average values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st 2nd 3rd</td>
<td>fund. 2nd 3rd</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Loading amplitude (N)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>± 10 ± 15 ± 20 ± 30</td>
<td>± 10 ± 15 ± 20</td>
</tr>
<tr>
<td>one standing man</td>
<td>12</td>
<td>3.02 3.21 -</td>
<td>3.12</td>
</tr>
<tr>
<td>two standing men</td>
<td>12</td>
<td>4.34 4.66 -</td>
<td>5.05</td>
</tr>
<tr>
<td>equiv. mass of one man</td>
<td>12</td>
<td>1.42 1.50 -</td>
<td>1.46</td>
</tr>
<tr>
<td>equiv. mass of two men</td>
<td>12</td>
<td>1.40 1.46 -</td>
<td>1.43</td>
</tr>
<tr>
<td>five cantilever partitions and</td>
<td>9</td>
<td>1.50 1.52 2.89</td>
<td>1.51 2.89</td>
</tr>
<tr>
<td>dead load at 0.25kN/m²</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>five cantilever partitions and</td>
<td>9</td>
<td>3.06 3.11 -</td>
<td>3.08</td>
</tr>
<tr>
<td>standing man</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>false floor layout1</td>
<td>18</td>
<td>1.52 1.56 -</td>
<td>1.54</td>
</tr>
<tr>
<td>false floor layout1 and one standing man</td>
<td>8</td>
<td>3.00 2.92 -</td>
<td>2.96</td>
</tr>
<tr>
<td>false floor layout1 and equiv.</td>
<td>8</td>
<td>1.51 1.53 -</td>
<td>1.52</td>
</tr>
<tr>
<td>mass of one man</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>false floor layout2</td>
<td>10</td>
<td>1.54 1.51 -</td>
<td>1.53</td>
</tr>
<tr>
<td>false floor layout2 and one standing man</td>
<td>8</td>
<td>2.87 3.25 -</td>
<td>3.06</td>
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</table>
### Steady-state acceleration responses

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Load amp. (N)</th>
<th>RMS accel. (m/s²)</th>
<th>( \frac{m/s^2}{N} )</th>
<th>Slab configuration</th>
<th>Load amp. (N)</th>
<th>RMS accel. (m/s²)</th>
<th>( \frac{m/s^2}{N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100kN post-tension force</td>
<td>±20</td>
<td>0.25</td>
<td>0.013</td>
<td>5 cant. &amp; one full-height partitions</td>
<td>±30</td>
<td>0.05</td>
<td>0.0017</td>
</tr>
<tr>
<td></td>
<td>±10</td>
<td>0.64</td>
<td>0.006</td>
<td>first screed layer (screed 1)</td>
<td>±15</td>
<td>0.54</td>
<td>0.036</td>
</tr>
<tr>
<td></td>
<td>±10</td>
<td>0.18</td>
<td>0.018</td>
<td>second screed layer (screed 2)</td>
<td>±15</td>
<td>0.30</td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>±20</td>
<td>0.32</td>
<td>0.016</td>
<td></td>
<td>±15</td>
<td>0.46</td>
<td>0.033</td>
</tr>
<tr>
<td>100kN force with dead load</td>
<td>±20</td>
<td>0.79</td>
<td>0.04</td>
<td>screed 1 + man</td>
<td>±15</td>
<td>0.11</td>
<td>0.0073</td>
</tr>
<tr>
<td></td>
<td>±20</td>
<td>0.45</td>
<td>0.023</td>
<td></td>
<td>±20</td>
<td>0.17</td>
<td>0.0085</td>
</tr>
<tr>
<td>175kN force with dead load</td>
<td>±6</td>
<td>0.35</td>
<td>0.068</td>
<td>screed 1 + 5 cant. partitions</td>
<td>±15</td>
<td>0.30</td>
<td>0.02</td>
</tr>
<tr>
<td>false floor layout 1</td>
<td>±10</td>
<td>0.41</td>
<td>0.041</td>
<td></td>
<td>±20</td>
<td>0.46</td>
<td>0.023</td>
</tr>
<tr>
<td>false floor layout 2</td>
<td>±20</td>
<td>0.62</td>
<td>0.041</td>
<td></td>
<td>±20</td>
<td>0.15</td>
<td>0.0087</td>
</tr>
<tr>
<td></td>
<td>±20</td>
<td>0.36</td>
<td>0.018</td>
<td></td>
<td>±20</td>
<td>0.15</td>
<td>0.0089</td>
</tr>
<tr>
<td>one man near slab centre</td>
<td>±10</td>
<td>0.05</td>
<td>0.005</td>
<td>screed 2 + equiv. mass of man</td>
<td>±15</td>
<td>0.41</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>±20</td>
<td>0.13</td>
<td>0.0065</td>
<td></td>
<td>±20</td>
<td>0.51</td>
<td>0.026</td>
</tr>
<tr>
<td>equiv. mass of man</td>
<td>±10</td>
<td>0.31</td>
<td>0.010</td>
<td>screed 2 + 5 cant. parts.</td>
<td>±20</td>
<td>0.22</td>
<td>0.011</td>
</tr>
<tr>
<td>five cantilever partitions</td>
<td>±15</td>
<td>0.54</td>
<td>0.036</td>
<td></td>
<td>±20</td>
<td>0.55</td>
<td>0.0275</td>
</tr>
<tr>
<td></td>
<td>±20</td>
<td>0.65</td>
<td>0.0325</td>
<td></td>
<td>±20</td>
<td>0.24</td>
<td>0.012</td>
</tr>
<tr>
<td>5 perp. &amp; 3 trans. partitions</td>
<td>±15</td>
<td>0.34</td>
<td>0.023</td>
<td>screed 2 + floor layout 1</td>
<td>±20</td>
<td>0.47</td>
<td>0.0235</td>
</tr>
<tr>
<td></td>
<td>±20</td>
<td>0.49</td>
<td>0.0245</td>
<td></td>
<td>±20</td>
<td>0.15</td>
<td>0.0073</td>
</tr>
<tr>
<td>one full-height partition</td>
<td>±30</td>
<td>0.07</td>
<td>0.0023</td>
<td></td>
<td>±20</td>
<td>0.05</td>
<td>0.005</td>
</tr>
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</table>

### Table A.36: RMS steady-state accelerations at slab second natural frequency

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Load amp. (N)</th>
<th>RMS accel. (m/s²)</th>
<th>( \frac{m/s^2}{N} )</th>
<th>Slab configuration</th>
<th>Load amp. (N)</th>
<th>RMS accel. (m/s²)</th>
<th>( \frac{m/s^2}{N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100kN post-tension force</td>
<td>±30</td>
<td>0.22</td>
<td>0.0073</td>
<td>one man near slab centre</td>
<td>±20</td>
<td>0.21</td>
<td>0.0105</td>
</tr>
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<td></td>
<td>±20</td>
<td>0.11</td>
<td>0.0055</td>
<td>equiv. mass of man</td>
<td>±20</td>
<td>0.22</td>
<td>0.011</td>
</tr>
<tr>
<td>100kN force with dead load</td>
<td>±50</td>
<td>0.40</td>
<td>0.008</td>
<td>five cantilever partitions</td>
<td>±25</td>
<td>0.16</td>
<td>0.0064</td>
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<td></td>
<td>±20</td>
<td>0.16</td>
<td>0.008</td>
<td></td>
<td>±25</td>
<td>0.22</td>
<td>0.0085</td>
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<td>false floor layout 1</td>
<td>±25</td>
<td>0.22</td>
<td>0.0088</td>
<td>first screed layer</td>
<td>±10</td>
<td>0.05</td>
<td>0.005</td>
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<tr>
<td>false floor layout 2</td>
<td>±25</td>
<td>0.16</td>
<td>0.0064</td>
<td>second screed layer</td>
<td>±10</td>
<td>0.04</td>
<td>0.004</td>
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### Table A.37: RMS steady-state accelerations at slab third natural frequency

<table>
<thead>
<tr>
<th>Slab configuration</th>
<th>Load amp. (N)</th>
<th>RMS accel. (m/s²)</th>
<th>( \frac{m/s^2}{N} )</th>
<th>Slab configuration</th>
<th>Load amp. (N)</th>
<th>RMS accel. (m/s²)</th>
<th>( \frac{m/s^2}{N} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100kN post-tension force</td>
<td>±30</td>
<td>0.34</td>
<td>0.011</td>
<td>equiv. mass of man</td>
<td>±30</td>
<td>0.32</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>±30</td>
<td>0.38</td>
<td>0.0127</td>
<td>five cantilever partitions</td>
<td>±30</td>
<td>0.36</td>
<td>0.012</td>
</tr>
<tr>
<td>100kN force with dead load</td>
<td>±50</td>
<td>0.56</td>
<td>0.0112</td>
<td>5 perp. &amp; 3 trans. partitions</td>
<td>±30</td>
<td>0.32</td>
<td>0.011</td>
</tr>
<tr>
<td></td>
<td>±30</td>
<td>0.31</td>
<td>0.010</td>
<td>first screed layer</td>
<td>±20</td>
<td>0.03</td>
<td>0.0015</td>
</tr>
<tr>
<td>false floor layout 1</td>
<td>±30</td>
<td>0.35</td>
<td>0.012</td>
<td>second screed layer</td>
<td>±20</td>
<td>0.04</td>
<td>0.002</td>
</tr>
<tr>
<td>one man near slab center</td>
<td>±30</td>
<td>0.37</td>
<td>0.0123</td>
<td></td>
<td>±20</td>
<td>0.04</td>
<td>0.002</td>
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</table>
Appendix B

Model slab dimensions and properties

This appendix lists the model slab dimensions and concrete properties used throughout the dissertation. The values of slab generalised mass, $M_1^*$, and stiffness, $K_1^*$, are also calculated in the fundamental mode.

$L = 5.1\text{m}$ \hspace{1cm} $\rho = 2340\text{kg/m}^3$

$b = 1.0\text{m}$ \hspace{1cm} $E_c = 25.6\times10^9 \text{N/m}^2$

$h = 135\text{mm}$ \hspace{1cm} $E_s = 200\times10^9 \text{N/m}^2$

$d = 115\text{mm}$ \hspace{1cm} $A_s = 6.28\times10^{-4} \text{m}^2$ (8×T10 rebar)

Generalised mass

Taking a simply supported slab with mode shape, $\Phi(x) = \sin \frac{\pi x}{L}$, the generalised mass is:

$$M_1^* = \int_0^L \rho bh \Phi^2 dx = \rho bh \int_0^L \sin^2 \frac{\pi x}{L} dx = \rho bh \frac{L}{2} = 806\text{kg} \quad (B.1)$$

Generalised stiffness

Taking the same $\Phi(x)$, the generalised stiffness is:

$$K_1^* = \int_0^L EI \Phi'' dx = EI \int_0^L -\frac{\pi^2}{L^2} \sin^2 \frac{\pi x}{L} dx = \frac{EI \pi^4}{2L^3} = 2.02 \times 10^6 \text{N/m} \quad (B.2)$$
Appendix C

**Effect of cracking on concrete section**

In this appendix, typical calculations for the derivation of effective second moment of area, \( I_e \), for uncracked, cracked, and partially cracked sections are presented. These calculations apply to the model post-tensioned slab in its bare state with full prestress, no prestress, and partial prestress. Slab dimensions and concrete properties are given in appendix B.

**General formula**

For a cracked section, the effective second moment of area is given by: (as in section 6.2)

\[
I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_u + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr}
\]  

(C.1)

This formula applies if \( I_e \) is smaller than \( I_u \) (the uncracked second moment of area). Here, \( M_{cr} \) is the bending moment at the onset of cracking and \( I_{cr} \) is the cracked second moment of area at the most heavily loaded section. The maximum possible bending moment, \( M_a \), is:

\[
M_a = \frac{pbhgL^2}{8} = 10.08 kNm
\]  

(C.2)

**Fully prestressed section**

First, let's consider the slab in its uncracked state. Here, the effective concrete section is \( bh \), and the equivalent section is as shown in figure C.1. Taking moments about the neutral axis gives:

\[
bh \left( x - \frac{h}{2} \right) = \alpha (d - x) \quad \Rightarrow \quad x = 69 \text{ mm}
\]  

(C.3)

The second moment of area of the uncracked equivalent section, \( I_u \), is:

\[
I_u = \frac{bh^3}{12} + bh \left( x - \frac{h}{2} \right)^2 + \alpha (d - x)^2 = 2.15 \times 10^{-4} \text{ m}^4
\]  

(C.4)
Figure C.1: Actual uncracked and its transformed (equivalent) section

With the concrete tensile strength, $\sigma_t$, derived as 3.0N/mm² (from cylinder splitting tests), equation 6.15 (page 107) can be used to find $M_{cr}$. Here, the third term in equation 6.15 is ignored, as the tendon eccentricity for the model slab was very small and difficult to achieve accurately for such a shallow slab.

\[
\sigma_t = \frac{M_{cr} y_t}{I_u} - \frac{P_t}{bh} \quad \Rightarrow \quad M_{cr} = 26.02 \text{kNm} \tag{C.5}
\]

where $P_t = 700$ kN corresponding to 175 kN on each tendon. Hence, the moment required to initiate cracking is 26.02 kNm, whereas the maximum possible moment in the slab is 10.08 kNm (see equation C.2). Therefore, the slab remains uncracked, equation C.1 does not apply for this case and $I_e = I_u$.

Reinforced section

Now consider the slab in its reinforced state, with no post-tensioning, (see figure 6.3). Here, the concrete is assumed to be cracked up to the neutral axis and no tensile strength exists in the concrete below it. The transformed section is drawn as before assuming that the neutral axis of the cracked section passes through the centroid of the transformed section, (see figure C.2), [Kong and Evans (1989)].

Taking bending moments of the transformed section about the neutral axis gives:

Figure C.2: Actual cracked and its transformed (equivalent) section
\[ b x \left( \frac{x}{2} \right) = \alpha (d - x) \implies x = 29\text{mm} \quad (C.6) \]

The second moment of area of the cracked section is:

\[ I_{cr} = \frac{bx^3}{3} + \alpha (d - x)^2 = 4.44 \times 10^{-5}\text{m}^4 \quad (C.7) \]

Assuming that the cracked section is reinforced only, the bending moment at the onset of cracking is given by equation 6.14 (page 107), where:

\[ \sigma_t = \frac{M_{cr} y_t}{I_u} \implies M_{cr} = 9.54\text{kNm} \quad (C.8) \]

With \( M_a = 10.08\text{kNm}, \) (equation C.2), this means that the bending moment necessary for formation of cracks is less than the maximum possible value. Therefore equation C.1 applies where:

\[ I_e = \left( \frac{9.54}{10.08} \right)^3 \times 2.15 \times 10^{-4} + \left[ 1 - \left( \frac{9.54}{10.08} \right)^3 \right] \times 4.44 \times 10^{-5} = 1.89 \times 10^{-4}\text{m}^4 \quad (C.9) \]

**Partially prestressed section**

In this case, some prestressing force is present and application of equation 6.15 gives:

\[ \sigma_t = \frac{M_{cr} y_t}{I_u} - \frac{P_t}{bh} \implies M_{cr} = 18.95\text{kNm} \quad (C.10) \]

where \( P_t = 400\text{kN} \) corresponding to 100kN on each tendon. For this case, \( M_{cr} \) lies between the values for cracked and uncracked sections. However, it is still higher than the maximum possible moment, \( M_a, \) (equation C.2). Therefore, the slab must remain uncracked, equation C.1 does not apply and \( I_e = I_u. \)
Appendix D

Estimations of fundamental frequency

This appendix presents typical fundamental frequency calculations for the model slab by the EBM, Static Deflection, and Concrete Society methods given in section 2.5.3. The slab is assumed to be uncracked, hence its second moment of area is taken as $I_u$. For consideration of cracked sections, this can be replaced by the effective value, $I_e$, as calculated in appendix C. The slab dimensions and concrete properties used below are as given in appendix B.

Equivalent beam method (EBM)

Taking a strip of beam one meter wide with $I_u = 2.15 \times 10^{-4} \text{m}^4$ (equation C.4), the fundamental natural frequency of the slab, when simply supported, is: (see equation 2.14)

$$f_0 = \frac{\pi}{2} \sqrt{\frac{E_c I_u}{\rho bh L^4}} = 7.96 \text{Hz}$$  \hspace{1cm} (D.1)

Static deflection method

The static deflection of the uncracked equivalent beam, simply supported, is:

$$\Delta_s = \frac{5 \rho bh g L^4}{384 E_c I_u} = 4.96 \text{mm}$$  \hspace{1cm} (D.2)

Hence, the beam fundamental frequency is given as:

$$f_0 = 0.18 \sqrt{\frac{g}{\Delta_s}} = 8.01 \text{Hz}$$  \hspace{1cm} (D.3)

The factor 0.18 can be calculated or derived from the conversion factor $\lambda$ taken as 1.3 for a simple supported beam (see equation 2.15, page 27). Hence, expressing equation 2.5, for a lumped mass-spring system, and incorporating $\lambda$ gives:

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1.3g}{\Delta_s}} = 8.07 \text{Hz}$$  \hspace{1cm} (D.4)

The small difference is due to arithmetical rounding errors.

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Concrete Society method

This method is basically an extension of the EBM for two-way slabs. For the model slab, which is one-way spanning, the method is expected to give a similar result as in the EBM. The following calculations are in the long span, y, direction with $l_x=1\text{m}$, $l_y=5.1\text{m}$, $n_x=1$ and $n_y=1$. Also, $I_x$ and $I_y$ are taken over a one metre length strip and hence are equal where $I_x=I_y=I_u=2.15\times10^{-4}\text{m}^4$.

Hence, with reference to equations 2.16-2.20 on page 28,

$$\lambda_y = \frac{n_y l_y}{l_x} \left( \frac{E_c I_x}{E_c I_y} \right)^{\frac{1}{2}} = 5.1 \quad k_y = 1 + \frac{1}{\lambda_y^2} = 1.04 \quad (D.5)$$

By taking the model floor in the category of slabs without perimeter beams,

$$f'_y = \frac{k_y \pi}{2} \sqrt{\frac{E_c l_x}{\rho b h I_x}} = 215 \quad (D.6)$$

$$f_b = \frac{\pi}{2} \sqrt{\frac{E_c l_x}{\rho b h I_y}} \frac{n_y}{l_y^2} = 7.95\text{Hz} \quad (D.7)$$

$$f_y = f'_y - (f'_y - f_b) \left[ \frac{1}{2n_y} + \frac{1}{2n_x} \right] = 7.95\text{Hz} \quad (D.8)$$

And hence the fundamental natural frequency is given as:

$$\Rightarrow f_0 = 7.95\text{Hz} \quad (D.9)$$

This result is similar to the EBM as expected.
Appendix E

Derivation of gypsum partition stiffness

The gypsum plasterboard used to simulate full-height partitions were seen to undergo lateral bowing under the vertical load imposed by the slab during vibration (see figures 6.6 and 6.7, page 112). Hence, to impose similar conditions to those of the model slab tests, the axial stiffness of the gypsum partitions, with bowing action, was found experimentally by vertical compression tests on sample specimens. During these tests, an initial lateral deflection was given to the sample specimen at its midspan and the axial and lateral deflections were measured due to axial load increments. The lateral deflections were measured at the sample midspan by dial gauges and the axial displacements were recorded at the top and bottom edges of the specimen. The samples tested were 200mm in length and were loaded under simply supported conditions. Each test was conducted in the elastic region of the load-deflection curve where on unloading, the sample returned to its original shape without permanent lateral deflections.

From figure E.1 it can be seen that the stiffness obtained from a 200mm long partition, under simply supported conditions, is comparable to the stiffness of a 400mm long partition under fixed conditions (as used in the model slab tests). This can be shown by simple beam deflection formulae.

![Figure E.1: Representation of a 400mm long partition, under fixed-fixed conditions, as a 200mm partition under simply supported conditions](image)

During the axial compression tests, non-linearities were observed in the results at approximately the first three loading-unloading cycles. This was attributed to the sample 'bedding' itself into the compression chamber and also to the compression of the plaster at the micro-level. After the initial
non-linearities, the load-deflection curves for each sample were in good agreement between the tests. Figure E.2 shows these curves for six tests on a given sample. Similar results were obtained with the other specimens.

Figure E.2: Gypsum partition axial compression test results

The gradient of each curve in figure E.2 represents the stiffness of the sample for the particular test. For these tests, the gradient was calculated at the linear portion of the graph. The average stiffness obtained from all the tests was $7.52 \times 10^6$ N/m with an error range of ±5%. With the initial non-linearities, the stiffness was $7.30 \times 10^6$ N/m with an error of ±12%. To substantiate these results, samples of different lengths were tested under identical conditions and the derived stiffnesses, when transformed to a 200mm sample, were within 15% of the above average value.

Hence, all calculations in this thesis take $7.52 \times 10^6$ N/m as the axial stiffness (with bowing) of the 400mm long, 9mm thick, gypsum partition under fixed-fixed conditions. However, note must be made of the relatively large error range, which is mainly due to the non-uniform nature of plasterboard. Assuming a linear relationship between the axial stiffness and partition dimensions, the obtained value would increase with an increase in partition thickness and width, and would decrease with an increase in partition height.
Appendix F

Example of plywood TMD design

In chapter 7, a guideline for the design of plywood TMDs was proposed. Here, an example is given of a problematic floor, for which a plywood TMD is designed and its effectiveness evaluated. The steps given are in accordance with those listed in section 7.6.1. The description of the problem is as follows:

An open-plan office floor consists of a one-way spanning post-tensioned flat concrete slab supported on each side by edge-beams. A typical panel has dimensions of 9.5x8m with a slab thickness of 250mm. While in use, the occupants of the office sometimes complain of vibration annoyance during times of intense activity on the floor, namely during walking or running in the office. Instrumented hammer tests on the slab have concluded that the slab fundamental frequency is 6.2Hz, with average damping of the first mode of around 2% critical. Assess the viability of utilising a plywood TMD system to alleviate the serviceability vibration annoyance of the given floor. Assume the concrete properties to be the same as given in appendix B.

SOLUTION

step 1 The model has to be adapted such that it is tuned to the natural frequency of the given slab. Assuming simple supports and using the principle of generalised coordinates to predict fundamental frequency, the above dimensions give a natural frequency of around 4.1Hz, which is an underestimate of the measured value. This would be expected as the boundary conditions are not fully simply supported and some restraints exist at the supports. By trial and error, a value of 7.7m was taken as the equivalent simply supported span for the slab, which gives a fundamental frequency of 6.17Hz.

step 2 Since the floor is used as an office environment, it is necessary to adopt a fairly low value of acceptable amplitude limit. Bachmann et al. (1995) suggest a limiting amplitude for offices of
2%g. As explained in section 7.4.4, the walking simulations adopted in the model are based on successive heeldrops which are more severe than normal walking, hence in this example a limiting value of 4%g is adopted to account for this severity. Figure F.1 shows the response of the slab (as simulated by the model) to a single heeldrop. It shows that the initial amplitudes are at the limit of acceptance chosen, hence a single heeldrop is not expected to be annoying to the occupants. However, the problem arises by people walking on the slab. With an equivalent span of 7.7m, taking each walking stride to be approximately 0.7m, it can be assumed that the maximum imposed load at the centre of the slab would be caused due to around six consecutive steps. As this is an office environment, the frequency of walking is expected to be fairly low at around 2.3Hz. However, the problem also mentions incidents of annoyance due to running or fast walking, which would be expected to be at a frequency of around 3Hz. With the slab natural frequency being 6.2Hz, resonance with the second harmonic of fast walking at this frequency may be the cause of excessive vibrations. Figure F.2 shows the slab response to six consecutive steps at 2.3Hz and 3Hz, giving a maximum peak amplitude of 16%g at 3Hz, which is far above the chosen acceptance limit of 4%g.

![Figure F.1: Simulation of heeldrop on example slab](image)

**step 3** Having decided that a plywood TMD system is to be utilised, assume initial values for TMD damping and mass ratio. Since the case of 3Hz resonance causes the highest response, the proceeding analysis will only consider this form of loading. Keeping the TMD damping constant at 3% critical, figure F.3 shows the slab response at a range of mass ratios. From this figure, a suitable value of \( \mu \) for the slab would be around 0.02. Higher values can also be considered but may prove to be impractical due to a high TMD mass.

**step 4** Keeping the mass ratio constant at 0.02, figure F.4 shows the slab response at a range of damping ratios. It can be seen that initially the lower values of \( \zeta_2 \) are actually more effective, but after the completion of walking, the higher damping values lead to faster decay.
of vibrations. By taking practical considerations into account, table 7.2 (page 157) shows that the maximum derived value of $\zeta_2$ for the half-scale model test was 4.3% critical. Hence, although it may prove to be a conservative assumption, it would be wise not to assume a value higher than 5% critical for this case. It must be mentioned that some purposely designed TMD systems can achieve damping capacities of over 60% critical. Here, a value of 4% is selected for the plywood system.

**step 5** Having arrived at optimum values for $\zeta_2$ and $\mu$, figure F.5 shows the optimum slab response, with the designed TMD installed, due to worst case loading of 3Hz. The amplitudes of vibration are seen to have decreased significantly with the utilisation of the TMD system. Note that these responses are very conservative and in real cases the TMD damping would be expected to be much higher than 4% critical, leading to increased reduction of vibration response, and hence better performance of the TMD than illustrated in figure F.5.
Figure F.4: Walking simulation of six steps with a range of TMD damping values

Figure F.5: Walking simulation of six steps on example slab with optimised plywood TMD system

**Step 6** The mass ratio to be used is 0.02, which with the generalised mass of the slab being 18.4 tonnes, gives a generalised TMD mass of 368kg. The largest sheets of plywood in market today are of the size 2.44m×1.22m×24mm, with a mass of approximately 55kg. Having decided on TMD damping of 4% critical, table 7.2 (page 157) shows that the best TMD configuration would be two layers of plywood with polythene inbetween. However, the mass of the TMD is too large to be supported by two layers of plywood, hence four layers can be supplied. This gives a value for the mass needed to tune the TMD of 258kg, uniformly distributed along the centreline of the plywood system. Although this is rather large, calculations showed that it can safely be supported by the combined 96mm thick plywood sheets. Note that by supplying four layers of plywood, it is expected that the TMD damping will be higher than the case of two layers, due to the increased number of friction surfaces.
References

ACI (1995). Building code requirements for reinforced concrete. ACI 318-95, American Concrete Institute, Detroit, Michigan, USA.


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References


Moss, R. M. (1994). Assessment of stiffness increase due to non-structural screeds. The Structural Engineer, 72(14), 221–228.


References


