Combined Loading of Spudcan Foundations on Clay: Numerical Modelling

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SYNOPSIS

The load–displacement response of a spudcan foundation on clay is described by means of an incremental plasticity model with three degrees of freedom (vertical, rotational, horizontal). The model is termed "Model B" and employs a yield surface and flow rule which are derived from a programme of carefully controlled small-scale laboratory tests (Martin & Houlsby, 2000). Behaviour inside the yield surface is defined by a set of elastic stiffness factors for embedded conical footings, determined from three-dimensional finite element analysis. The hardening law, which defines the vertical bearing capacity as a function of plastic spudcan penetration, is based on a set of theoretical lower-bound bearing capacity factors for embedded conical footings. Care is needed to ensure consistent treatment of partially and fully penetrated spudcans under combined loading. Full details of the elastic and elastoplastic stiffness matrices needed to predict incremental load–displacement behaviour are provided, and the performance of the model is demonstrated by simulating some of the laboratory calibration tests numerically. While Model B succeeds in capturing many important features of the observed spudcan behaviour, its prediction of sudden, rather than gradual yielding is a noticeable weakness, particularly in the simulation of cyclic loading events. Practical applications of the foundation model are discussed with reference to the soil–structure interaction analysis of independent leg jack-up units.

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period studies it was formerly regarded as acceptable to model the foundations using linear elastic springs (Hambly & Nicholson, 1989; Carlsen et al., 1986). However it is now widely acknowledged that predictions of structural performance, especially for extreme storm loading, should take some account of non-linear spudcan behaviour. This can be achieved using either an iterative analysis, with secant foundation stiffnesses (Noble Denton & Associates, 1987; Arnesen et al., 1988; SNAME, 1997) or an incremental analysis, with tangent foundation stiffnesses (Schotman, 1989; Hambly et al., 1990; Martin, 1994; Thompson, 1996; Cassidy, 1999; Martin & Housbly, 1999). An important result in dynamic analysis is that the rotational stiffness at the spudcan has a major influence on the natural period of the unit (Cassidy, 1999).

The jack-up unit analyses of Schotman (1989) marked a significant development because the tangent stiffness matrix for each spudcan was derived from a work-hardening plasticity formulation. In the simplest form of this type of model, footing behaviour is taken to be linear elastic as long as the $(V, M, H)$ load combination lies within a three-dimensional yield surface. Only when the load state reaches and remains on the yield surface does elastoplastic deformation occur, with incremental plastic displacements determined by a flow rule (associated or otherwise) and a hardening law (typically linking the size of the yield surface to the current value of plastic vertical displacement). This idea of applying "macroscopic plasticity" to shallow foundations under combined loading was originally suggested by Roscoe & Schofield (1957), although the more recent renewal of interest can largely be attributed to the papers by Butterfield (1980, 1981). There have since been many experimental, theoretical and numerical investigations of the shape of the $(V, M, H)$ yield surface for various footing geometries and soil types, but relatively few authors have gone on to propose complete incremental plasticity models capable of predicting the load–displacement behaviour beyond initial yielding. Of those who have, Schotman (1989) stressed that the parameters used in his models for spudcans on sand and clay were only
preliminary, and had not been calibrated or validated experimentally. Bransby & Martin (1999) added linear elasticity and a simple hardening law to Murff’s (1994) rigid / perfectly plastic model for suction caissons on clay, but again this was primarily for demonstration purposes and there was no detailed experimental verification. The conical footing model developed by Tan (1990) and the strip footing models proposed by Nova & Montrasio (1991) and Gottardi & Butterfield (1995) did include certain features derived from laboratory or centrifuge tests, but the testing in each case involved surface foundations on sand. The authors are not aware of any published work of a similar nature involving foundations on cohesive soil.

This paper describes "Model B", a plasticity-based method for predicting the incremental load-displacement response of a spudcan footing on clay under combined \((V, M, H)\) loading. Some components of the model are based on theoretical solutions, while others are derived from the experimental calibration tests reported by Martin & Housby (2000). The model is primarily intended for application to monotonic, undrained loading, for example when a jack-up unit is struck by a single extreme wave. Model B footings can easily be incorporated into the structural analysis of a complete jack-up when simulating loading events of this type (Thompson, 1996; Martin & Housbly, 1999). Here, however, attention is confined to the development of the numerical model and an assessment of its predictive capabilities.

FEATURES OF MODEL B

Preliminary remarks

Although Model B is intended for use with spudcan-shaped footings, two of its components (elasticity and hardening law) are based on solutions for an embedded conical footing. It is therefore convenient to treat the penetrated portion of a spudcan, or more precisely the penetrated portion of its underside, as an "equivalent conical footing" (see Fig. 2). This footing has the same contact radius as its parent, \(r\) or \(R\) as appropriate, and an apex angle
\( \beta_{\text{equiv}} \) chosen to give equality of the two embedded volumes. Note that both the geometry of the equivalent conical footing and its embedment \( D \) are determined from the plastic component of vertical displacement \( w_p \), not the total vertical displacement \( w_p + w_c \). As indicated in Fig. 2, these calculations of geometry and embedment take no account of the soil displaced during spudcan penetration. This assumption is regarded as conservative. In this Section the features of Model B are first described with reference to a fully penetrated spudcan, Fig. 2(b). Because the loads \( (V, M, H) \) and displacements \( (w, \theta, u) \) are always referred to the footing origin O, some minor modifications are needed to cope with the partially penetrated case, Fig. 2(a). Details of these modifications are given at the end of the Section. Finally it should be pointed out that the version of Model B used by Martin & Houlsby (1999) featured a simplified three-parameter yield function, rather than the full six-parameter version which is incorporated here.

**Elastic response**

Jack-up unit foundations frequently undergo very deep penetrations on soft clay soils (Endley et al., 1981) and in these circumstances it is no longer appropriate to use the familiar closed-form elastic solutions for \( (V, M, H) \) loading of a rigid circular footing at the soil surface. The definition of elasticity in Model B is based on the work of Bell (1991a), who used three-dimensional finite element analysis to study the behaviour of a rough, rigid circular footing subjected to combined \( (V, M, H) \) loading. He considered footing embedments \( D \) of up to two diameters and a range of Poisson's ratios for the soil, although only the undrained situation \( (\nu = 0.5) \) is relevant here. Bell's numerical investigations highlighted the existence of an elastic cross-coupling effect between the rotational and horizontal degrees of freedom, so an appropriate expression for the incremental elastic response in Model B is

\[
\begin{bmatrix}
\delta V \\
\delta M \\
\delta H
\end{bmatrix} =
\begin{bmatrix}
K_1 GR & 0 & 0 \\
0 & K_2 GR^3 & K_4 GR^2 \\
0 & K_4 GR^2 & K_5 GR
\end{bmatrix}
\begin{bmatrix}
\delta w_c \\
\delta \theta_c \\
\delta u_c
\end{bmatrix}
\]

(1)
where

$$M_0 = m_0 \cdot 2RV_0, \quad H_0 = h_0 \cdot V_0, \quad \bar{e} = e_1 + e_2 \left( \frac{V}{V_0} \right) \left( \frac{V}{V_0} - 1 \right), \quad \bar{\beta} = \left( \frac{\beta_1 + \beta_2}{\beta_1 + \beta_2} \right)^{\frac{1}{2}} \left( \frac{\beta_1^{\frac{1}{2}}}{\beta_1^{\frac{1}{2}}} \right)^{\frac{1}{2}}$$

and $V_0$ is the bearing capacity under pure vertical load. The six parameter values obtained from regression analysis of the experimental data were $m_0 = 0.083$, $h_0 = 0.127$, $e_1 = 0.518$, $e_2 = 1.180$, $\beta_1 = 0.764$ and $\beta_2 = 0.882$. The fitted yield surface is illustrated in Fig. 3.

For various reasons it proves advantageous to manipulate equation (2) and define the Model B yield function as

$$f(V, M, H, w_p) = \left[ \left( \frac{M}{M_0} \right)^2 + \left( \frac{H}{H_0} \right)^2 \right] \left( 1 - \frac{V}{V_0} \right) \left( 1 - \frac{V}{V_0} \right) = \bar{\beta}^{\frac{1}{2}} \left( \frac{V}{V_0} \right) \left( 1 - \frac{V}{V_0} \right)$$  \hspace{1cm} (3)

Note that this gives a yield surface ($f = 0$) which is identical to that of equation (2). The plastic component of vertical displacement $w_p$ is listed as an argument of the function because, as described below, it directly determines the reference vertical bearing capacity $V_0$.

It should be reiterated that this yield function is based on a detailed programme of physical modelling performed at unit gravity. An important difference in the full-scale situation is that the overburden pressure $\gamma'D$ (taking into account buoyancy effects) makes a significant contribution to the vertical bearing capacity of an embedded footing. In these circumstances it would seem appropriate to divide $V_0$ into components due to cohesion and due to overburden, and make $H_0$ proportional to just the cohesive component, thus:

$$H_0 = h_0 \cdot \left( V_0 - \gamma'D \cdot \pi R^2 \right)$$  \hspace{1cm} (4)

There should, however, be no need to modify the definition of $M_0$ when applying Model B to real spudcan footings.
Flow rule

At any point on the \((V, M, H)\) yield surface, an outward-pointing normal may be found by evaluating the gradient vector \(\nabla f\). With an associated flow rule the vector of incremental plastic displacements would be parallel to this vector:

\[
\begin{pmatrix}
\delta w_p \\
\delta \theta_p \\
\delta u_p
\end{pmatrix} = \Lambda \begin{pmatrix}
\frac{\partial f}{\partial V} \\
\frac{\partial f}{\partial M} \\
\frac{\partial f}{\partial H}
\end{pmatrix}
\]  

(5)

with \(\Lambda\) being a non-negative scalar determining the magnitude of the plastic increment. The validity of assuming such a flow rule for spudcan footings on clay was investigated in a second set of laboratory tests reported by Martin & Houlsby (2000). The observed rotational and horizontal displacements confirmed the hypothesis of normality to the yield surface in the \((M, H)\) plane, but the vertical displacements \(\delta w_p\) were found to be consistently smaller than those predicted by an associated flow rule. This trend can be captured by including an empirical "association parameter" \(\zeta\) in the flow rule for Model B,

\[
\begin{pmatrix}
\delta w_p \\
\delta \theta_p \\
\delta u_p
\end{pmatrix} = \Lambda \begin{pmatrix}
\zeta \cdot \frac{\partial f}{\partial V} \\
\frac{\partial f}{\partial M} \\
\frac{\partial f}{\partial H}
\end{pmatrix}
\]  

(6)

From regression analysis of their experimental results, Martin & Houlsby (2000) obtained a best-fit value of 0.645 for \(\zeta\). There was, however, considerable scatter in the data, and only two vertical load levels \((V/V_o = 0.75\) and \(V/V_o = 0.2\) were employed in the testing programme. Fig. 3 shows the pattern of incremental plastic displacement vectors obtained from equation (6). The selective adjustment of the vertical component \(\delta w_p\) is closely analogous to Burd's (1986) treatment of plastic volumetric strains when formulating a constitutive law for sand.
Hardening law

The reference vertical bearing capacity \( V_0 \) is calculated from the standard expression

\[
V_0 = \left( N_c s_{u0} + \gamma'D \right) \cdot \pi R^2
\]

(7)

where \( N_c \) is a dimensionless bearing capacity factor and \( s_{u0} \) is the undrained shear strength of the clay at depth \( D = w_p \) for a fully penetrated spudcan. An additional contribution from skin friction is included if the spudcan has a vertical perimeter wall of significant height (Martin & Houlsby, 1999). Model B uses a set of theoretical \( N_c \) factors which are particularly suitable for estimating spudcan bearing capacity in offshore clay deposits. These factors are based on the work of Houlsby & Wroth (1983), who used the method of characteristics to obtain lower bound collapse loads for axisymmetric footings on clay with a linearly increasing undrained strength profile. Houlsby & Martin (1992) extended this study to examine the effect of the footing embedment ratio \( D/2R \) in combination with Houlsby & Wroth's original parameters of cone apex angle \( \beta \), interface friction coefficient \( \alpha \) and dimensionless undrained strength gradient \( \rho 2R/s_{um} \) (here \( s_{um} \) denotes the mudline strength and \( \rho \) the rate of increase of strength with depth). The parameter ranges considered were as follows: \( \beta = 30^\circ \) to \( 180^\circ \), \( D/2R = 0 \) to \( 2.5 \), \( \alpha = 0 \) to \( 1 \), \( \rho 2R/s_{um} = 0 \) to \( 5 \). The clay was idealised as a weightless, rigid-plastic Tresca material and the space above the footing was assumed to be occupied by a smooth-sided shaft. Results of the numerical study, in the form of tables of \( N_c \) factors, can be found in Martin (1994) and in SNAME (1997). For the special case of a flat circular footing embedded in uniform soil, Martin (1994) gives a detailed comparison between the theoretical bearing capacity factors and those calculated from various semi-empirical formulae (Skempton, 1951; Brinch Hansen, 1970; Vesic, 1975).

Fig. 4 shows a typical field of stress characteristics for an embedded conical footing. It can be shown that stress fields of this type are extensible throughout the semi-infinite soil mass, and hence the bearing capacity factors used in Model B represent rigorous lower bound
collapse loads. The validity of introducing an auxiliary smooth-sided shaft does, however, remain open to question. Lower bound calculations of the type shown in Fig. 4 are felt to be appropriate for the relatively shallow embedments \((0 \leq D/2R \leq 2.5)\) considered in the parametric study, but extrapolation of the tabulated \(N_c\) factors to greater depths is not recommended. This is because deep failure is more likely to involve localised flow around the spudcan, and the assumption of rigid-plastic soil behaviour is also likely to become increasingly inadequate.

**Handling of partial penetration**

Consider now the partially penetrated spudcan of Fig. 2(a). The equivalent conical footing has a radius \(r\) and its origin \(O'\) lies a distance \(h = -w_p\) below the overall footing origin \(O\). Assuming that displacements are small, it can be shown that the incremental elastic response at \(O\) is given by a modified version of equation (1):

\[
\begin{bmatrix}
\delta V \\
\delta M \\
\delta H
\end{bmatrix} =
\begin{bmatrix}
K_1'Gr & 0 & 0 \\
0 & K_2'Gr^3 & K_3'Gr^2 \\
0 & K_2'Gr^2 & K_3'Gr
\end{bmatrix}
\begin{bmatrix}
\delta w_c \\
\delta \theta_c \\
\delta u_c
\end{bmatrix}
\tag{8}
\]

where the effective stiffness factors \(K_i'\) are related to the basic stiffness factors for \(D = 0\) (given in Table 2) by the transformations

\[
K_1' = K_1, \quad K_2' = K_2 + \left(\frac{h}{r}\right)^3 K_3 - 2\left(\frac{h}{r}\right)K_4, \quad K_3' = K_3 \quad \text{and} \quad K_4' = K_4 - \left(\frac{h}{r}\right)K_3
\]

The yield function, equation (3), must also be modified during partial penetration because the moment acting on the equivalent conical footing is now \(M + hH\) (rather than \(M\)) and the reference moment capacity \(M_0\) is now \(m_0 \cdot 2rV_0\) (rather than \(m_0 \cdot 2RV_0\)). These are the only changes required in equation (3) so the revised version will not be given in full. Note that as long as the flow rule of equation (6) is applied to the modified yield function, incremental plastic displacements at \(O\) will be calculated correctly (Dean et al., 1992). Finally a revised
version of the hardening law, equation (7), is needed during partial penetration. The required expression is

\[ V_0 = N_c s_{um} \cdot \pi r^2 \]  \hspace{1cm} (9)

where the \( N_c \) factor relates to a conical footing with embedment \( D = 0 \) and a dimensionless undrained strength gradient \( \rho 2r/s_{um} \) (rather than \( \rho 2R/s_{um} \)).

**INCREMENTAL LOAD–DISPLACEMENT FORMULATION**

It is convenient to introduce the notation

\[ \sigma = \begin{bmatrix} V \\ M \\ H \end{bmatrix} \quad \text{and} \quad \varepsilon = \begin{bmatrix} \sigma \\ \theta \\ u \end{bmatrix} \]  \hspace{1cm} (10)

The incremental load–displacement response during elastic or elastoplastic deformation is defined by the tangent stiffness relationship

\[ \delta \sigma = K_{el} \delta \varepsilon \quad \text{or} \quad \delta \sigma = K_{ep} \delta \varepsilon \]  \hspace{1cm} (11)

as appropriate. The elastic stiffness matrix \( K_{el} \) can be obtained directly from equation (1) for a fully penetrated spudcan or equation (8) during partial penetration. Construction of the elastoplastic stiffness matrix \( K_{ep} \) is more involved. The load derivatives of the yield function \( f \) are required:

\[ \frac{\partial f}{\partial \sigma} = \begin{bmatrix} \frac{\partial f}{\partial V} \\ \frac{\partial f}{\partial M} \\ \frac{\partial f}{\partial H} \end{bmatrix} \]  \hspace{1cm} (12)

A notional plastic potential function \( g \) is also introduced, even though it is only defined in terms of its derivatives (cf. equation (6)):  

12
\[
\begin{align*}
\frac{\partial g}{\partial \sigma} &= \left\{ \frac{\partial g}{\partial V}, \frac{\partial g}{\partial M}, \frac{\partial g}{\partial H} \right\} = \left\{ \xi \cdot \delta f / \partial V, \delta f / \partial M, \delta f / \partial H \right\}
\end{align*}
\]  

(13)

A feature of the present problem is that the elastic stiffness matrix \( K_e \) is not constant, but depends on the current value of plastic vertical penetration \( w_p \). This phenomenon is called "elastic-plastic coupling", and requires careful treatment of the components of the displacements. In particular, it is essential that the elastoplastic stiffness matrix \( K_{ep} \) account for the "coupled" elastic displacements which arise from the change of \( K_e \) during an elastoplastic increment (Maier & Hueckel, 1979). The appropriate expression for \( K_{ep} \), derived in full by Martin & Houlsby (1999), is

\[
K_{ep} = K_e - \frac{1}{K_e} \frac{\partial g}{\partial \sigma} \left( \frac{\partial g}{\partial V} \frac{dK_e^{-1}}{dw_p} \sigma \right) \frac{\partial f^T}{\partial \sigma} K_e - \frac{\partial f}{\partial w_p} \frac{\partial g}{\partial \sigma} \frac{\partial f^T}{\partial \sigma} K_e \left( \frac{\partial g}{\partial \sigma} \frac{dK_e^{-1}}{dw_p} \sigma \right)
\]  

(14)

This formulation of the tangent elastoplastic stiffness is suitable for use in an explicit (forward Euler) integration scheme, as discussed below. Note that in general the matrix \( K_{ep} \) is fully populated and unsymmetric.

**Computational aspects**

Some features of a FORTRAN implementation of Model B will now be discussed. Input to the model can take the form of a load increment \( \delta \sigma \), a displacement increment \( \delta \varepsilon \), or a mixed increment (e.g. one involving combined rotation \( \delta \theta \) and horizontal displacement \( \delta u \) at constant vertical load \( \delta V = 0 \)). Regardless of the chosen control method, equation (11) can readily be solved for the three conjugate unknowns once the appropriate stiffness matrix \( K_e \) or \( K_{ep} \) has been formed. If the current state of the system is elastic \( (f < 0) \) it is assumed that the next increment will also be elastic, so a trial solution is made using the matrix \( K_e \). If the
trial load state proves to be inadmissible ($f > 0$) a simple bisection algorithm is used to isolate the elastic portion of the increment, and the remainder is applied elastoplastically using the matrix $K_{ep}$. Likewise if the current state of the system is elastoplastic ($f = 0$) it is assumed that the next increment will also be elastoplastic, and a trial solution is made using the matrix $K_{ep}$. The scalar plastic multiplier $\Lambda$ is then evaluated (see Martin & Houlsby, 1999) and its sign is examined. If the trial solution indicates attempted unloading from the yield surface ($\Lambda < 0$), the increment is reanalysed elastically using the matrix $K_e$.

During elastoplastic deformation, all terms in equation (14) are evaluated with reference to the load state $\sigma$ and the plastic vertical displacement $w_p$ at the start of a new increment. While closed form expressions for $\partial f / \partial \sigma$ and $\partial g / \partial \sigma$ can easily be worked out from the definitions above, the derivatives of $f$ and the elastic flexibility matrix $K_e^{-1}$ with respect to $w_p$ must be evaluated numerically. This is because (i) calculations of $f$ and $K_e^{-1}$ both involve interpolation from tables; (ii) during partial penetration the geometry of the equivalent conical footing does not evolve in a straightforward manner. Drift of the solution during elastoplastic deformation is controlled by correcting the final loads $\sigma$ back onto the yield surface (in a direction normal to the surface) after each increment. Any residual differences between the corrected and externally applied loads are then removed using the Newton-Raphson technique, with an updated elastoplastic matrix $K_{ep}$ for every iteration. As with any incremental plasticity model, it is often necessary to use many small increments to obtain a converged solution for a complete loading step (or sequence of steps).

It should be pointed out that certain details of the incremental formulation and solution procedure described above differ from those used in the original implementation of Model B (Martin, 1994). The changes are purely for reasons of improved efficiency and compactness of expression, and have no effect on the numerical predictions obtained.
MODEL B SIMULATIONS OF SELECTED TESTS

A total of 24 model spudcan tests were used to define the yield surface and flow rule of Model B. As a first step towards evaluating the performance of the model, retrospective simulations have been performed for five of these tests (the same ones described in detail by Martin & Houlsby, 2000).

The spudcan tests were performed on heavily overconsolidated samples of kaolin ($\sigma_{\text{vmax}}' = 200$ kPa followed by full unloading to $\sigma'_{\text{v}} = \gamma'D$, where $\gamma' \approx 6.85$ kN/m$^3$). The following power-law expression (Martin & Houlsby, 2000) was found to give a good description of the variation of $s_u$ with depth:

$$\frac{s_u}{\sigma'_{\text{v}}} = 0.25 \text{OCR}^{0.75}$$  (15)

This equation is plotted in Fig. 5, together with undrained strength measurements from the five samples of interest (each data point represents the average of four miniature vane tests). Because these individual strengths all fall close to the "average" strength profile, equation (15) was used to define the increase of $s_u$ with depth in all of the Model B analyses reported below. At any given footing embedment, values of $s_{um}$, $\rho$ and $s_{u0}$ appropriate to the local variation of undrained strength were chosen by fitting a least-squares straight line to the $s_u$ profile between $D$ and $D + R$ (or between the surface and a depth $r$ in the case of a partially penetrated spudcan); cf. Young et al. (1984). This was necessary because the bearing capacity factors used to calculate $V_0$ have only been tabulated for linearly increasing strength profiles. Full soil–footing adhesion ($\alpha = 1$) was assumed when determining the $N_r$ factors, and the rigidity index $G/s_u$ was arbitrarily taken to be 100. The geometry of the footing used in the Model B simulations was identical to that of the spudcan used in the tests, with a shape as shown in Fig. 1 and a diameter $2R = 125$ mm.

Each numerical simulation commenced from a plastic penetration $w_{p,\text{int}}$ chosen such that the tip of the spudcan was just embedded in the soil, thus allowing an initial yield surface
to be established. After initialising all loads and displacements to zero (with the exception of \( w = w_{p,\text{init}} \)), each Model B analysis proceeded using exactly the same sequence of control steps as the test being simulated. The basic pattern was the same in all tests, with displacement-controlled vertical penetration being interrupted for a series of "swipe" or constant-\( V \) events at nominal embedments of 0, 20, 50, 100 and 200 mm, as well as vertical unload-reload events at depths of 35, 75 and 150 mm. The swipe events were fully displacement-controlled, with rotation or horizontal displacement (or a combination of the two) being applied at constant embedment \( w \), commencing from either maximum vertical load (\( V/V_0 = 1 \)) or very low vertical load (\( V/V_0 = 0.01 \)). Constant-\( V \) events involved the application of rotation and/or horizontal displacement at constant vertical load, and were performed after unloading to an intermediate level of \( V/V_0 \) (0.75 or 0.2). Because the experimental procedures and results have already been described by Martin & Houlsby (2000), the focus here is on the comparison between observed and predicted behaviour in the selected tests.

**Vertical load–penetration**

Fig. 6 illustrates the performance of Model B in predicting the vertical load–displacement response from Test AH1. This test contained swipe events involving pure rotation (AB to IJ), and unload-reload events involving full unloading to \( V = 0 \) (UR1 to UR3). Fig. 6(a) shows how each of these events was preceded by a short period of load-controlled penetration at constant \( V \), designed to minimise the effects of soil creep. Note however that Model B has no facility for modelling time-dependent behaviour, so these load holding stages were ignored in the simulation and treated as straightforward vertical displacement increments.

Comparing Figs 6(a) and 6(b), there is good agreement during the initial period of penetration (\( w < 0 \)) as full contact is established between the soil and the underside of the spudcan. The theoretical bearing capacity factors then give an excellent prediction of the
more sharply defined "critical states" in which all three loads \( (V, M, H) \) approach constant values. Unfortunately the latter stages of the spudcan test at \( w_{\text{nom}} = 200 \text{ mm} \) were affected by spurious load cell readings, indicated by dashed lines in Figs 7(a) and 7(b). Martin & Houlsby (2000) give a more detailed discussion of this problem, which arises when the waterproof shaft surround attached to the spudcan makes contact with the side of the hole formed by spudcan penetration.

Fig. 8 shows the \((V, M)\) and \((V, H)\) load paths from Test AL2, in which the swipe events involved unloading to \( V/V_0 = 0.01 \) followed by horizontal displacement at constant \( w \). In the Model B simulation there is a short period of elastic response at the start of each event, with the vertical load remaining constant until the yield surface is reached. Subsequently the increase in \( V \) during each swipe event is substantially overpredicted, possibly suggesting that there is a need to refine the Model B flow rule at very low vertical loads. While the numerical simulation of Test AL2 captures the general character of the observed results, including the increase in moment and horizontal load capacities with additional penetration, the prediction is not as accurate as that of Test AH3.

The measured and predicted bearing capacities \( V_0 \) listed in Figs 7 and 8 generally agree well, with the exception of the events at \( w_{\text{nom}} = 0 \text{ mm} \). At this level the soil–footing contact area changes very rapidly, and both experimental and numerical results are very sensitive to the exact depth at which penetration is interrupted. It will also be recalled that the baseline for vertical displacements is the undisturbed soil surface level, so when the real spudcan reaches a penetration of (say) \( w = -2 \text{ mm} \) it may already have made full contact with the clay. By contrast Model B does not account for the soil displaced during earlier penetration, so full contact is only achieved when \( w_p = 0 \text{ mm} \). The effects of this discrepancy can also be seen in the early stages of penetration in Figs 6(a) and 6(b).
Constant vertical load events

Fig. 9 shows the key experimental data from Test BH1 (footing rotations with $V$ held constant at $0.75V_i$) together with corresponding excerpts from the Model B simulation of the test. Comparison between Figs 9(a) and 9(d) shows that a rigidity index $G/s_o = 100$ is much too low for modelling the very high initial rotational stiffness observed in the experiments. A better prediction was obtained with $G/s_o \approx 400$, but the quality of the test data did not warrant a detailed investigation of spudcan behaviour at very small rotations, where apparatus flexibility was a source of considerable uncertainty. In the experimental moment–rotation curves it is difficult to distinguish post-yield hardening from the gradual process of yielding itself, but in most events there does appear to be a significant "plastic rotational stiffness" which is underestimated by Model B. This implies that a hardening law based purely on the plastic vertical penetration $w_p$ may be overly simplified. In Fig. 9(e) the predicted responses show the effect of using a negative cross-coupling stiffness factor $K_s$ in equation (1). Within the elastic domain, a positive applied rotation results in a positive moment but a negative horizontal load. While there is no experimental evidence of this phenomenon in Fig. 9(b), this does not mean that the negative values of $K_s$ obtained by Bell (1991a, 1991b) should be rejected. The most likely explanation for the observed behaviour is that significant plastic deformations are occurring right from the start of each rotation event. There are similar differences between Figs 9(c) and 9(f), with vertical settlement beginning immediately in each experimental events but only after yielding in the Model B predictions. Improvements allowing Model B to capture this gradual onset of full plasticity (discussed in more detail below) would certainly be more worthwhile than empirical adjustment of the elastic stiffness factors. The calculated settlements in Fig. 9(f) are significantly smaller in the first rotation ($w_{nom} = 0$ mm) than in subsequent events, a consequence of the steep slope $dV/dw$ of the virgin load–penetration curve as the footing makes full contact with the soil (see Fig. 6).
Largely analogous comments can be made with respect to Fig. 10, which shows measured and predicted behaviour for the constant-$V$ events in Test BL2 (horizontal footing displacement with $V$ held constant at $0.2V_0$). Fig. 10(d) resembles Fig. 9(e), with the negative stiffness factor $K_4$ predicting that a positive horizontal displacement should result in a negative footing moment prior to yielding. This behaviour is certainly not manifested in the experimental curves of Fig. 10(a), and clearly there is a need for further experimental study of the footing response at small displacements (using a stiffer testing rig and more sensitive instrumentation) in conjunction with appropriate refinement of this aspect of the numerical model. At larger (post-yield) horizontal displacements the Model B predictions in Figs 10(d) and 10(e) generally show good agreement with the test results. The calculated vertical displacements in Fig. 10(f) agree quite well with the observed ones until about $u = 2$ mm, but thereafter too much heave is predicted. This reflects the fact that the empirical association parameter $\zeta$ in equation (6) was determined on the basis of incremental plastic displacements at first yield. A more complicated flow rule would be needed to improve this aspect of the model. Although the Model B prediction of plastic vertical heave is associated with post-yield softening of the load–displacement responses in Figs 10(d) and 10(e), this is only discernible in the event at $w_{\text{nom}} = 0$ mm.

**LIMITATIONS OF MODEL B**

*Applicability*

A number of specific shortcomings of Model B have already been highlighted. One concern of a general nature is that the component features of the model (whether based on theory or small-scale experiment) all pertain to conditions where there is negligible infilling behind the penetrating spudcan. By contrast at a typical offshore soft clay site, deep spudcan installation is usually associated with a substantial amount of infilling (Endley *et al.*, 1981; Le Tirant & Pérol, 1993). While it is easy to account for the additional vertical load acting on the footing
(Martin & Houlsby, 1999), infilling may have other influences on spudcan behaviour such as increased moment and horizontal load capacities, increased elastic stiffnesses and increased capacity under tensile vertical loading. A further consequence of infilling is that significant passive resistance can be mobilised by the embedded portion of a jack-up leg under combined \((V, M, H)\) loading (Springman & Schofield, 1998). Under these circumstances it may no longer be sufficient to consider the behaviour of the spudcan footing alone. In passing it should be noted that the empirical yield surface and flow rule used in Model B are specific to the representative spudcan shape (Noble Denton & Associates, 1987) used in the experiments. This evidently limits the applicability of the model to footings having a similar geometry.

**Non-monotonic loading**

The comparisons in the previous Section concentrated on the performance of Model B in simulating monotonic loading events. The only instances of cyclic behaviour considered were the unload-reload events in Fig. 6, where the calculated response was linear and elastic but the experiments gave hysteretic loops. It is not surprising that similar limitations become apparent when Model B is used to simulate reversals of moment or horizontal loading. Fig. 11, for example, shows a complete rotation cycle from Test BH1 (only the monotonic part of this event appeared in Fig. 9). As discussed above, the Model B analysis with \(G/s_u = 100\) underpredicts the initial moment–rotation stiffness during primary loading, but more seriously it fails to capture the gradual process of yielding. Identical comments apply to the reverse cycle, where the observed yielding process is even more gradual. Model B does at least give a good prediction of the ultimate moment loads attained during the outward and inward rotations, but more serious discrepancies between the experimental and calculated results would be expected to develop if further cycles were performed.

Fig. 12 illustrates another situation involving moment and horizontal load reversals. The spudcan tests featuring "loop events" of the type depicted here are not described by
Martín & Houlbys (2000), and they were not used to calibrate any features of Model B. It is nevertheless interesting to compare the results of a typical event, performed under constant vertical load at $V/V_0 = 0.4$, with the corresponding numerical prediction. In Fig. 12 the imposed $(\theta, u)$ displacement path commences with pure horizontal displacement in a positive direction. The elastic stiffness factors used in Model B predict an initial load path direction which is dominated by $H$ but with a slight negative component of $M$. With the switch to elastoplastic deformation the calculated $(M, H)$ load path begins to track around the yield surface (there is a very slight degree of softening, or yield surface contraction, but this is not discernible in Fig. 12). Apart from the expected "rounding off" of the transition from elastic to elastoplastic behaviour, the early part of the experimental curve shows reasonable qualitative and quantitative agreement with the Model B prediction. This agreement deteriorates in the latter stages of the loop event as the footing returns to previously disturbed soil, possibly suffering some loss of contact as well. Martin (1994) gives further details of three complete spudcan tests incorporating loop events of various types.

Byrne & Houlbys (2000) discuss some recent theoretical developments which allow much improved simulations of gradual yielding and load–displacement behaviour under cyclic $(V, M, H)$ loading, while still retaining the overall framework of a macroscopic plasticity model for the foundation response. These new "continuous hyperplasticity" methods appear to offer many advantages over the ad hoc interpolations between elastic and elastoplastic stiffness adopted by authors such as Schotman (1989) and Hambly et al. (1990). The new approach is based on the concept of a continuous field of yield surfaces, and therefore results in smooth transitions between elastic and plastic behaviour.

**CONCLUSIONS**

A complete incremental plasticity model for describing the behaviour of spudcan footings on clay under combined $(V, M, H)$ loading has been described. "Model B" uses analytically-based definitions of elasticity and the variation of vertical bearing capacity with embedment,
with an empirically-determined yield surface and a flow rule based on normality (but with one empirical adjustment). Several of the small-scale spudcan tests used to calibrate the latter two features have been simulated *a posteriori* with the numerical model. The theoretical hardening law gives good predictions of the virgin load-penetration behaviour, but vertical unload-reload loops cannot be replicated in detail because of the assumption of linear elasticity. Similarly, while the model is very successful in capturing the general patterns of behaviour in combined loading events, its "hard" yield surface does not allow realistic prediction of the gradual yielding observed experimentally. This is perhaps the major weakness of Model B, and one which is also responsible for its relatively poor performance in tests involving cyclic reversals of moment and horizontal load.

The main reason for developing a compact plasticity-based model of spudcan behaviour is to allow more rational analysis of the performance of complete jack-up units on clay. Martin (1994) and Martin & Houlsby (1999) have incorporated Model B footings into some simplified structural analyses of three-legged jack-up units subjected to monotonic, quasi-static lateral loading. Thompson (1996) and Cassidy (1999) have performed similar work, but with more detailed modelling of wave loading and the inclusion of structural dynamics. These analyses have afforded many interesting insights into the complex processes of soil–structure interaction (*e.g.* post-yield load redistribution between spudcans) which determine factors such as the critical stress resultants in the structure and the \((V, M, H)\) load paths followed at individual footings. Model B footings offer an additional benefit over traditional pinned or linear spring idealisations in that the ultimate lateral capacity of the soil–structure system emerges directly from the requisite incremental loading analysis. In an effort to check the validity of such predictions, experimental research involving pushover testing of a model three-legged jack-up unit on clay is currently in progress.

**ACKNOWLEDGEMENT**

The first author gratefully acknowledges the support of the Rhodes Trust.
**NOTATION**

\( D \)  \quad \text{depth below soil surface; footing embedment}

\( e_1, e_2 \)  \quad \text{yield surface fitting parameters}

\( e \)  \quad \text{(subscript) elastic component}

\( ep \)  \quad \text{(subscript) sum of elastic and plastic components}

\( f \)  \quad \text{yield function}

\( g \)  \quad \text{plastic potential function}

\( G \)  \quad \text{shear modulus}

\( h \)  \quad \text{height of footing origin \( O \) above effective origin \( O' \) (partial penetration)}

\( h_0 \)  \quad \text{dimensionless maximum horizontal load parameter}

\( H \)  \quad \text{horizontal load}

\( H_0 \)  \quad \text{maximum horizontal load capacity when } M = 0

\( K \)  \quad \text{stiffness matrix}

\( K_1 - K_4 \)  \quad \text{elastic stiffness factors (full penetration)}

\( K_1' - K_4' \)  \quad \text{elastic stiffness factors (partial penetration)}

\( m_0 \)  \quad \text{dimensionless maximum moment parameter}

\( M \)  \quad \text{moment load}

\( M_0 \)  \quad \text{maximum moment capacity when } H = 0

\text{nom}  \quad \text{(subscript) nominal}

\( N_c \)  \quad \text{bearing capacity factor with respect to } s_{u0}

\( p \)  \quad \text{(subscript) plastic component}

\( r \)  \quad \text{soil–footing contact radius (partial penetration)}

\( R \)  \quad \text{overall footing radius}

\( s_u \)  \quad \text{undrained shear strength}

\( s_{um} \)  \quad \text{\( s_u \) at mudline level}

\( s_{u0} \)  \quad \text{\( s_u \) at footing level}
\( u \)  
horizontal displacement

\( V \)  
vertical load

\( V_0 \)  
maximum vertical load capacity when \( H = M = 0 \)

\( w \)  
vertical displacement

\( \alpha \)  
soil–footing interface friction coefficient

\( \beta \)  
conical footing apex angle

\( \beta_1, \beta_2 \)  
yield surface fitting parameters

\( \delta \)  
(prefix) incremental value

\( \Delta \)  
(prefix) change from initial or reference value

\( \varepsilon \)  
vector of displacements

\( \theta \)  
rotational displacement

\( \Lambda \)  
scalar plastic multiplier

\( \nu \)  
Poisson's ratio

\( \rho \)  
rate of increase of \( s_u \) per unit depth

\( \sigma \)  
vector of loads

\( \zeta \)  
flow rule association parameter
REFERENCES


Table 1. Elastic stiffness factors for a rough, rigid, flat circular footing embedded in incompressible soil ($\nu = 0.5$) (after Bell, 1991a)

<table>
<thead>
<tr>
<th>Cone apex angle $\beta$</th>
<th>Embedment $D/2R$</th>
<th>Dimensionless elastic stiffness factors $\dagger$</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°</td>
<td>0.0</td>
<td>$K_1$</td>
</tr>
<tr>
<td></td>
<td>8.00</td>
<td>5.33</td>
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<tr>
<td>180°</td>
<td>0.25</td>
<td>8.58</td>
</tr>
<tr>
<td>180°</td>
<td>0.5</td>
<td>9.21</td>
</tr>
<tr>
<td>180°</td>
<td>1.0</td>
<td>9.98</td>
</tr>
<tr>
<td>180°</td>
<td>2.0</td>
<td>11.18</td>
</tr>
</tbody>
</table>

Table 2. Elastic stiffness factors for a rough, rigid, conical footing on incompressible soil ($\nu = 0.5$) (after Bell, 1991b)

<table>
<thead>
<tr>
<th>Cone apex angle $\beta$</th>
<th>Embedment $D/2R$</th>
<th>Dimensionless elastic stiffness factors $\dagger$</th>
</tr>
</thead>
<tbody>
<tr>
<td>180°</td>
<td>0.0</td>
<td>$K_1$</td>
</tr>
<tr>
<td>150°</td>
<td>0.0</td>
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<tr>
<td>120°</td>
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</tr>
<tr>
<td>90°</td>
<td>0.0</td>
<td>8.22</td>
</tr>
</tbody>
</table>

$\dagger$ Note that Bell (1991a, 1991b) adopted a ($V, H, M$) load ordering, so the $K_2$ values appearing here correspond to his original $K_3$ values, and vice versa.
CAPTIONS

Fig. 1 Notation and sign convention for spudcan loads and displacements

Fig. 2 Partially and fully penetrated spudcan footings (equivalent conical footings shown dotted)

Fig. 3 Model B yield surface and flow rule

Fig. 4 Typical field of stress characteristics for embedded conical footing ($\beta = 150^\circ$, $D/2R = 0.5$, $\alpha = 0.8$, $\rho 2R/s_{om} = 5$). Bearing capacity factor $N_c = 7.93$.

Fig. 5 Measured values of undrained shear strength in selected model spudcan tests

Fig. 6 Test AH1: vertical load–penetration plot: (a) measured (b) calculated

Fig. 7 Test AH3: load paths observed in swipe events (combined rotation and horizontal displacement, initial $V/V_0 = 1$): (a,b) measured (c,d) calculated

Fig. 8 Test AL2: load paths observed in swipe events (horizontal displacement, initial $V/V_0 = 0.01$): (a,b) measured (c,d) calculated

Fig. 9 Test BH1: results of constant $V$ events (rotation at $V/V_0 = 0.75$): (a,b,c) measured (d,e,f) calculated

Fig. 10 Test BL2: results of constant $V$ events (horizontal displacement at $V/V_0 = 0.2$): (a,b,c) measured (d,e,f) calculated

Fig. 11 Measured and calculated results for typical rotation cycle at constant $V$

Fig. 12 Measured and calculated results for typical loop event at constant $V$
Fig. 1. Notation and sign convention for spudcan loads and displacements

(a) Partial penetration ($w_p < 0$)

$D = 0$ for equivalent conical footing

(b) Full penetration ($w_p \geq 0$)

$D = w_p$ for equivalent conical footing

Fig. 2. Partially and fully penetrated spudcan footings (equivalent conical footings shown dotted)
Fig. 3. Model B yield surface and flow rule

Fig. 4. Typical field of stress characteristics for embedded conical footing (\(\beta = 150^\circ, D/2R = 0.5, \alpha = 0.8, \rho 2R/s_{um} = 5\)). Bearing capacity factor \(N_r = 7.93\).
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