Analysis of Jack-Up Units Using a Constrained NewWave Methodology

by

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Abstract
There is a steadily increasing demand for the use of jack-up units in deeper water and harsher environments. Confidence in their use in these environments requires jack-up analysis techniques to reflect accurately the physical processes occurring. This paper is concerned with the models appropriate for the dynamic assessment of jack-ups, with a balanced approach taken in considering the non-linearities in the structure, foundations and wave loading. A work hardening plasticity model for spud-cans on sand is used for the foundation model. The spectral content of wave loading is considered using NewWave theory, and the importance of random wave histories shown by constraining the deterministic NewWaves into a completely random background. A method for determining short-term extreme response statistics for a sea-state using Constrained NewWaves is detailed.

Keywords: NewWave; Constrained NewWave; Jack-up; Spudcan foundations; Work hardening plasticity; Random wave.

1. Introduction

As jack-up units are used for most offshore drilling operations in water depths up to 120m, they are of significant economic importance within the offshore industry. Since their first employment in relatively shallow sections of the Gulf of Mexico, there has been a steadily increasing demand for their use in deeper waters and in harsher environments [1]. There is also a desire for longer commitment of a jack-up at a single location, especially in the role of a production unit [2].

Before a jack-up can operate at a given site, an assessment of its capacity to withstand a design storm, usually for a 50-year period, must be performed. In the past, with jack-ups used in relatively shallow and calm waters, it has been possible to use overly simplistic and conservative analysis techniques for this assessment. However, as jack-ups have moved into deeper and harsher environments, there has been an increased need to understand jack-up behaviour and develop analysis techniques. The publication of the ‘Guidelines for the Site Specific Assessment of Mobile Jack-Up Units’ [3] is an example of the offshore industry’s desire both to standardise and to develop jack-up assessment procedures.

The most significant developments in the modelling of jack-ups have been in the areas of
- dynamic effects,
- geometric and other non-linearities in structural modelling,
- environmental wave loading, and
- models for foundation response.

There has been considerable diversity in this development, with studies using varying levels of complexity for different aspects of the analysis. This is shown in Table 1, which details a representative set of published studies from the last fifteen years. The four areas of development highlighted have been broken into components of increasing degrees of sophistication (and accuracy). Table 1 shows that single studies often employ complex
methods in one or two of the areas but use the simplest of assumptions in the others. This is especially true for foundation modelling, with many studies using detailed structural models or advanced wave mechanics whilst still using the simplest of foundation assumptions (i.e. pinned footings).

In calculating wave loading on jack-ups deterministic regular wave theories, such as Airy and Stokes V, together with the Morison equation are still widely used. However, by assuming all wave energy is concentrated in one frequency component rather than the broad spectrum of the ocean environment regular wave theories give an unrepresentative dynamic response. For jack-ups, which are dynamically responding structures, it is important to simulate the random, spectral and non-linear properties of ocean loading. The extreme dynamic response depends not only on the load currently being applied, but is also affected by its load history. Therefore, the most accurate methods of estimating extreme response are based on time domain simulation of a pseudo-random ocean surface and calculation of the corresponding kinematics.

Acquiring confidence in random time domain simulation results is, however, computationally time consuming, with only a few of the waves in each time series capable of producing the extreme result. For this reason the majority of the studies in Table 1 used simplifying assumptions. In contrast to this, by constraining a NewWave with a predetermined large crest in an arbitrary random time series, a method for calculating extreme statistics without the same degree of computational burden can be devised [4].

The purpose of this paper is twofold. Firstly, a more balanced approach to the modelling of jack-up platforms will be described. The major non-linearities in the analysis of jack-up platforms are considered using a dynamic structural analysis program, named JAKUP. The principle characteristics of the structural, foundation and wave loading models used in JAKUP are given in this paper, however, further details can be found in Thompson [5], Williams et al. [6] and Cassidy [7]. Secondly, a methodology for using Constrained NewWaves for the evaluation of extreme response statistics of jack-up units is outlined. Example numerical calculations of a jack-up unit subjected to short-term (three-hour) storms are shown.

2. Outline of the Model Used

2.1. Example Structure

Figure 1 shows a schematic diagram of the idealised plane frame jack-up used in this paper. The mean water depth was assumed to be 90 m with the rig size typical of a three-legged jack-up used in harsh North Sea conditions. Figure 1 represents an equivalent beam model, with the corresponding stiffnesses and masses of the beams shown. The hull is also represented as a beam element with a rigid leg/hull connection. Though non-linearities in the leg/hull jack houses are recognised as significant [8,9], they were not included in the analyses. Example structural node locations and hydrodynamic modelling coefficients for the leg sections are shown in Figure 1.

2.2. The Wave Loading Model

NewWave Theory and Constrained NewWave have been implemented in JAKUP to evaluate the surface elevations and wave kinematics, with the extended Morison equation used to calculate the hydrodynamic loads on the jack-up legs. Horizontal particle kinematics are calculated at the undeflected beam position, with the equivalent nodal loads found by
integrating the distributed load with the corresponding shape function (using Gaussian integration techniques).

NewWave theory, a deterministic method described by Tromans et al. [10], accounts for the spectral composition of the sea, and can be used as an alternative to both regular wave and full random time domain simulations of lengthy time histories. By assuming that the surface elevation can be modelled as a Gaussian random process, the expected elevation at an extreme event (for example a crest) can be derived theoretically. The surface elevation around this extreme event is modelled by the statistically most probable shape associated with its occurrence. It is shown by Tromans et al. [10] that the surface elevation is normally distributed about this most probable shape, with the surface elevation described by two terms, one deterministic and one random. As a function of time, the surface elevation can be written as

\[ \eta(\tau) = \alpha r(\tau) + g(\tau) \]  \hspace{1cm} (1)

where \( \tau = t - t_1 \), the time relative to the initial position of the crest (i.e. \( t_1 \) is the time when the extreme event occurs). In Equation 1, term \( (1) \) describes the most probable value, where \( \alpha \) is the crest elevation defined as the vertical distance between the wave maximum and the mean water-level, and \( r(\tau) \) the autocorrelation function for the ocean surface elevation. The autocorrelation function is proportional to the inverse Fourier Transform of the surface energy spectrum, allowing the surface elevation to be determined efficiently.

Term \( (2) \) of Equation 1 is a non-stationary Gaussian process with a mean of zero and a standard deviation that increases from zero at the crest to \( \sigma \), the standard deviation of the underlying sea at a distance away from the crest. Therefore, as the crest elevation increases, term \( (1) \) becomes dominant and can be used alone in the derivation of surface elevation and wave kinematics near the crest.

The continuous time autocorrelation function is defined as

\[ r(\tau) = \frac{1}{\sigma^2} \int_{-\infty}^{\infty} S_{\eta \eta}(\omega) e^{i\omega \tau} d\omega \]  \hspace{1cm} (2)

with the time history of the extreme wave group proportional to \( r(\tau) \) at the region around \( \tau = 0 \). For a time lag of \( \tau = 0 \), the autocorrelation function of Equation 2 reduces to one, allowing the surface elevation of the NewWave to be scaled efficiently. This is shown below in Equation 3.

The NewWave shape as defined by the autocorrelation function (Equation 1) can be discretised by a finite number \( (N) \) of component sinusoidal waves. As there exists a unique relationship between wave number and frequency, spatial dependency can also be included, leading to the discrete form:

\[ \eta(X, \tau) = \frac{\alpha}{\sigma^2} \sum_{n=1}^{N} [S_{\eta \eta}(\omega_n)\omega_n] \cos(k_n X - \omega_n \tau) \]  \hspace{1cm} (3)

where \( k_n \) is the wavenumber of the \( n^{th} \) component. (An approximate solution method for the dispersion relation for plane waves in a constant water depth, as outlined by Newman [11], has
been implemented in JAKUP.) As defined previously, $\alpha$ is the crest elevation, $S_{\eta \eta} (\omega) d\omega$ the surface elevation spectrum and $\sigma$ the standard deviation corresponding to that wave spectrum. $X = x - x_0$ is the distance relative to the initial position with $X = 0$ representing the wave crest. This allows the positioning of the spatial field such that the crest occurs at a user-defined position relative to the structure, a useful tool for time domain analysis.

This deterministic formulation of NewWave allows it to be conveniently and efficiently implemented into structural analysis programs, such as JAKUP. Furthermore, because it is based on linear theory, the water wave particle kinematics can be easily obtained once the water surface has been established. However, care must be taken in describing their values in a crest with a number of stretching approaches commonly used. These include Delta stretching [12] and Wheeler stretching [13]. Although there are numerous other stretching or extrapolation methods described in the literature, there is no clear preferred option. Due to the substantial separation of jack-up legs the ability to evaluate accurately the time varying surface elevation and kinematics at two spatial positions is important.

NewWave theory provides a realistic deterministic alternative to regular wave theories. For structures which respond quasi-statically, NewWave theory has been used successfully in the prediction of global response [14]. It has also been validated against both measured global loading and conventional random wave modelling on a real platform by Elzinga and Tromans [15] and on standard column examples [10]. However, when used as a time-history for analysing dynamically sensitive structures, NewWave's single profile does not account for the randomness of the sea in which a large crest is found and therefore the effect this randomness has on the structure's dynamic response. By constraining a NewWave with a pre-determined height within a completely random background, NewWave theory can be used to produce a time series of a random surface elevation. This is performed in a mathematically rigorous manner such that the constrained sequence is statistically indistinguishable from the original random sequence. The details of a procedure achieving this are outlined by Taylor et al. [4].

Figure 2 illustrates the surface elevation of a NewWave with a crest elevation of 15 m embedded in a random sea-state characterised by $H_s = 12$ m and $T_z = 10$ s. The wave has been constrained such that its peak occurs at about 60 s. It is shown in Figure 3 that, for this example, the influence of the NewWave on the surface elevation is contained to within 40 s of the constrained peak.

Constrained NewWave allows for the easy and efficient evaluation of extreme response statistics, provided the required extreme response correlates, on average, with the occurrence of a large wave within a random sea-state. Constrained NewWave has been used for the calculation of extreme response by Harland [16] and Harland et al. [17]. The application of Constrained NewWave to the study of jack-up response is discussed further here.

2.3. The Structural Model

Structural non-linearities ($P - \Delta$ and Euler effects) are considered in JAKUP by using Oran’s [18] path-dependent formulation of beam column theory to specify an incremental stiffness matrix. Additional modifications to produce the additional end rotations on the beam due to the presence of shear have also been implemented [19]. Both the mass and damping matrix are time invariant, with the former derived using cubic Hermitian polynomial shape functions and
the latter by use of Rayleigh damping. Structural damping coefficients are defined for the lowest two modes and have been set at 2% of critical.

Conventionally, jack-up assessments have used the same quasi-static analysis methods employed for fixed structures [20]; however, the need to consider dynamic effects has long been acknowledged [21,22,23]. With use in deeper water, the contribution of dynamic effects to the total response has become more important as the natural period of the jack-up approaches the peak wave periods in the sea-state. Due to their ability to model all non-linearities, time domain techniques using numerical step-by-step direct integration provide the most versatile approach to the analysis of the extreme response of jack-ups. Within JAKUP, the Newmark $\beta = 0.25$, $\delta = 0.5$ method is used, since it is an unconditionally stable and highly accurate algorithm. Further details of the structural formulations and dynamic solution techniques can be found in Martin [19] and Thompson [5].

2.4. The Foundation Model

The use of a strain hardening plasticity theory is seen as the best approach to model soil behaviour with a terminology amenable to numerical analysis. This is because the response of the foundation is expressed purely in terms of force resultants. Accounting for a level of foundation fixity reduces critical member stresses (usually at the leg/hull connection) and other critical response values [24,25]. Furthermore, the natural period of the jack-up is also reduced, usually improving the dynamic characteristics of the rig.¹

JAKUP has the capabilities of modelling pinned, fixed or linear springs as the foundations of a jack-up. Furthermore, a strain hardening plasticity model for spudcan footings on dense sand has been developed and its numerical formulation implemented into JAKUP. It is called Model C and its main features are discussed further here, however, for a detailed description reference can be made to Cassidy [7]. Model C is based on a series of experimental tests performed at the University of Oxford and described in Gottardi et al. [26] and follows “Model B”, a strain hardening plasticity model described by Martin [19] for spudcans on clay.

Model C has four major components:

1. An empirical expression for the yield surface in three dimensional vertical, moment and horizontal loading space ($V, M/2R, H$) (note: moment is normalised by the diameter of the spudcan, $2R$). This envelope represents a yield locus defining permissible load states and its ‘rugby ball’ shaped surface is shown in Figure 3. The shape is best described as an eccentric ellipse in section on the planes of constant $V$, and approximately parabolic on any section including the $V$-axis. Once the yield surface is established, any changes of load within this surface will result only in elastic deformation. Plastic deformation can result, however, when the load state touches the surface.

2. An empirical strain-hardening expression to define the variation of the size of the yield surface with the plastic component of vertical displacement. Though the shape of the yield surface is assumed constant, it expands with vertical plastic penetration (as the footing is

¹ It is still widely accepted practice to assume pinned footings (infinite horizontal and vertical and no moment restraint) in the analysis of jack-ups [28,29]. This results in over-conservative analyses. Another approach often used in jack-up analysis, and an improvement on use of pinned footings, is the use of linear springs. Unfortunately, linear springs, while easy to implement into structural analysis programs, do not account for the complexities and non-linearities of spudcan behaviour, and this simplistic method can render unrealistic results, which may be unconservative.
pushed further into the soil). A strain hardening law for spudcans has been derived by combining the experimental observations for flat circular footings and the theoretical predictions of conical footings. Details of this “combined” strain hardening law will not be given here, but can be found in Cassidy [7] or Cassidy and Houlsby [27].

(3) A model for elastic load-displacement behaviour within the yield surface. Finite element work has shown that cross coupling exists between the horizontal and rotational footing displacements [30], with a linear elastic incremental force-displacement relationship of the form

\[
\begin{pmatrix}
\frac{dV}{dM/2R} \\
\frac{dM/2R}{dH}
\end{pmatrix} = 2GR \begin{bmatrix}
k_v & 0 & 0 \\
0 & k_m & k_c \\
0 & k_c & k_h
\end{bmatrix} \begin{pmatrix}
dw^e \\
d\theta^e \\
du^e
\end{pmatrix}
\]

where \( w, \theta \) and \( u \) are the vertical, rotational and horizontal displacements respectively. \( G \) is a representative soil shear modulus, estimated as

\[
\frac{G}{p_a} = g \sqrt{\frac{2R\gamma'}{p_a}}
\]

where \( p_a \) is atmospheric pressure, \( \gamma' \) the submerged unit weight of sand and \( g \) a non-dimensional shear modulus factor. Equation 4 represents elastic behaviour in Model C with typical values of the stiffnesses \( k_v, k_m, k_h, k_c \) and \( g \) given in Table 2.

(4) A suitable flow rule to allow prediction of the footing displacements during yield. The experimental evidence did not support the application of associated flow, and a plastic potential function \( g \) was defined to relate the ratio between the plastic displacements. Details are given in Cassidy [7].

3. Example Response of a Jack-Up Subjected to a Constrained NewWave

An example jack-up analysis for the structure shown in Figure 1 subjected to a Constrained NewWave will be outlined. As in the rest of this paper, wave loading is the only environmental force, with no current or wind applied. Figure 4 displays the wave as in Figure 2 constrained such that its peak is located at the upwave leg of the jack-up. The surface elevation time history at the downwave leg, as evaluated in JAKUP, is also displayed. The corresponding deck displacements with time are shown in Figure 5 for the cases of linear springs, Model C and pinned foundations. Pinned footings represent infinite horizontal and vertical, but no rotational stiffness. Model C is the strain hardening plasticity model described above, whilst linear springs represents the elastic region of the Model C case (Equation 4) with plastic strains excluded.

The peak displacements shown in this example are not necessarily what would be evaluated for an analysis considering just the equivalent NewWave. This is because of the random background and the structural memory caused, indicating that for dynamically sensitive structures, such as jack-ups, the response is not only dependent on the present applied load, but also on the load history.
The assumption of pinned footings is clearly shown in Figure 5 to be overly conservative. The linear springs can be seen to yield lower displacements than the Model C footing, due to the greater stiffness exhibited. In addition, Model C indicates a permanent horizontal displacement occurring in the jack-up. There is a little less than a factor of four difference between the pinned and the linear springs cases. This is expected as non-linearities in both the structure and loading are included, and the linear springs case, though very stiff, is not fully fixed. Theoretically, if non-linear and dynamic amplification effects were not considered, there would be a factor of four difference between a pinned and fixed case for a jack-up with a very stiff hull and a rigid leg/hull connection.


Though only one Constrained NewWave example has so far been shown here, one of the main benefits of the constraining technique is that the probability distribution of the extreme response can be estimated without the need to simulate many hours of real time, most of which is of no interest. For a storm associated with one sea-state, shorter time periods can be used with a logical combination of crest elevations to simulate responses for the expected wave sizes within that sea-state. Convolution with the probability of occurrences of crest elevations allows for the compilation of response statistics. A method used for evaluating short-term extreme response statistics will be described here.

Within this paper, simulations of 75 s duration, with the crests constrained to occur after 60 s, are used to estimate the time history of response associated with one crest height. A lead-time of 60 s was considered to be long enough to give the same statistical response properties (due to the crest) as longer lead-time periods. For five discrete crest elevations representative of the full range of wave heights in a three-hour storm, 200 simulations per crest elevation were performed using JAKUP. The largest response caused by the constrained peak was recorded for each simulation. The extreme response distributions were evaluated by convolving responses from the five constrained crest elevations with the Rayleigh distribution of crest heights. This was accomplished numerically using Monte Carlo techniques. The steps followed are outlined below and are also shown in Figure 6:

Step 1: For a short time period (for example three hours), crest heights may be randomly derived assuming a Rayleigh distribution.\(^2\) One crest elevation is predicted as

$$\alpha_{\text{pred}} = \sqrt{-2\sigma^2 \ln(1 - rm)}$$  \hspace{1cm} (6)

where \(rm\) is a random number generated from a uniform distribution between 0 and 1. Therefore, a set of wave crests \(\alpha_i, i = 1, 2, ..., N_{\text{crests}}\), can be estimated from a Monte Carlo simulation (\(N_{\text{crests}}\) is the number of wave crests, assumed as the time period divided by \(T_{\gamma}\)).

Step 2: Using the response information from the 5 sets of 200 JAKUP runs, lines of constant probability of exceedence are constructed by firstly sorting the population of responses at each of the five NewWave crest elevations into order from the lowest (or \(1^\text{st}\)) response to the highest (or \(200^{\text{th}}\)) response. Following this, polynomial curve fitting of the five responses representing the five NewWave elevations at each response level (\(1 \rightarrow 200\)) gives

\(^2\) This is based on the assumption of a Gaussian sea and a narrow-banded process (see Longuet-Higgins [31]). The Gaussian sea assumption depends on the water depth of the jack-up and for many shallow water cases it would not be appropriate.
200 lines of constant probability. These lines allow response values to be estimated for crest elevations between the five NewWave amplitudes used in JAKUP. Construction of the lines of constant probability will be shown later in the example calculation.

For each crest elevation in step 1, i.e. \( \alpha_i, i = 1, 2, ..., N_{crest} \), a response corresponding to its elevation is “randomly” chosen. By generating a random number between 1 \( \rightarrow \) 200 (the size of the population of JAKUP runs per NewWave crest elevation), interpolation along that number’s line of constant probability gives the “random” response for that one crest. Repeating for all \( N_{crest} \) elevations completes a set of responses for that time period. This is shown in step 2 of Figure 6. Therefore, for one random three-hour event, the distribution of responses within that sea-state have been calculated and the extreme event can be extracted.

**Step 3:** By repeating steps 1 and 2, responses for different three-hour events are evaluated and, by using the maximum of each, the distribution of extreme response can be compiled. This is shown in step 3 of Figure 6. In the example numerical experiments in this paper, distributions are based on 2500 of these samples. Note that this technique allows the results of 75000 s of simulation (21 hours) to be used to generate statistics for 7500 hours of storms.

5. Example Numerical Results for One Sea-State

An example of the methodology described to compile short-term extreme response statistics is outlined here. A sea-state described by the JONSWAP wave energy spectrum with parameters \( H_s = 12 \) m and \( T_z = 10.805 \) s has been chosen, conditions which could be representative of the 100 year sea-state in a central North Sea location.

Figure 7 shows the deck displacements calculated by JAKUP for the five crest elevations: 3.5, 7, 10, 12 and 15 m. Though Figure 7 seems to illustrate larger variation of deck displacements as the NewWave crest elevation increases, this is not the case. When the coefficient of variation (CoV) is calculated, defined in the usual way as the standard deviation divided by the mean of the random set, there is actually a reduction in CoV with increasing crest elevation. This implies that as the crest elevations become higher, they also become more dominant in the calculation of global response, thereby reducing the response’s variation.

 Constructed by simple polynomial curve fitting, Figure 8 shows four example lines of constant probability between the levels of evaluated response. The actual number of lines of constant probability used is not four, as implied in Figure 8, but is the number of simulations at each crest elevation performed by JAKUP (in this case 200).

The extreme response distribution for deck displacement, evaluated using the convolution procedure described above, is shown in Figure 9. The mean and 50% exceedence values are 0.251 and 0.241 m respectively and the distribution has a CoV of 22.26%.

5.1. Verification of Short-Term Extreme Response Results

The accuracy of this method has been examined by repeating the calculations. Four new sets of 200 extreme responses per crest elevation were evaluated by JAKUP, and the convolution procedures repeated. With little difference between the resulting statistics, uniformity in

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3 The 200 lines of constant probability need to be constructed only once and contain all of the response information from the JAKUP runs.
successive tests is shown in Figure 10. The question of whether 200 responses per crest elevation is a large enough sample has also been addressed. With convolution performed on all the data (1000 JAKUP responses per crest elevation), the extreme response statistics evaluated are consistent with results evaluated from the 200 JAKUP response data. This is shown in Figure 10 and validates the use of only 200 extreme responses per crest elevation.

Consistency is also shown in Figure 10 for a very extreme sea-state: \(H_s = 16.45\) m and \(T_c = 12.66\) s, corresponding to a one in a million years. This is an important outcome, as the method used is validated for cases that involve large amounts of plasticity and non-linearities in the Model C calculation. The results have also been verified by “brute force” random wave simulation for both sea-states. As shown in Figure 11, one hundred full three-hour simulations correspond well to the Constrained NewWave results.

6. Comparison of Different Footing Conditions

Figure 12 shows the extreme hull displacement statistics for a three-hour time period for the same two sea-states with three footing assumptions: linear springs, Model C and pinned. The mean and CoV values of the extreme response are also presented. Again, the pinned case is giving overly conservative results when compared to the Model C predictions. The yielding effects in Model C are giving only a slightly larger mean extreme response than the assumption of linear springs for the \(H_s = 12\) m and \(T_c = 10.806\) s case. However, for the larger sea-state the effects of the plasticity model are clearly observed. There is an interesting difference in CoV values for the three cases, with extra variation in the Model C case due to the use of a plasticity model.

7. Conclusions

A method for determining short-term extreme response statistics using the Constrained NewWave was demonstrated. The peak responses due to five Constrained NewWave elevations (for 200 random backgrounds each) were used to evaluate numerically the short-term extreme statistics of that sea-state using Monte Carlo methods. This was found to be computationally efficient and the results comparable with 100 full three-hour simulations of random seas, even with the inclusion of significant amounts of foundation non-linearities.

With knowledge of long-term conditions, convolution of short-term statistics into long-term probabilities can be performed. The methodology of evaluating short-term statistics outlined here allows this procedure to be performed efficiently. Long-term extreme exceedence probabilities for response design properties of interest in the reliability of jack-ups (for example lower leg-guide moments or deck displacement) can therefore be evaluated.

Model C, as implemented into the plane-frame structural analysis program JAKUP, represents a significant advance to the response analysis of jack-up units. When compared with techniques widely used in the jack-up industry, a significantly different response is found, as was shown in this paper. Model C together with the structural model and the Constrained NewWave technique outlined here reflects a balanced approach to the design uncertainties and achieves what is believed to be a realistic modelling of a jack-up.
Acknowledgements
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References


Table 1 - Level of complexity used in the analysis of jack-up units

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a. investigated Dynamic Amplification Factors for different levels of damping for 1000 waves (representing one 3hr event)
b. uncoupled springs
c. for random time domain simplified the structure to a stick model
d. fitted extreme response of 3hr storm with 3 parameter Weibull distribution from one 40min and two 20min simulations
e. ten 2.5hr simulations
f. ten 3hr simulations
g. used single degree of freedom stick model
h. used Constrained NewWave
i. six 0.57hr simulations
j. implemented the non-linear stiffness model recommended in the SNAME (1994) procedures
k. used a 200-second segment of one 3hr storm to scale and represent the extreme case
l. used dynamic analysis of a simplified stick model to evaluate Dynamic Amplification Factors at specific Hc values for use in a quasi static push over analysis
(R) studies on jack-up reliability
Table 2 - Example values used in the foundation model

<table>
<thead>
<tr>
<th>Constant</th>
<th>Dimension</th>
<th>Explanation</th>
<th>Typical value</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>L</td>
<td>Footing radius</td>
<td>(various)</td>
<td></td>
</tr>
<tr>
<td>$\gamma'$</td>
<td>F/L$^3$</td>
<td>Unit weight of soil</td>
<td>10kN/m$^3$</td>
<td>Saturated sand</td>
</tr>
<tr>
<td>$g$</td>
<td>-</td>
<td>Shear modulus factor</td>
<td>4000</td>
<td></td>
</tr>
<tr>
<td>$k_v$</td>
<td>-</td>
<td>Elastic stiffness factor (vertical)</td>
<td>2.65</td>
<td></td>
</tr>
<tr>
<td>$k_{bh}$</td>
<td>-</td>
<td>Elastic stiffness factor (moment)</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>$k_h$</td>
<td>-</td>
<td>Elastic stiffness factor (horizontal)</td>
<td>2.3</td>
<td></td>
</tr>
<tr>
<td>$k_e$</td>
<td>-</td>
<td>Elastic stiffness factor (horizontal/moment coupling)</td>
<td>-0.14</td>
<td></td>
</tr>
</tbody>
</table>
Values:

For a single leg:

\[ E = 200 \text{GPa} \]
\[ I = 15 \text{m}^4 \]
\[ A = 0.6 \text{m}^2 \]
\[ M = 1.93 \times 10^4 \text{kg} \]
\[ A_s = 0.04 \text{m}^2 \]
\[ G = 80 \text{GPa} \]
\[ D_K = 8.44 \text{m} \]
\[ A_h = 3.94 \text{m}^2 \]
\[ C_d = 1.1 \]
\[ C_m = 2.0 \]

For hull:

\[ I = 150 \text{m}^4 \]
\[ A_s = 0.2 \text{m}^2 \]
\[ M = 16.1 \times 10^4 \text{kg} \]

**Figure 1** - General layout of idealised jack-up used in the analyses

**Figure 2** - Surface elevation of a NewWave of 15m elevation constrained within a random background
Figure 3 - Shape of yield surface in Model C

Figure 4 - Surface elevations at the upwave and downwave legs for a Constrained NewWave

Figure 5 - Horizontal deck displacements due to the Constrained NewWave
Step 1

For a narrow banded Gaussian sea, crest elevations are Rayleigh distributed. A set of elevations ($\alpha_i$) can be generated from a Monte Carlo simulation.

Step 2

The crosses represent the 5 sets of 200 JAKUP runs (sorted into ascending order) from which lines of constant probability are constructed.

For each set of $\alpha$, a set of responses $R_i$ is generated by selecting the response value for each elevation from a randomly generated number (between 1 → 200) and then its line of constant probability.

Step 3

One Monte Carlo simulation of steps 1 and 2 produces one extreme response (the largest in the set $R_i$). Repeating the steps gives an expected distribution of extreme response for the period in that sea-state.

Figure 6 - Explanation of methodology adopted to evaluate the extreme response statistics for one short-term sea-state
Figure 7 - Deck displacements calculated by JAKUP for five Constrained NewWave elevations

Figure 8 - Lines of constant probability

Figure 9 - Extreme deck displacement distributions for a three hour period of a sea-state represented by $H_s = 12$ m and $T_s = 10.805$ s
Figure 10 - Repetition of the calculation of extreme response statistics

Figure 11 - Comparison of extreme response statistics evaluated using the Constrained NewWave method with 100 three-hour random simulations
Figure 12 - Extreme deck displacement distributions for three footing conditions