Conference papers on numerical modelling of tunnelling and settlement-induced damage to masonry buildings

by


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INTRODUCTION

This report contains eight papers presented at conferences between 1994 and 1999. These papers present many of the findings of an ongoing project on the understanding of the interaction between tunnelling processes, the associated ground movements and possible damage to adjacent buildings.

The central concept behind the whole research programme has been that tunnels and buildings cannot be examined in isolation. If the complex problem of soil-structure is to be understood, then tunnel and building have to be incorporated into a single analysis. The analysis is, of necessity, three-dimensional, and must take into account a number of different types of nonlinearities. The research at Oxford is recorded in three theses (Chow (1994), Liu (1997), Augarde (1997)), and two further research students (Bloodworth and Wisser) are currently working in this area.

The program has been divided into three broad phases:
- Preliminary work (Chow) using two-dimensional analysis.
- Development of 3-D analyses of tunnels (including lining), the surrounding ground and masonry buildings (Augarde, Liu)
- Calibration of the methods against case records (Bloodworth), and development of tunnel installation procedures and analysis of compensation grouting (Wisser).

The first paper (Burd et al. 1994) sets out the general approach that was planned for the subsequent research, drawing on the preliminary study by Chow (1994). The second (Augarde et al. 1995) provides more details on the numerical procedures adopted in the research. The third (Augarde et al. 1998) reports the experience in using these methods to analyse typical tunnelling problems.

The fourth paper (Houlsby et al. 1999) is a general report on the results of the second phase of research.

Bloodworth and Houlsby (1999) report one comparison of the analyses with a case history (of a shaft construction rather than a tunnel).

Augarde et al. (1999) present more detail on the numerical aspects of the calculations, particularly developments in tunnel installation modelling.

Houlsby (1999) presents the details of the non-linear model used for undrained clay in the analyses: this was developed primarily for this project, and the nonlinearity is important for the pattern of settlements predicted.

Finally Bloodworth et al. (1999) report some of the experience of the transfer of the FE code to the Oxford Supercomputer OSCAR for the later analyses.

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References


Contents of this report:


Analysis of settlement damage to masonry structures


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ABSTRACT: The methods used in practice for the assessment of damage to buildings caused by settlements due to tunnelling are generally based on two separate and distinct procedures. A geotechnical analysis is first carried out to estimate likely ground movements, and then a separate method is used to assess the resulting damage to the existing structure. This paper describes progress made at Oxford University on the development of a unified approach to the problem of damage to buildings induced by tunnelling. Finite element analysis of the combined geotechnical and structural problem will be used. A preliminary study is described in which suitable soil models for the prediction of settlements above plane strain tunnels are investigated. Details are given of proposals to extend this analysis to three-dimensions. A study has also been carried out of suitable constitutive models for masonry and details are given of a simple elastic-no tension model.

1 INTRODUCTION

A number of new tunnelling projects in London are currently planned (for example the Jubilee line extension and the CrossRail project). Since these proposed tunnelling activities will take place in close proximity to existing structures, it is necessary to predict the probable amount of structural damage as part of the design process.

Current approaches taken to assess the effect of tunnel construction on an existing building generally consist of two separate parts. Firstly, an estimate is made of the ground surface movements associated with the construction of the tunnel. This is usually achieved using semi-empirical approaches based on experience from previous tunnelling projects (e.g. Peck (1969), Rankin (1988) and Mair et al. (1993)). Secondly, the effect that these ground movements would have on neighbouring structures is assessed.

A common method to estimate the response of a structure is the approach proposed by Burland and Wroth (1975), where the strains in the building due to combined bending and shear (making the further assumption of simple beam theory) are calculated. The effects of these strains and of the relative angular distortion of extremes of the building are then assessed by comparison with previous field measurements.

An important omission in the above procedure is the influence of the building on the surface settlement profile. The stiffness of the building would be expected to reduce surface differential settlements while its self-weight would tend to increase the total settlement.

Previous investigations of tunnel-induced settlements using numerical models have largely been restricted to two-dimensional studies. To capture fully the mechanisms of deformation around tunnels, however, it is generally accepted that a three-dimensional analysis is required. Some previous work on three-dimensional analysis of tunnels has been reported (e.g. Lee and Rowe (1990)) but to the authors’ knowledge no three-dimensional study has investigated the interaction between a tunnel and a neighbouring masonry building.

A research project is currently underway at Oxford University on the development of three-dimensional finite element procedures to study this problem. An initial two-dimensional study, which concentrated on the choice of a soil model that is able to capture correctly the deformation field around a tunnel, has been completed. This work is currently being extended to three-dimensions, including also the implementation of numerical procedures to model a masonry building.
2 PRELIMINARY NUMERICAL STUDIES

An initial study of the use of finite element methods to predict ground surface settlements associated with tunnelling is described by Chow (1994). This study deals with the case of a single, unlined tunnel beneath a horizontal soil surface and does not consider the effect of nearby structures. The analyses were all carried out under plane strain conditions for the case of undrained loading.

It is well known that unless care is taken in the choice of soil model, the general shapes of predicted surface settlement troughs caused by tunnelling can differ significantly from those observed in practice. This study was primarily concerned with the identification of soil models that are able to capture correctly the patterns of deformation that occur around tunnels. These calculations were concerned only with the prediction of the shape of settlement trough. No attempt was made to calculate accurately the magnitude of the settlement which would depend on details of soil stiffness and construction procedures.

The analyses were carried out as follows. Finite element meshes were generated to include the tunnel geometry; initial stresses, corresponding to the in-situ state prior to tunnel construction, were then assigned to all of the mesh Gauss points. Equivalent nodal forces were then generated and applied. To model the construction of the tunnel, calculations were performed in which the nodal forces around the perimeter of the tunnel were balanced by the application of equal, opposite, nodal forces.

Initial studies were carried out in which the soil was modelled as a homogeneous, linearly elastic material. A variety of values of the ratio of tunnel depth, $z$, to tunnel diameter, $D$, were adopted (where $z$ is the depth of the tunnel axis below the ground surface). For each value of tunnel depth, calculations were performed using three separate meshes, each with different overall dimensions. All of the meshes involved six-noded triangles with a three Gauss point integration rule. Poisson’s ratio was set to 0.49. For the case where $z/D$ is 2, the mesh with the smallest overall dimensions Mesh A, is shown in Figure 1.

Meshes B and C were similar except that the overall dimensions were increased to widths $20D$ and $40D$ respectively and heights $12D$ and $22D$ respectively. The total number of elements in these meshes was also increased in order to ensure similar element densities.

Surface settlements calculated using these meshes are shown in non-dimensionalised form in Figure 2, where $\delta$ is the vertical displacement at the soil surface, $x$ is the horizontal distance from the tunnel centre-line and $\gamma$ is the soil self-weight. The results obtained from Mesh A show a shallow trough extending over the width of the mesh whereas the two other meshes indicate that heave occurs immediately above the tunnel centre-line. Empirical correlations proposed by Mait et al. (1993) suggest that for a tunnel in clay at this depth, the settlement curve could be approximated by a Gaussian curve with a point of inflection at a distance of 0.5 $z$ from the tunnel centre-line (in this case this corresponds to a value of $x/D$ of unity).

![Figure 1; Mesh A](image1)

![Figure 2; Surface Settlements](image2)
The vertical displacement profiles obtained from these elastic analyses clearly differ significantly from the profiles that would be expected in practice. It is concluded, therefore, that a simple elastic model of this sort is unsuitable for this type of analysis. A similar conclusion was reached by Gunn (1992).

These elastic results may be interpreted in terms of two competing mechanisms. Installation of the tunnel causes settlements associated with local loss of soil support. This mechanism tends to produce downward surface movements. Removal of soil within the tunnel, however, causes stress relief and this tends to produce heave. As the bottom boundary of the mesh becomes more distant from the tunnel, then the heave effect begins to dominate. If the mesh size were to be increased indefinitely then the magnitude of these heave displacements would also increase without limit for the special case of a plane strain tunnel.

Numerical experiments were carried out in order to investigate the use of an elastic - perfectly plastic soil model for the analysis. The use of this soil model did not appear to lead to a significant improvement in the quality of the results. It was discovered in a further study, however, that if a linearly elastic model in which the soil stiffness increased linearly with depth was used, then the calculated surface profiles were more realistic and the problem of heave displacements for large meshes did not occur.

An important feature of recent work on the modelling of clays is the increasing realisation of the importance of correct modelling of soil non-linearity at small strains where, conventionally, the soil is thought of as elastic. Several approaches have been adopted to model these small strain non-linearities, for example the non-linear elastic model adopted by Gunn (1992), the kinematic yield hardening model developed by Atkinson and Stallebrass (1992) and the 'bricks-on-strings' model described by Simpson (1993). In view of this recent activity, a multi-surface kinematic yield hardening model was formulated and implemented in the finite element code to investigate its use for the analysis of tunnel settlements. This model is similar in several respects to the models proposed by Atkinson and Stallebrass (1992) and Simpson (1993).

A full description of this kinematic yield hardening model is given by Houlshby and Chow (1994). Briefly, the model allows an arbitrary number of von Mises type yield surfaces. Strain rates are decomposed into elastic and plastic parts where the plastic parts, are obtained by assuming a fully associated flow rule. The centre of each yield surface in stress space is allowed to translate in accordance with a set of linear strain hardening relationships. The model also includes an outer von Mises bounding surface.

It was found that this work-hardening plasticity model produced numerical solutions that were considerably more realistic than those obtained using a simple linear elastic model. A typical set of surface settlements, obtained using Mesh C, is shown in Figure 3. In this particular analysis, a total of ten nested yield surfaces were adopted with the parameters chosen to provide a realistic variation of shear stiffness with shear strain. The outer bounding surface corresponds to an undrained shear strength of 80 kPa. The shear modulus, \( G \), (corresponding to the shear stiffness at zero strain) was set to 33557 kPa. The tunnel depth, \( z \), was taken to be 10m, Poisson's ratio was taken to be 0.49 and \( \gamma \) was 20 kPa/m.

![Figure 3; Settlement Profiles Obtained Using Kinematic Yield Hardening Model](image)

Also plotted on Figure 3 is a Gaussian curve representing the expected form of the settlement trough. The position of the inflection point is taken to be at a distance of 0.5 \( z \) from the tunnel centre-line (Mair et al. (1993)) and the magnitudes of the expected settlements are scaled to match the finite element results on the tunnel centre-line. The finite element prediction is seen to provide a slightly flatter response than the empirical curve but, nevertheless, the numerical results are substantially better than those obtained using a linear elastic model.

These preliminary calculations are thought to indicate that a kinematic yield hardening model of the sort described here would be a suitable basis for more detailed three-dimensional calculations of tunnel settlements. It is accepted, however, that these
preliminary calculations represent a simplification of the three-dimensional processes that occur in reality. In particular, it is expected that the amount of ground loss will have some influence on the shape of the settlement curve. The ground loss cannot be modelled accurately using a two-dimensional model.

3 FORMULATION OF A THREE-DIMENSIONAL FINITE ELEMENT MODEL

Work is currently in progress on the development of a three-dimensional finite element model of a shallow tunnel in undrained clay. The following procedures are proposed.

3.1 Element Types

In any undrained analysis the plastic volumetric strains are constrained to be zero. It is well known that care needs to be taken in the choice of finite element to solve problems of this type. In this project, three-dimensional finite element meshes will be based on the use of ten-noded isoparametric tetrahedral elements. These elements were shown to be efficient for this type of analysis by Bell et al. (1993); they have the further advantage that they may be used to model irregular boundaries, although this feature is not being exploited in this research. Conforming shell elements will be used to model the tunnel lining and the walls of a masonry building.

3.2 Mesh Generation and Post-Processing

The finite element meshes required for this project will be highly complex and will be generated by proprietary software. A set of initial meshes have been generated using I-DEAS, which is a commercial package designed primarily for solid modelling and computer-aided design. This package also has powerful mesh generation and visualisation features that are highly suitable for this project. Experience of the use of this software package for geotechnical problems is described by Grabinsky and Curran (1993).

It is clear that the finite element analysis will generate large amounts of data, and a suitable post-processing system is required to interpret and display these data graphically.

3.3 Modelling the Tunnelling Process

The project is concerned primarily with the prediction of displacements caused by shield tunnelling. Other important techniques such as NATM (New Austrian Tunnelling Method) will not be considered until a later stage in the research.

In shield tunnelling, a shield is advanced at the same time that the tunnel is excavated. The face of the excavation may be stabilised, if necessary, by one of a variety of methods, for example compressed air working, slurry shields or earth pressure balance machines. It has been suggested by Clough et al. (1983) that the main source of settlements induced by shield tunnelling is the ground loss at the tail void behind the shield. A component of ground loss will also occur at the face, but this is usually less important in terms of settlements provided that suitable tunnelling procedures are adopted.

The precise procedures that will be used to model tunnel installation have not yet been decided. It is expected, however, that a procedure will be used in which soil elements are removed from the mesh to model the advance of the tunnel, and shell elements activated to model the installation of the lining.

4 MASONRY MODELS

Masonry is a well established structural material; its engineering behaviour, however, is complex. Masonry consists of individual blocks connected by mortar joints and is intrinsically a non-homogeneous, and often an anisotropic, material. Masonry has a high compressive strength which makes it suitable for use in load-bearing elements in buildings. Its tensile strength is relatively small, and it is brittle, which means that cracks have a tendency to form in buildings subjected to differential settlement. Ground surface settlements associated with tunnelling are not generally large enough to cause complete structural failure of masonry buildings. However, settlements may lead to visible patterns of cracks that are unsightly and of concern to the owners and occupants of the building.

Two approaches have been adopted in the past for the numerical modelling of masonry structures. One approach is to model each brick and mortar joint separately. This procedure would lead to an excessive number of elements to model a complete building and so is not appropriate for this project. The alternative approach is to treat the masonry as a homogeneous material and to use a constitutive model that reflects the combined behaviour of the
masonry blocks and the joints. This second approach has the advantage that the finite element mesh used for the analysis may be independent of the actual arrangement of the masonry blocks. This second approach is adopted for the project described in this paper.

A considerable amount of literature exists on the subject of constitutive modelling of masonry. Loo and Yang (1991), for example, suggest that an appropriate plane stress failure surface for a masonry composite might take the form shown in Figure 4. This surface is plotted in principal stress space and consists of a von Mises type surface in the compressive region combined with suitable tensile cut-off behaviour. The parameters $f_t$ and $f_c$ are the tensile and compressive strengths respectively.

![Failure surface for masonry](image)

**Figure 4: Failure surface for masonry (after Loo and Yang (1991))**

It is thought that in order to predict settlement damage in masonry structures, it is only necessary to deal with the possibility of the failure of the material in tension. An elegant masonry model that deals with this type of failure is described by Di Pasquale (1992); this model is currently being developed and implemented in the finite element program.

In this model, the material is assumed to have zero tensile strength and is infinitely strong in compression. When the stresses are compressive the material is assumed to be elastic.

To illustrate this model, consider an element of material under plane stress conditions. The stress rate $\dot{\sigma}$ and the strain rate $\dot{\varepsilon}$ are related by the equation:

$$ \dot{\sigma} = D \dot{\varepsilon} $$

where:

$$ \dot{\sigma} = \begin{pmatrix} \dot{\sigma}_x \\ \dot{\sigma}_y \\ \dot{\tau}_{xy} \end{pmatrix} $$

and

$$ \dot{\varepsilon} = \begin{pmatrix} \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \\ \dot{\gamma}_{xy} \end{pmatrix} $$

(2)

When both principal stresses are compressive, the material is elastic and the material stiffness matrix, $D$, is given by:

$$ D = \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix} $$

(3)

If the strains in the material cause the minor principal compressive stress to fall to zero, then tension cracks will form as shown in Figure 5, where $\theta$ is the inclination of the direction of the major principal stress to the x-axis.

![Cracked elastic no-tension material](image)

**Figure 5: Cracked elastic no-tension material**

In this case, the stiffness of the material in a direction normal to the crack field is set to zero and the material stiffness matrix, $D^*$, written in a coordinate frame rotated anti-clockwise by angle $\theta$ is:

$$ D^* = \begin{pmatrix} E & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} $$

(4)

The material stiffness matrix referred to the $(x,y)$ coordinate system is:

$$ D = T^T D^* T $$

(5)

where $T$ is the transformation matrix.
\[ T = \begin{pmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2-s^2 \end{pmatrix} \] (6)

and \( s = \sin \theta \) and \( c = \cos \theta \). If the values of both principal stresses fall to zero then two orthogonal crack fields form and all of the terms in the material stiffness matrix fall to zero.

5 CONCLUSIONS

(a) Existing methods for the assessment of settlement damage to existing structures do not consider the interaction between the tunnel and a neighbouring building. This interaction is thought to be significant.

(b) In order to capture correctly the pattern of deformation that occurs around tunnels in clay, it is necessary to take great care in the choice of soil model. It is shown that a simple linear elastic model is quite unsuitable for this purpose. A kinematic yield hardening model, however, is shown to provide realistic results for a two-dimensional form of the problem.

(c) In order to capture fully the deformation mechanisms around tunnels is necessary to use a three-dimensional model. The use of proprietary software is a convenient way to generate the required finite element meshes.

(d) A numerical model for masonry should include the cracking behaviour of the material in tension. It is suggested that a simple elastic no-tension model may suitable for this purpose.

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REFERENCES


A three-dimensional finite element model of tunnelling

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ABSTRACT: A numerical model is under development at Oxford University to investigate the effects of tunnelling induced settlements on existing masonry buildings. This work applies a unified approach to this problem incorporating the effects of the building self-weight and stiffness on the ground surface settlement profile, using a numerical model that includes both the soil mass and the building. Previously published numerical models have generally not included incremental excavation and have therefore neglected the transient effects of the passage of a tunnel which are thought to have a significant influence on the consequent structural damage. A procedure for modelling incremental excavation using finite elements is discussed here in detail. A three-dimensional finite element analysis of an unlined tunnel under a greenfield site using a simple constitutive model for the soil is used to illustrate this numerical procedure.

1. INTRODUCTION

The prediction of the effects of tunnel construction induced settlements on existing buildings is usually based on a two-stage process. Firstly, the settlements due to the construction of the tunnel beneath a greenfield site are calculated for specified tunnel dimensions and expected ground loss for the soil conditions. The surface displacement profile assumed is usually that of a reverse Gaussian curve. These deflections are then imposed on a model of the building and the resulting strains are calculated. This analysis may be carried out conveniently using elastic deep beam theory (Burland & Wroth, 1975) or by more detailed numerical analysis. The degree of damage is then estimated on the basis of the magnitudes of the tensile strains induced in the building.

It is important to note that this process ignores any effect that the building itself may have on the settlement profile. This may be to deepen the response due to the building's self-weight or force a shallower and wider response as a result of the stiffening effect of the building. The application of a greenfield site settlement profile to a building may therefore lead to an inaccurate assessment of the resulting damage.

In addition, determination of the shape and size of the settlement curve requires an estimate of the expected ground loss. This is the volume of soil excavated above that required to house the tunnel and its permanent lining and is generally determined from experience of the tunnelling method and the ground conditions.

Research into the numerical analysis of settlement damage caused by tunnelling is underway at Oxford University; this work is prompted by various tunnelling schemes planned or under construction in London such as the Jubilee Line Extension and CrossRail. The numerical model includes both the soil and the building in an attempt to determine the effect of the building on the response of the ground and hence obtain a realistic assessment of structural damage. The study is limited to tunnels in clay (typically London Clay) and to masonry structures. The modelling of the incremental excavation of the tunnel and the construction of a lining are important aspects of the model as they allow the transient effects of tunnelling to be captured. The overall scope of the numerical model has already been described (Burd et al., 1994); this paper describes the development of the excavation modelling.

Two-dimensional finite element analysis is now used routinely for design and research problems in geomechanics but the use of three-dimensional
analysis is not yet established. Although the use of three-dimensional methods to study ground movements caused by tunnelling has the appeal that some of the modelling errors are reduced, it suffers from the important disadvantages that the solution times and the complexity of the analysis are substantially increased.

In some cases, the generation of a three-dimensional finite element mesh is relatively straightforward, for example a two-dimensional mesh of quadrilateral elements containing a tunnel could be extended in the third dimension to produce a three-dimensional mesh of hexahedral elements. In other cases, such as the unstructured tetrahedral meshes used in this study, more complex procedures are required. The results of three-dimensional analyses generally require graphical interpretation; this may be achieved relatively conveniently by using one of the commercially available graphics packages.

Investigations of the numerical modelling of this problem have shown the choice of the soil model to be of great importance (Chow, 1994). The use of a linear elastic model gives mesh-dependent results where heaves above the tunnel rather than settlements are predicted for large meshes. However, reasonable results can be obtained when linear elasticity is combined with increasing stiffness with depth; this model is adopted in the analysis described later in this paper. Lee & Rowe (1990) demonstrate that realistic predictions can be made using an anisotropic elastic soil model. It is not clear, however, to what extent the use of an anisotropic elastic model is generally appropriate for the soil types of interest in this study.

There is considerable interest in the accurate modelling of the non-linear behaviour of clays at small strains where experimental data show large reductions in shear stiffness for small increases in strain (Atkinson & Stallebrass, 1991). A suitable elasto-plastic soil model which includes this aspect of the behaviour of London Clay is described by Houltsby & Chow (1994). The model, which involves multiple surface kinematic yield hardening has been used in plane strain analyses of a shallow tunnel (Burd et al., 1994) and found to agree well with the standard Gauss curve predictions. It has been extended to cover three-dimensional problems and will be used in later analyses once lining installation has been incorporated into the model.

Parallel research has been undertaken to develop an elastic-no tension constitutive model for masonry modelling.

The buildings modelled will consist of two-dimensional facades tied to the three-dimensional model of the ground and tunnel using elements which constrain the nodes on the base of the building to follow the deflections of the same points on the ground surface.

2. MODELLING TUNNEL INSTALLATION

To model tunnel installation using finite elements it is important to simulate both the removal of elements and the installation of a support lining. When considering the loading due to excavation it should be noted that although an excavated face is stress-free this does not mean that the nodal loads on that surface should be zero. This would neglect components of body forces from elements still present in the mesh. The following procedure is derived from Brown & Booker (1985) and determines the correct nodal loads to ensure equilibrium at each stage of excavation. We consider an initial mesh volume \( V_0 \) of linear elastic soil which is excavated to \( V_i \) where the applied loading is limited to body forces, \( f_b \).

For the initial volume \( V_0 \), a set of displacements \( d_0 \) can be obtained from the standard finite element equation:

\[
Kd_0 - f_s = 0 \tag{1}
\]

where is \( K \) the structure stiffness matrix and \( f_s \) is the force vector. For the case of body forces only, the force vector is calculated as,

\[
f_b = \int_{V_i} N^T f_b \, dV \tag{2}
\]

where \( N \) is the matrix of element shape functions.

The stresses \( \alpha \) within the volume resulting from the effects of these body forces can be calculated from

\[
\alpha = DBd_0 \tag{3}
\]

where \( B \) and \( D \) are the strain-displacement and material stiffness matrices in standard finite element notation. Now consider the excavation to volume \( V_i \); the final displacement vector \( d_{net} \) of the nodes in the mesh volume is,

\[
d_{net} = d_0 + d_{exc} \tag{4}
\]
where \( d_{exc} \) is the vector of incremental displacements occurring during the excavation. The finite element method leads to the solution of,

\[
K_{i}(d_{e} + d_{exc}) = \int_{V_{i}} N^{T} f_{e} \, dV
\]

(5)

where \( K_{i} \) is the structure stiffness matrix for the new volume (i.e. neglecting the elements removed in the excavation) then,

\[
K_{i}d_{exc} = \int_{V_{i}} N^{T} f_{e} \, dV - K_{i}d_{e}
\]

(6)

\[
K_{i}d_{exc} = \int_{V_{i}} N^{T} f_{e} \, dV - \int_{V_{i}} B^{T}DBd_{e} \, dV
\]

(7)

\[
K_{i}d_{exc} = \int_{V_{i}} N^{T} f_{e} \, dV - \int_{V_{i}} B^{T}\sigma_{e} \, dV
\]

(8)

Therefore, to determine the correct displacements the applied loading is an external loading term representing the body forces on the new volume less an internal loading term associated with the stresses obtained for the unexcavated mesh. Adjacent to the excavation the incremental loads are non-zero while away from the excavation the internal and external load terms cancel. The analysis software used for this study, \( QIFEM \) which has been developed primarily for geotechnical problems treats the loading due to excavation similarly to any other applied load.

The use of the frontal solution technique to solve Equation 1 considerably simplifies the assembly of the stiffness matrices of the elements in the reduced volume. The analysis program maintains a list of all elements in the mesh, ordered to minimise the size of the frontal matrix. Elements in this list may be temporarily or permanently de-activated to represent their removal (excavation) from the mesh. Once the mesh is generated, the geometry data is searched for elements which lie within the swept volume of the user-specified tunnel. These are added to another list which controls de-activation during the analysis. A check is also made to detect any nodes which become disconnected from an active element. The forces and displacements associated with these nodes are automatically set to zero.

While lining installation remains to be implemented in the model it is clear that the activation of "dormant" shell elements can follow a similar procedure to that described for excavation. Procedures to model the soil movement into the gap between the lining and the excavated surface (the ground loss) are currently under investigation. One option is to use two-dimensional interface elements linking the soil and lining which are initialised with a gap corresponding to the specified ground loss. An alternative, avoiding the complications of interface elements, is to set the lining to be oversized and then to apply an internal hydrostatic pressure to reduce the diameter in accordance with the required value of ground loss. In both cases, the numerical model does not automatically determine the value of ground loss which must be estimated in the same manner as in a traditional analysis.

3. AN ELASTIC ANALYSIS

To demonstrate the modelling of tunnel excavation the results of an analysis using an elastic model for the soil and an unlined tunnel are now given. As stated above, a soil model incorporating increasing stiffness with depth has provided adequate results in two-dimensional analyses and is now adopted here.

![Figure 1: Finite element mesh used in the elastic analysis](image)

The soil parameters used in this analysis are those given by Gunn (1993) which are thought to be representative of London Clay. The variation of shear modulus, \( G \) with depth, \( z \) for the soil is,

\[
G(z) = G(0) + \omega z
\]

(9)

with \( G(0) = 0 \) and \( \omega = 3355.7 \) kPa/m. Undrained conditions are approximated by a Poisson's ratio of 0.49 and the soil pressure coefficient at rest, \( K_{0} \) is unity. The density of the soil is taken to be 20 kPa/m.
The mesh used in this analysis is generated using the commercial package I-DEAS and consists of 2407 ten-node, isoparametric tetrahedral elements with a total of 3831 nodes (Figure 1) to model one-half of a symmetrical tunnel excavation in a cube of sides 60 metres. The tunnel is 5 metres in diameter with its axis at 10 metres depth. A smooth curved tunnel wall is obtained by defining a separate tunnel volume during mesh generation. Tetrahedral elements are used because of their improved performance over hexahedral elements for analysis of incompressible materials (Bell et al., 1993).

The tunnel excavation is split into ten increments of equal length. The elements to be removed each increment are those whose centroids lie within the swept volume formed by the tunnel face and the axis incremental length. This means that the semi-circular face at the head of each stage of excavation following removal of elements is not necessarily smooth.

However, the same feature was found when the analysis was repeated using a full mesh and its cause remains unclear.

![Graph](https://via.placeholder.com/150)

**Figure 3a; Longitudinal settlements (along AC) for stages 2 and 5**

The longitudinal plots show good agreement to the expected cumulative normal distribution curve shape predicted by the traditional approach although again there are some irregularities which may be due to the coarseness of the mesh.

![Graph](https://via.placeholder.com/150)

**Figure 3b; Longitudinal settlements (along AC) for stages 6 and 9**

In addition, contours of the z displacements on the mesh surface are shown for stages 3, 6 and 9 (Figure 4). The contours are evenly spaced at intervals of 0.3 mm. The plots are obtained using the post-processing tools in I-DEAS and demonstrate one format for visualising three-dimensional analysis data. Solution time for this analysis which involves only one load step for each of the ten increments of excavation using a SUN Sparc 10 is three hours.
4. PROPOSALS FOR MODELLING A TUNNEL LINING

A tunnel lining provides rigidity to resist out-of-plane loading from the soil mass to preserve the tunnel shape and to prevent collapse. Therefore, shell finite elements, having both bending and membrane stiffness are required.

It is difficult to formulate robust curved shell elements since the curvature couples the bending and membrane effects (Astley, 1992). A simplification which involves the geometrical approximation of a curved surface by a collection of flat facets has been used to model curved shells successfully and is the approach proposed in this research.

The tetrahedral elements representing the soil mass have six-node triangular faces on the excavated surface of the tunnel which can be "lined" with four, three-node triangular shell elements. The bending resistance for the shell can be modelled using a well-developed plate element such as the discrete Kirchoff triangle (DKT) and the membrane action using constant strain triangles.

There is a problem with this approach in that it produces elements with three translational and two rotational degrees-of-freedom (d.o.f.) per node, one less than required for completeness in three dimensions. The absence of a local, in-plane rotational d.o.f. (the drilling d.o.f.) causes problems where elements are close to coplanar. In this case and where the plane of the elements is close to a coordinate plane the row of the structure stiffness matrix corresponding to the in-plane rotation at a node is filled with zeroes leading to an indeterminate solution. Where the element plane is tilted with respect to a coordinate plane the coefficients form singular equations since the three global rotations at a node have been determined by two independent local ones. To fix this, a fictional in-plane rotational stiffness can be applied to the coplanar elements. A number of authors have published formulations for constant strain triangle elements which incorporate this d.o.f. (e.g. Allman, 1984) although their goal has been to formulate two-dimensional elements which give improved results for in-plane bending. The adoption of such an element with a DKT plate element then removes this problem at the element stiffness matrix formation level.

Alternatively, the drilling degree of freedom problem may be avoided by using the formulation for overlapping plate elements given by Phaal & Calladine (1992). This element has only translational degrees of freedom as shown in Figure 5. The bending effects are modelled by internal springs.

Figure 4; Surface settlement contour plots for stages 3, 6 & 9
whose rotations are converted kinematically to the displacements of the nodes. The element uses complete quadratic polynomials for displacement interpolation leading to constant curvature and moment in an element. The disadvantage of this approach is the need for the elements to overlap although this appears to be outweighed by the absence of any rotational degrees of freedom. This aspect makes this plate formulation especially suitable for combination with the solid elements used in this model which have no rotational degrees of freedom themselves.

5. CONCLUSION

The modelling of excavation using finite elements requires some care in the assessment of the correct loads on the mesh due to excavation. Three-dimensional analyses allow the modelling of incremental excavation without ambiguity and provide a full set of results for the surface and subsurface displacements adjacent to the tunnel. However, the need for automatic mesh generation and post-processing of analysis data makes three-dimensional analyses more complex and time-consuming even before the large additional solution times required are taken into account.

A simple elastic soil model has been used to demonstrate the finite element analysis of an unlined tunnel excavation and shows reasonable agreement with the methods currently used to predict settlement trough shape.

Further enhancements to improve the constitutive model of the soil mass and the addition of a lining should improve the results although the adoption of the former will lead to considerable solution times for the three-dimensional analysis. The model will be extended to include a simple masonry structure modelled as a collection of two-dimensional facades.

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SOME EXPERIENCES OF MODELLING TUNNELLING IN SOFT GROUND USING THREE-DIMENSIONAL FINITE ELEMENTS

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ABSTRACT: A three-dimensional finite element model has been developed at Oxford University to study the effects of subsidence from soft ground tunnelling on adjacent surface structures. Simulation of excavation and the ground loss associated with tunnelling are incorporated in the model. Surface buildings are also included, as groups of interconnected two-dimensional façades composed of an elastic no tension material, to model masonry. This paper describes the development, implementation and performance of procedures to model the tunnelling processes. A description is also given of the methods used to generate the finite element meshes and to post-process the data.

1 INTRODUCTION

The construction of a tunnel in soft ground usually leads to subsidence of the ground surface. The size and shape of the settlement trough at a greenfield site may be estimated by a well-tested semi-empirical approach, described in many references (e.g. Mair et al., 1996). The presence of a surface structure usually changes the settlement profile, due to the interaction between the ground and the building. If differential settlements are significant they may damage the building. It is important, therefore, that reliable methods for settlement and damage prediction are available if urban tunnelling schemes are to be successfully promoted.

Numerical methods, principally the finite element method, have been applied for some years to this problem, although most studies have been two-dimensional and have not
included a realistic model of a building (e.g. Potts & Addenbrooke, 1997). It is thought that realistic modelling of this problem can be achieved only with three-dimensional models. In order to respond to this need, research has been underway at Oxford University since 1993 to develop a three-dimensional numerical model for the prediction of the effects of soft ground tunnelling on surface structures.

A paper presented at the last in this series of conferences described the proposed composition of a three-dimensional finite element model of tunnelling (Burd et al., 1994). The model has now been implemented and is described in detail in Augarde (1997) and Liu (1997). Some initial results using the model are given in Burd et al. (1998). This paper describes some of the challenges met during its development and subsequent use and discusses some of the issues that face developers of complex numerical models of this type. The paper is concerned particularly with the procedures adopted to model tunnel installation.

2 DESCRIPTION OF THE NUMERICAL MODEL

The numerical model uses finite elements to represent the ground, a tunnel and a surface structure. A typical problem is shown in Figure 1, where a straight circular tunnel of 5m diameter, with its axis at a depth of 10m, is constructed beneath four connected façades of a large masonry building of plan dimensions 10m by 20m. The building is unsymmetrical with respect to the tunnel centreline. The tunnel is assumed to be installed with a ground loss of 2%.

A non-linear, elasto-plastic material formulation is used to model an overconsolidated clay deposit with increasing stiffness and strength with depth. The building is constructed from masonry, modelled as an elastic no tension material. Tunnel linings are linear elastic. Tunnel installation is simulated in the model in discrete stages, each of which consists of the simultaneous removal of soil and activation of shell elements to simulate the liner. Four stages of tunnel installation are used in the analysis shown in Figure 1. Elements that are to be removed in the current stage are ignored when the structure stiffness is formed. Nodal loads are imposed on elements surrounding the excavated block to render the exposed faces free of surface tractions (Brown & Booker, 1985). The calculation of these loads is described in Augarde et al. (1995).

An important requirement of a finite element model of tunnelling is the ability to model ground loss. Ground loss arises in practice from two sources; radial movement of the soil around the tunnel liner (tail loss) and inward movement of the soil at the tunnel heading (face loss). The amount of ground loss that occurs in practice is determined mainly by the installation method and the quality of the construction procedures. In this model, the excavated face of the tunnel is unsupported, thus permitting movement to represent face loss. Tail loss is simulated by uniform shrinkage of the lining elements in a plane normal to the tunnelling centreline. This occurs at the same time as excavation and lining activation.

While large masonry buildings are three-dimensional structures, their most significant feature, with respect to settlement damage, is the in-plane response of the main façades. The façades are represented by meshes of six-noded triangular plane stress elements, each
façade having its own local co-ordinate system. The façade meshes are joined to each other, and to the surface of the ground mesh, using a novel system of tie elements that implement the kinematic constraints linking each local two-dimensional co-ordinate system with the three-dimensional global system (Liu, 1997).

The various components of the model described above have been implemented in OXFEM, which is a finite element program written in Fortran 90 and developed at Oxford University. Non-linearity is accommodated in an incremental scheme, the Modified Euler method. Loads are applied in steps, followed by an equilibrium check to determine the out-of-balance force at each nodal degree-of-freedom. This out-of-balance force is then corrected in the next step. Incremental techniques such as this require the user to specify the numbers of steps, a decision that requires some experience of the non-linearities involved.
3 NUMERICAL MODEL OF TUNNEL INSTALLATION

A numerical model of tunnelling requires consideration of the structural behaviour of the liner, and the ground loss that invariably occurs in practice during tunnel installation. A variety of modelling procedures have been developed for 2D analysis (e.g. Rowe, 1983; Potts & Addenbrooke, 1997); no established procedures are, however, available for 3D analysis. In the numerical model described in this paper, the liner is modelled using shell elements, and the effect of one component of ground loss (tail loss) is included by numerical shrinking of the liner elements. These procedures are discussed below.

3.1 The choice of lining element formulation

The choices of element formulations for the ground and building are relatively straightforward. The choice of a suitable shell element for the tunnel lining is complicated by the fact that a large number of different approaches are proposed in the literature. Little work appears to have been published, however, on the behaviour of shell elements when connected to continuum elements. It is possible, for example, that the use of these incompatible elements may lead to numerical problems.

In determining a suitable element a choice has to be made from the two main classes of shell element:

- true curved elements based on classical shell theory or derived from the degeneration of a solid continuum element,
- faceted elements, where bending stiffness is attributed to a plate element and membrane stiffness to a plane stress, continuum element.

Many formulations in the first category have been proposed. Yang et al. (1990), for example, reviewed over 280 publications relating mainly to curved shell elements. Degenerated solid shell elements appear to be more commonly used than those based directly on shell theory, and are available in various commercial packages. Faceted formulations are usually less complicated both to code and to operate, since membrane and bending effects are uncoupled. This has the advantage that separate formulations for bending and membrane actions can be adopted (Hughes et al., 1995).

The element used in this project was a faceted shell with displacement, but no rotational, degrees of freedom. The formulation was developed by Phaal & Calladine (1992) and is novel in that bending stiffness is provided by a six-noded plate element having four flat triangular facets while membrane stiffness is provided by three-noded triangular elements which overlay each facet. Thin plate elements must fulfil the Poisson-Kirchhoff requirement (i.e. continuity of displacement gradient) and this is met in conventional plate formulations by the inclusion of rotational degrees of freedom at nodes, in addition to the translational degrees of freedom. In Phaal & Calladine’s shell element, the facets of adjacent plate component elements overlap to achieve this continuity without the need for rotational degrees of freedom (Figure 2). This unconventional feature leads to many difficulties in the inclusion of this element in a standard finite element program.
Fig. 2: Overlapping shell elements

This element was chosen because it appeared to offer reasonable behaviour and accuracy without the need for rotational degrees of freedom at the element nodes. The lack of rotational degrees of freedom was thought to have three advantages:

- The number of global degrees of freedom in the mesh is less than that required for conventional shell formulations,
- all nodes in the model all have the same number of degrees of freedom. This makes the element more convenient to implement.
- There is no need to consider the in-plane rotational degree of freedom (the drilling degree-of-freedom).

The Phaal & Calladine (1992) formulation is, at first sight, highly complex. The formulation is presented in a general form, and further work is needed to cast it in a form that may be implemented in a finite element program. The original formulation was described in the context of the analysis of linear shell problems. The model described in this paper is non-linear, however, and this required some minor further development of the shell element formulation.

The linear elastic material model for the tunnel lining requires three properties to be specified: Young’s modulus, $E$, Poisson’s ratio, $\nu$, and thickness, $t$. These parameters are used to derive the flexural stiffness of the plate component of the shell, $D$. Where
\[ D = \frac{E t^3}{12(1-v^2)} \]

and the membrane stiffness component as \( E t \) (per unit length of lining). The elements performed reasonably well when used to model tunnel linings. There are, however, some unresolved difficulties using the elements in combination with the non-linear elasto-plastic material model for the ground. It is occasionally found that the analysis fails to converge with the out-of-balance force at the equilibrium check (see Section 2) increasing with each step. The problem may be avoided by increasing the flexural stiffness, \( D \), but this leads to a tunnel liner that is substantially stiffer than would be expected in practice. While the need to impose unrealistic properties on the liner is unfortunate, since it is difficult then to interpret the stress resultants, it does not unduly affect the surface response of the model, which is of primary interest in this study.

3.2 Modelling ground loss in 3D

Procedures for modelling ground loss in 2D finite element analysis are well established. Potts & Addenbrooke (1997), for example, use a technique in which the unloading caused by soil removal is carried out in a series of small increments. At the end of each increment the amount of ground loss is recorded; when the ground loss reaches the specified value then the calculation is terminated. A more complex procedure is described by Rowe (1983). In this procedure separate meshes of beam elements to model the lining and continuum elements to model the soil are used. The soil mesh contains a zone of elements that are removed to simulate excavation and the lining elements are activated to simulate the installation of the lining. The mesh of lining elements is separated from the surrounding ground by a specified gap. After the soil has been excavated and the liner elements activated, the ground closes onto the lining mesh as the soil around the tunnel unloads. Numerical procedures are used to model the contact between the soil and liner after the gap between the two meshes closes.

The above methods would be complex to implement in a 3D analysis; this is particularly the case for the gap parameter approach of Rowe (1983). A third approach was therefore selected in which the elements within the tunnel are removed (to model excavation) and, simultaneously, lining elements are activated to model the liner. At the end of this procedure the lining elements are subjected to a uniform hoop shrinkage to develop the required amount of ground loss. The hoop shrinkage is achieved by the application of a suitable set of radial forces within the tunnel liner. The process leads to fictitious stresses within the liner, but the liner is elastic and so this does not affect the way in which the ground and the liner interact. An important feature of this finite element model is that the tunnel is installed in separate stages. During any stage of tunnel installation (except the first one) it is important to ensure that the applied numerical shrinkage does not cause additional shrinkage in the liner installed during the previous stage. This was achieved by constraining to zero the shrinkage deformation applied at the tunnel heading (except for the final
tunnelling stage). The nodal forces associated with these constraints were then removed during the shrinkage phase of the next tunnelling stage. This was a somewhat artificial, but necessary, device to achieve satisfactory modelling of volume loss for incremental tunnel construction in a 3D model.

Figure 3 shows the lining elements at each stage of the analysis associated with the mesh in Figure 1. The shrinkage at each stage, which is displayed to an exaggerated scale, follows that specified by the user.

4 PERFORMING ANALYSES WITH THE MODEL

4.1 Preparation of input data and interpretation of output

The meshes used with the numerical model are unstructured, to permit localised refinement around the tunnel and below the building. This feature precludes hand generation, and an automated method must be used. Unstructured mesh generation has been the subject of considerable research in other areas, notably computational fluid dynamics, and guidance is available from the literature in this area. It is recommended, however, that a commercial package is used, since the effort involved in writing a robust generator for three-dimensional unstructured meshes is considerable. The package used in this research is I-DEAS, primarily a solid modelling package but with excellent mesh generation tools.

It is vital that the output from the analysis is checked carefully. This is a complex task, however, because of the large amount of computed data. Apart from checking that the
rodal forces are in equilibrium, which is an integral part of the solution phase of the analysis (Section 2), it is important to check that the tunnel installation procedures produce the specified amount of ground loss. This finite element model is based on the assumption of undrained soil behaviour, and therefore that the soil is incompressible. Although the soil model adopted in the analysis ensures incompressible plastic deformation, it is not possible to specify perfect incompressibility in the elastic region. (In these calculations a Poisson’s ratio of 0.49 was adopted). It is possible that this loss of incompressibility may lead to the volume of the surface settlement trough being less than the total volume of ground loss generated by tunnel installation. To check that volumetric deformation of the soil did not reduce the volume of the surface settlement trough by an appreciable amount, careful comparisons were made between these two volumes. The ground loss achieved in the tunnel is computed from the tunnel liner displacements. The procedure is based on the computation of the volume of a mesh of tetrahedra within the tunnel formed by 3D triangulation. The total volume of the settlement trough at the ground surface was computed by numerical integration of the surface settlements. It was found that the ground loss could be controlled to within 0.1%, and that the volume of the surface trough correlated very closely with the ground loss.

The final stage of an analysis is the post-processing of output data into a useful format. The large volume of data precludes any hand manipulation and the options are either in-house or commercially available software. In this research, use is made of both approaches with I-DEAS and an in-house program, 2CAN, used for generation of contours of surface settlements. 2CAN is also used to process and display the cracking of building façades. This is a feature not available in I-DEAS, and was one of the specific post-processing tools to be written for this project. Figure 4 shows the predicted crack pattern for each tunnel stage for the front façade of the building in the analysis of Figure 1. The intensity and inclination of cracks in the masonry model can be obtained from the OXFEM output. These data are displayed as collections of lines, at the given inclination, centred on the element integration point. The number of lines indicates the magnitude of the crack strain. (Crack strain is closely associated with principal strain normal to the crack and provides a convenient measure of the intensity of cracking, Liu, [1997]). This display gives an immediate indication of the damaged areas of the building façades.

Figure 4 indicates that the location of the most severe damage to the front façade changes as the tunnel approaches and passes beneath the building. At stage 1, the tunnel heading is closest to the large door opening in the façade, and transverse settlements cause cracking adjacent to and above this region. There is little effect in the façade directly ahead of the tunnel at this stage. The greatest damage is predicted at the end of stage 2, when the tunnel heading is situated between the front and side façades. At this point, the building is subjected to the combined effects of differential settlements along the tunnel axis and in a transverse direction. Much of the cracking seen at stage 1 has closed up by stage 2 due to the changing settlement profile along the base of the façade. Stages 3 and 4 are similar since the tunnel heading has progressed beyond the building. The differential settlements along the tunnel axis have reduced, leaving slight damage from transverse differential settlements.
Fig. 4: Predicted crack patterns in the front façade of the building analysed using the model shown in Figure 1

5 ANALYSIS COSTS

Typical hardware used to run analyses with the model described in this paper is currently a Sun Microsystems UltraSparc 2 machine with processors running at 200MHz and 256Mb of RAM. Runs of the analysis shown in Figure 1 with this hardware are completed in approximately five days. During solution, virtual memory use rises to approximately 250Mb, which can be accommodated entirely within the RAM, provided other processes are small or prevented from running via a queue system. While this level of hardware configuration is of a high specification, it is not excessively expensive, considering the value of the information it is able to provide. Equally, while runtimes are long they are not excessively so, considering the complexity of the model.

6 FUTURE DEVELOPMENTS

Further work is underway to increase the scope of the model and to improve its efficiency. In particular, the modelling of compensation grouting, using interface finite elements, is under development. It is also intended to develop an effective stress model for soil in order to study the effect of consolidation settlements. The behaviour of the shell elements in the model has prompted an investigation of other methods of modelling volume loss. It appears possible to use thin continuum elements with high stiffness for the lining, which should remove any difficulties associated with element non-compatibility.
A parallel project began in 1996 to compare predictions from the model with field data from tunnelling schemes in London. This, and further studies, should serve to validate the model and its components and indicate areas for improvement. Work has recently begun on porting the code to run on OSCAR, the Oxford Supercomputer where significant reductions in solution times are anticipated.

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Analysis of tunnel-induced settlement damage to surface structures
Analyse des dégâts créés par tunnels sur les constructions en surface

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Keywords: analysis, finite elements, masonry structures, settlement, tunnelling

ABSTRACT: Transport developments in cities often involve tunnelling, which inevitably leads to ground movements. These must be carefully predicted if there is a risk of settlement damage to nearby structures. Tunnel-induced settlements may be predicted empirically for greenfield sites, but surface structures modify these movements. Two-dimensional models, often used in practice, neglect the effect of transients as the tunnel is excavated, and do not allow realistic models of buildings. Research on a 3-D numerical model of tunnelling is described. This includes a building and a simulation of tunnel construction processes. Interactions between the building and the ground are investigated. Settlement, and structural damage, is studied as the tunnel installation proceeds. An analysis of a building unsymmetrical in plan to the tunnelling direction is presented.


1 SUMMARY OF THE RESEARCH PROGRAMME

The purpose of the research described here is to develop a comprehensive numerical model, able to provide realistic modelling of the interaction between tunnelling processes and buildings. The long-term intention is that such a model should become a useful predictive tool for design. Central to the analysis is the recognition that the tunnel, the ground and any adjacent buildings are inextricably linked, and that the analysis must take into account their mutual interaction. This contrast with many current techniques (some even involving quite sophisticated numerical methods) in which an analysis of the ground deformation is made without accounting for the influence of buildings. It is further recognised that, in spite of the attractions of simplified 2-dimensional analysis, it is only possible to model the interaction if the 3-dimensional nature of the problem is accounted for.

The research programme has been divided into three main phases as detailed below.

1.1 Preliminary studies

These included:
- 2-D finite element analysis examining the effect of different material models on the shape of greenfield settlement troughs (Chow, 1994). This showed that the most realistic (i.e. narrowest) settlement troughs were predicted when the nonlinearity of soils at small strains was modelled.
- Modelling of masonry structures using a commercially available package (ABAQUS). Simple displacement profiles were applied to building façades, using a variety of different material
models (Parry-Jones and Cline, 1993). Some information on likely patterns of damage for buildings was obtained, and further research was needed on the modelling of the masonry.

- Preliminary 3-D studies investigating methods of coupling the ground and building (Hurst, 1994, Curtis, 1995). These proved the value of commercially available mesh generation software (I-DEAS) for this project.

1.2 The main research phase

This involved the development of a finite element model with the following features:

- A 3-D block of ground, modelled as undrained clay, using a specially developed soil model accounting for the nonlinearity of soil at small strains using a nested-surface plasticity approach. Tunnel construction is modelled by progressive removal of elements (Augarde, 1997).
- A model of a masonry building in which the structure is represented by a series of interconnected façades constructed from 2-D plane stress elements. The masonry behaves as elastic in compression, but as unable to sustain tension. Specially developed tie elements are used to connect the 2-D façades together, and to connect them to the ground (Liu, 1997).
- Modelling of the installation of a tunnel lining. The liner is modelled using special shell elements which use the overlapping facet technique to avoid the need for rotational degrees-of-freedom in the analysis (Phaal & Calladine, 1992, Augarde, 1997). The ground loss associated with imperfect installation of the liner is modelled by a method in which the liner diameter is shrunk by a controlled amount after installation.

The main conclusions from this phase of the research (some of which are illustrated below, and others reported by Bud et al., 1998) are:

- A building has an important effect on the settlements caused by nearby tunnelling operations.
- Façades subjected to sagging displacements are resistant to crack damage because of the restraint provided by the ground; they retain much of their bending stiffness and therefore suffer differential settlements that are less than would be estimated from greenfield analysis.
- Façades subjected to a hogging mode of displacements are highly susceptible to crack damage, with consequential loss of bending stiffness.

1.3 Development phase

The research is currently continuing, with developments being in four main areas:

- Confidence in any numerical analysis technique can only be achieved by careful comparison with case histories. Work is in progress comparing analyses with the results of monitoring exercises at a number of sites, principally in London, where tunnels and shafts have been excavated close to major masonry buildings. Analysis of particular sites inevitably leads to more complex models than those used in the earlier main research phase.
- The complex 3-D analyses using non-linear soil models are extremely demanding on computing resources (both in terms of memory and processor time), since finely detailed meshes and many analysis steps are necessary to capture the details of the problem. Some success has been reported in reducing computation time using iterative solution techniques, so these methods are being explored. Most computations are now being made on the Oxford Supercomputing Centre’s 84-processor Origin 2000 machine, so that parallel computing techniques can be exploited. Equally important is the machine’s exceptionally large RAM memory (21Gbytes), which makes possible large analyses which could not be attempted on a conventional machine.
- Further attention is being given to the details of the tunnel lining, and of the modelling of the processes of volume loss.
- In many cases where it appears that predicted settlement damage might be unacceptable, compensation grouting is becoming the favoured technique for alleviating the problem. It is necessary therefore to model compensation grouting in the analysis, so that alternative grouting schemes can be examined and their efficacy assessed. Modelling of grouting is in progress.
- The present analysis examines short-term (undrained) movements only, since these are viewed as being of primary importance. This will be extended to include time-dependent response, which will be of particular interest in the cases where compensation grouting is employed.
2 AN EXAMPLE CALCULATION

The example used here is drawn from the analyses in the main phase of the research, and is reported in detail by Liu (1997). It concerns a single tunnel of diameter 5m, excavated at a depth (to centreline) of 10m. The properties of the soil through which the tunnel is excavated are chosen to be typical of London Clay. Analyses of unlined tunnels are reported here, but in other analyses a liner is included (Augarde, 1997). The tunnel passes obliquely under the corner of a masonry building, 20m by 10m in plan, see Figure 1. The building is modelled as four planes of six-noded triangular elements to form the façades. The longer sides include a regular pattern of openings for windows and doors, whilst the shorter sides are plain gables. No internal structure of the building is modelled, since in a masonry building the internal structure is usually considerably lighter and more flexible than the main façades.

Two analyses are briefly reported here. In the first the tunnel is constructed as if it were under a greenfield site. The predicted settlements around the perimeter of the building are then applied to a separate model of the structure. In the second analysis the building is fully coupled to the foundation, so that there is an interaction between the stiffness of the building and the ground. Clearly the second analysis is expected to be the more realistic, and the comparison between the two analyses is made to highlight the importance of carrying out the coupled analysis.
The settlement profiles along the front façade of the building (i.e. the one facing the advancing tunnel) are given in Figure 2(a) for the greenfield analysis for four stages in the advance of the tunnel. The maximum settlement is at first near the left-hand end of the façade, since at first this is closest to the advancing tunnel, but shifts to the right as tunnelling progresses, since the centreline of the tunnel passes under the right hand end of the building. The corresponding results for the coupled analysis are shown in Figure 2(b). Note that the stiff façade maintains an almost straight base. At first it tilts slightly to the left, then more strongly to the right. The absolute value of the settlements is larger than for the greenfield. This is because the weight of the building triggers larger local downward movement.

Figure 3 shows schematically the predicted pattern of cracking damage to the front façade at the end of Stage 1 (Figure 3(a)) and the end of Stage 4 (Figure 3(b)). On these figures a line is drawn parallel to the crack direction for each integration point where cracking exceeds 500με, and two lines if cracking exceeds 1000με and so on. A cracking intensity of 1000με would represent one crack of 1.0mm every metre. Although an averaged indication of cracking intensity can be obtained, the analysis does not predict the precise size and location of individual cracks. It can be seen that early in the analysis some minor cracking at the left hand end of the façade is predicted, and later these cracks re-close and more major cracking occurs at the right hand end. Bearing in mind the fact that the cracks are parallel to the direction of major principal stress, it can be seen that there is arching action in the masonry over the settlement trough at the right hand side of the façade.

In contrast, Figure 4(a) shows the predicted crack damage for the front façade at the end of the coupled analysis. The cracking damage is much more localized, because the distortion of the masonry structure is much less than that implied by the greenfield analysis. The contrast between Figures 3(b) and 4(a) demonstrates clearly the need for the coupled analysis.

The cracking pattern predicted from the coupled analysis for the rear façade is shown in Figure 4(b). This part of the masonry structure is subjected to hogging deformation, which causes the top
of the structure to crack vertically, and results in almost total loss of bending stiffness of the rear façade. As a result it has much less influence on the local ground deformations than the front façade, so in this case the crack pattern is in fact rather similar to that predicted in the greenfield analysis. It is interesting to note that, in this case, the rear façade is subjected to more severe cracking than the front façade. This reflects a consistent feature of the numerical model that façades subjected to hogging deformations, such as the rear façade, are more prone to damage than façades subjected to a sagging mode, such as the front. This demonstrates that it is not sufficient to model the masonry as elastic: masonry structures show complex non-linear responses which depend on the nature of the loading.

Figure 5(a) and (b) show the horizontal movements of the ground surface predicted at the end of Stage 2 for the greenfield and coupled analyses respectively. Horizontal movements often do not receive as much attention as settlements, but they can be equally damaging to buildings. In the greenfield analysis it can be seen that the displacement vectors (much exaggerated) are all directed towards the tunnel centreline and are of course undisturbed by the building, the outline of which is shown on the figure. Close attention to Figure 5(b) shows that the pattern of horizontal movement around the building is significantly different, particularly near the front corners A and B. The stiffness of the building changes the local pattern of deformations.

Clearly the details of horizontal movements (as well as settlements) would depend on precise nature of the foundation of the building. In the analysis presented here the foundation was
effectively modelled as a strip surface footing, with no detail included. More advanced modelling of the foundation is clearly desirable.

3 CONCLUSIONS

Advanced numerical techniques are capable of modelling complex problems of soil-structure interaction involving the influence of tunnelling operations on masonry buildings. The model described here does not model the fine detail of the building or its foundations, and further work is needed to develop it as a practical design tool. It does, however, suggest some general mechanisms of interaction between the building and the ground. These are summarized as follows:

- The presence of the building modifies the pattern of ground movements both qualitatively and quantitatively
- The nature of any cracking damage changes as the tunnel progresses. Crack systems may open and close as the tunnel passes beneath the building.
- Façades subjected to hogging deformation are more prone to cracking damage than those subjected to sagging deformation.

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Three Dimensional Analysis of Building Settlement Caused by Shaft Construction

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ABSTRACT: An analysis of a case history of settlement damage to a masonry building, due to the construction of a nearby shaft, is presented. Three-dimensional finite element analysis is used, in which the non-linear behaviour of both the soil (a heavily overconsolidated clay) and the building is taken into account. Procedures are used to capture the construction process as realistically as possible. Comparisons are made between a variety of analyses (including separate analyses of the building and the soil, and a fully coupled analysis of the whole system), as well as with observations at the site. The principal comparisons are in terms of observed surface settlements and of the categories of cracking damage suffered by the masonry building. It is concluded that the analysis technique can model adequately the interaction between the building and the ground, but that details of settlement and damage patterns depend critically on assumptions about the structure and stiffness of the building and its foundation.

1 INTRODUCTION

Increasing numbers of tunnels are being constructed in urban areas for transportation and utility purposes. It is inevitable that structures of historic and economic importance will be affected. Precautions such as compensation grouting have been applied to reduce the impact on nearby structures, but are expensive and may not always be effective. Current semi-empirical methods of structural damage prediction are often conservative because they neglect the effect of ground/structure interaction in reducing the severity of settlements. There is a need for accurate prediction of the effects of tunnelling which fully considers the interaction.

Any numerical model addressing this issue must take into account a number of aspects. The behaviour of the soil, especially at small strains, is fundamental. Key aspects of the tunnelling process (advance of the heading, volume loss and the lining process), the ground/structure interaction and the behaviour of the structure itself should all be represented. The analysis must be in three dimensions to model the geometrical complexity of a real site and the progress of an advancing tunnel heading. Ground/structure interaction may be modelled by coupling the structure to the ground from the start of the analysis, rather than applying calculated ground displacements in a separate analysis of the structure. Important structures in urban areas are often constructed from masonry, a highly non-linear material, and the stiffness and self-weight of the building should be modelled realistically. Including all these components in the modelling procedure leads to large and complex analyses that will, however, be justified by the risk of damage due to tunnelling in certain key cases.

Research has been in progress at Oxford University since 1993 on the development of a 3-D finite element modelling procedure, including all of the above features. The detailed description of the approach has been the subject of past publications. A kinematic yield hardening constitutive model is used to describe over-consolidated clay (Chow, 1994, Houlby, 1999). A macroscopic elastic no-tension model is used for the masonry (Liu, 1997). An incremental excavation technique in which soil elements are removed from the mesh, tunnel liner elements are installed and a shrinkage technique is used to model volume loss is described by Augarde (1997). Tie elements are used to couple degrees of freedom between two-dimensional building facades and the soil (Liu, 1997). General conclusions from the project are presented by Augarde et al. (1998) and Houlby et al. (1999).

As part of this project, comparisons are being made between this procedure and case histories involving detailed monitoring of ground and
structure movements. The aim is to confirm the practicability and accuracy of the approach.

A case history is presented and analysed here of an excavation in London. It concerns the construction of a shaft and tunnel adjacent to a historic masonry church.

2 SITE DESCRIPTION

Howard Humphreys Consulting Engineers designed and supervised the construction of a tunnel in 1991/92 for a new electricity supply into the West End of London. The project required construction of a 2.5m tunnel, 12m to 25m below ground. Shafts were required along the route for access during construction. One such shaft was in Maddox Street, which is bounded on the North side by a terrace of four storey brick Georgian houses, and on the South side along its entire length by an 18th century stonelaid brick church. The 4m diameter 15m deep shaft was sunk 5m from the North East corner of the church (Fig. 1). No protective measures such as compensation grouting were used, but extensive monitoring of ground and structure movements and observations of damage were carried out.

Since the church was the most important building, it was modelled in the analysis. As the terrace on the North side and the church are on opposite sides of the settlement trough, it seemed reasonable to assume that there is little interaction between the two. The largest movements were predicted during shaft, rather than tunnel, construction, and so that was chosen for analysis.

A number of assumptions were required in the representation of the church. The first was whether it was valid to model the church as a rectangular box of four facades. The church structure consists mainly of masonry walls. There are columns within the church but these support only the roof and balcony, which both being of wooden construction have little stiffness or self weight compared to the masonry walls themselves, which are 1m thick.

There are windows at regular intervals around the church. All these openings could be modelled, but that would lead to rather complex meshes. The main reason for modelling discrete openings is to model crack growth from the corners. In regions of small movement this may be unnecessary, but it is still useful to capture the reduction of stiffness and self-weight due to the opening as it may have implications for the global structure response.

This reduction may be modelled in two ways. In a fully “smearred” approach, an average stiffness for the whole facade is used. A “semi-smearred” approach recognises that openings aligned vertically may affect the global behaviour by acting as vertical columns of reduced stiffness. This latter approach is similar to that of Simpson (1994) except that his “strata” of reduced stiffness represented rows of openings horizontally rather than columns. This may not model so well the important formation of cracking from the top of the facade, propagating downwards in a hogging region.

In this project, a combination of discrete openings, “smearred” and “semi-smearred” was used. The openings closest to the shaft were modelled in detail, the next as columns of reduced stiffness. The two more distant facades were modelled using a fully smeared stiffness. The masonry was modelled as an elastic, no tension, material with a Young’s modulus of 20kN/mm².

A borehole at the centre of the shaft showed 1.5m of pavement and made ground, followed by 3.8m of terrace gravels and then London clay to depth, a typical profile for that area of London. Since a foundation depth 3m below ground level was assumed, the response would be dominated by the clay, and so the soil was modelled as undrained London clay throughout. The undrained strength was taken as 120kPa at 3m depth, increasing at 6kPa/m, and the rigidity index was taken as 500.

3 INTERPRETATION OF THE ANALYSIS

Results from four analyses are presented and discussed, as follows:
1. A 3-D non-linear analysis of the shaft construction and the surrounding soil, i.e. a “greenfield” calculation without the building.
2. Analysis of the building alone, prescribing the displacements calculated from the above “greenfield” analysis.
3. Analysis of the building alone, prescribing the observed field displacements.
4. A full 3-D analysis including shaft, soil and building. A view of part of the finite element mesh for this analysis is given in Figure 2.

The results of the analyses are compared with the observed data. This comparison leads to a reassessment of the methods, and a fifth analysis is then introduced.

The important output from Analysis 1 is the “greenfield” soil surface settlement profile due to the construction of the shaft, particularly around the building footprint. It is a disadvantage that at this site the full settlement contours from the field are not available, due to the number of buildings in the vicinity of the shaft. The buildings other than the church were not modelled in the analysis.

Analysis 4 is of most interest. The soil settlements, especially in the vicinity of the building, would be expected to be different from Analysis 1. In addition, damage to the building is predicted, and comparison of this damage with site observations is a fundamental aim of this project.

The additional analyses are made to assist in gauging whether or not it is beneficial to undertake the full analysis (Analysis 4), or whether a less sophisticated analysis can be shown to give as good results. For example Analyses 1 and 2 model the shaft and soil first, and apply the resulting displacements to the building. Interestingly this approach gives little overall benefit in computing time, but it does have the advantage that, by dividing the problem into two parts, the memory requirements are smaller.

Analysis 3 was carried out for the purposes of verification of the modelling technique for the building. The applicability of the no-tension material model was assessed. Assumptions about the building weight and stiffness, and the way that openings are treated were tested. The application of the site-observed displacements to the building model did indeed give a similar damage distribution to that observed on site (which in this case involved no significant damage, apart from some possible cracking in the basement walls superimposed on pre-existent cracking). This supports the modelling approach for the building, and provides justification for the full model of soil and building.

The settlements in the vicinity of the building for the cases with and without the building present are shown in Figures 3(a) and 3(b). It can be seen that, as expected, the presence of the building modifies the shape of the settlement bowl, with the weight of the building appearing to drive settlements further. For example, the settlement at the North East corner of the building nearest the shaft increases from 3mm to 9mm with the building present.

The surface settlements are seen to vary in a somewhat random fashion close to the shaft, with a band of about 1m width around the shaft where heave of up to 4mm occurs. It is considered that these fluctuations are mainly due to numerical problems occurring when applying the ground loss to the shaft, and that they do not extend out far enough to affect the building significantly.

More detail on the settlement profile affecting the building may be obtained by plotting settlement with distance along the facades, and this is presented in Figures 4(a) and 4(b), for the East and North facades, which are those closest to the shaft. The South and West facades did not settle significantly, or experience any measurable damage in any of the analyses or in the field.

The profiles for the observed site data indicate that the building tilted almost as a rigid body towards the North, with a fairly constant settlement of 6–8mm along the whole of the North facade. Some of this settlement may have resulted from the construction of the tunnel, which was not modelled in the analysis, but may estimated as up to 3mm of the settlement according to current design approaches.

The results from the full analysis (Analysis 4) show the bases of both facades following half-troughs similar to a Gaussian shape, but with a somewhat pronounced point of maximum hogging. The maximum settlement of 9mm calculated at the North-East corner is just 2mm greater than that observed. The calculated “greenfield” settlements without the building also follow Gaussian-type half-
troughs, a bit smoother in general but with a reduced maximum settlement of only 3mm at the corner, and with signs of localised irregular behaviour (probably due to mesh discretisation) near the corner as noted on the surface settlement plots.

Both analyses used a value of ground loss due to the excavation of the shaft of 2%, which corresponds to the physical dimension of the overbreak due to the cutting edge on the caisson used to sink the shaft. There may have been other effects, for example heave of the soil into the base of the shaft, which could increase the ground loss above 2%. As already mentioned, it was not possible to measure the settlements over the entire area affected by the shaft and hence calculate the real ground loss. A figure of 2%, however, is not unreasonable.

Plots of damage category for the North and East facades of the building for Analyses 2 and 4 are presented in Figures 5 (for Analysis 4) and Figure 6 (for Analysis 2). The damage category is based on the value of cracking strain calculated in the no-tension masonry material model, in accordance with Boscardin & Cording (1989), using the classification of visible damage in masonry first presented by Burland, Broms & De Mello (1977). The facade has been divided into regions and the damage category based on the average cracking strain over all the Gauss points within the region. This may result in some loss of information on some localised cracking behaviour, but makes it easier in the first instance to compare the results of different analyses quantitatively (Liu, 1997). Some judgement is required on the division of the facades into regions. These were chosen to highlight regions adjacent to openings and areas of different stiffness. Reduced region sizes were used where more detail was required, *e.g.* in the North East corner of the building.

The distribution of the damage, in particular the position of the “slight” damage, is quite distinct for each run, and these will be discussed in turn. “Slight” damage implies typical crack widths of 1mm to 5mm. No specific new cracking of this magnitude was observed on site. The structure was, however, known to be cracked, due possibly to earlier nearby tunnel construction. This cracking particularly affected the East facade and the basement level. Thus the predictions cannot be
regarded as definitely proved accurate, but are entirely consistent with the observations in suggesting “slight” damage. They certainly do not under-predict the real damage.

Figure 5 shows the results for the full non-linear analysis. This predicts the largest amount of damage on the East facade, with “slight” damage around the larger opening, and further damage associated with the North East corner. The North facade experienced damage concentrated in one column of openings. It can be seen by comparison with Figures 4(a) and 4(b) that for both facades these areas of damage near openings occur in the hogging part of the settlement profile, which was quite pronounced for both facades. It appears that some interaction occurs. The hogging region of the settlement bowl lines up with an opening or openings; this causes damage of that part of the facade, thus reducing its stiffness, thereby magnifying the hogging profile, producing a slight “hinge” and saving the remainder of the facade from damage. The position of such hinges would be hard to predict without a 3-D analysis.

The results for Analysis 2 (Figures 6(a) and 6(b)) show damage in the “very slight” and “slight” categories concentrated at the corners of both facades nearest the shaft. This is consistent with the “greenfield” settlements applied, which were fairly small, and slightly irregular near the corner.

Analysis 3 (in which the observed settlements are imposed on the building) is not presented graphically here, but shows no damage greater than “negligible” on the East facade, and on the North facade “slight” damage aligned with the second column of openings. This low damage is consistent with the building tilting as a rigid body towards the North, with almost straight settlement profiles along both facades. The fact that “slight” damage was still registered between openings shows the important effect of the presence of openings on the behaviour of masonry facades, causing stress concentrations and sections of reduced shear strength and stiffness.

The fact that the building tilted almost as a rigid body is thought to be due to the foundation contributing more to the stiffness of the whole system than had been taken into account in the analyses. To test this hypothesis a further calculation, Analysis 5, was made in which a stiff foundation was added. The settlement contours from
this analysis are shown in Figure 7. Comparison with Figure 3(b) reveals how important the modelling of the foundation stiffness may be to the accurate prediction of settlement of buildings and of the associated damage.

4 CONCLUSIONS

The results obtained from Maddox Street appear to be reasonable in terms of the damage category of “slight” predicted for the building, which is consistent with the observed result, perhaps a little conservative, but not overly so. There is, however, scope for refinement of the assumptions made in the modelling process, and improvements can be made in future analyses.

The maximum calculated building settlement was similar to that observed on site, although the distribution around the building was different. The real building behaved more as a rigid body, indicating that its global stiffness was higher than in the model. This may be due to tensile strength in the masonry and, more importantly, a basement structure consisting of a vaulted roof supported on columns. Because of its complexity, this was not modelled, except approximately in Analysis 5.

Interaction phenomena between the building and the ground were observed. Cracking of the building in hogging regions reduces the effective stiffness, thus causing the cracking to localise. The importance of the building weight in increasing the settlements compared to the “greenfield” case was seen, as was noted by Liu (1997) in 2-D analyses. The situation would, however, have been quite different if the building had spanned across the whole settlement trough, so enabling arching action to take place. Thus the importance of the particular geometry of the problem is obvious.

The importance of representing openings to model the facade behaviour was shown. Openings were either modelled as holes or as vertical regions of reduced stiffness. The former is appropriate where detail is needed, but the latter seems to be a satisfactory approach for less critical zones.

5 ACKNOWLEDGEMENTS

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6 REFERENCES

Numerical modelling of tunnel installation procedures

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ABSTRACT: A complex 3D numerical model of tunnelling in soft ground has been developed at Oxford University for the study of tunnelling-induced settlement damage to large masonry structures. The model includes procedures for simulating tunnel installation, including excavation, lining and ground loss. These procedures involve increments of tunnel installation where ground loss is simulated by shrinking the tunnel lining for that increment. This procedure is seen to be an effective approach for modelling tunnel installation, but it does lead to deformations that are non-uniform above the tunnel axis. This paper describes investigations of alternative methods for modelling tunnel installation using 3D finite element analysis.

1 INTRODUCTION

There has been considerable interest in recent years into developing improved numerical models of shallow tunnel construction in soft ground for two different purposes. The first is the investigation of the behaviour of tunnel linings, and their interaction with the surrounding ground, to improve lining design. The second area of interest is the effect of tunnelling on adjacent structures.

In this second area, considerable recent research has been undertaken to investigate the damaging effects of settlements that arise from ground loss at the tunnel (e.g. Potts & Addenbrooke 1997, Burd et al. 1999). One example of research in this area is a project at Oxford University, UK where a three-dimensional (3D) finite element model of tunnelling, including excavation, ground loss and lining installation, has been developed. Ground loss is simulated by shrinking shell elements representing the tunnel lining. This model also includes a surface structure to study the effects of tunnelling-induced settlements on large masonry buildings. Only by using 3D techniques is it thought possible to achieve realistic modelling of this particular problem.

The Oxford model uses non-linear material models for the soil and also the masonry used to construct the building. The history of this research project is described in Houlshby et al. (1999) and the model itself is described in detail in Augarde (1997) and Liu (1997). The use of the model to analyse masonry buildings, subjected to ground movements caused by shallow tunnel construction is described by Burd et al. (1999). This model is under further development; it is also currently being validated against field data (Bloodworth & Houlsby 1999).

Previous research at Oxford was based on the use of shell elements to model the liner (Augarde 1997, Augarde et al. 1998). These elements (which are based on the use of overlapping displacement fields) are relatively complex to program; they suffer from the further disadvantage that the assumed displacement fields are incompatible with those adopted for the tetrahedral elements used to model the surrounding soil. This paper is concerned with the development of alternative procedures to model tunnel installation in which continuum elements (rather than shell elements) are used to model the liner. This leads to an increase in the number of degrees of freedom required to model the tunnel liner, but it is thought that this disadvantage may be outweighed by the advantages of using soil and lining elements that are based on compatible formulations.

2 MODELLING TUNNEL INSTALLATION

It appears necessary to include, as a minimum, three simulations in a finite element model of soft ground tunnelling: excavation, ground loss and lining. The first of these is relatively straightforward (Augarde et al. 1995). Simulating excavation involves the removal of stiffness contributions from elements in the current excavation stage and the imposition of nodal loads on the excavated surface to render them free of surface tractions.

It is less clear how ground loss should be simulated and how suitable elements for tunnel linings
should be selected. Various attempts have, however, been made to reproduce the surface effects of tunnelling using numerical methods, mostly using two-dimensional finite elements. Potts & Addenbrooke (1997), for example, describe a technique in which the unloading caused by soil removal is carried out in a series of small increments. When the ground loss reaches the specified value then the calculation is terminated. No lining is modelled in this work.

A more complex procedure is described by Rowe et al. (1983), for 2D analysis, in which separate meshes are used to model the lining and the surrounding soil, between which a gap is specified at the start of the analysis. Once the analysis is underway, contact between the soil and liner is monitored and the interaction between the two, after contact has been made, is modelled by suitable numerical procedures. Another 2D approach is described by Abu-Farsakh & Voyiadis (1999) where two orthogonal plane strain analyses are conducted: one transverse to and the other longitudinal to the tunnelling axis. None of these 2D analyses, however, include realistic models of surface structures.

Much less research has been published using 3D techniques to model tunnel installation. Akagi & Komiya (1996) describe three-dimensional finite element modelling of tunnelling where properties of elements within and ahead of the tunnel are changed to model the changes in stiffness due to excavation. Incremental excavation is carried out with the activation of stiffer continuum elements representing a lining. Swoboda et al. (1989) use 3D finite elements and boundary elements to study NATM liner behaviour. This work includes excavation and the activation of liner elements whose stiffness changes during the analysis, to model the curing of shotcrete. No allowance is made for ground loss in either of these analyses.

3 DEVELOPMENT OF NEW TECHNIQUES

3.1 Introduction

Demonstration analyses for tunnelling under greenfield sites, using a coarse mesh and shell elements for the tunnel lining, exhibit non-uniform deformations above the tunnel axis (Augarde 1997). In particular, “hotspots” of concentrated settlement are noted at the end of each tunnelling increment. The possible causes of this behaviour are thought to be:

- the effect of a relatively coarse mesh,
- incompatibility between the shell elements employed for the lining and the surrounding continuum elements, modellihg the soil, or
- the face loss for the whole increment being concentrated at the end of the tunnelling increment.

The first can be dealt with by moving to finer meshes. This has been made possible by the presence, since April 1998, of the Oxford Supercomputer (a Silicon Graphics Origin 2000 shared memory parallel computer available to a number of research groups in science and engineering departments at Oxford University). The benefits this has produced are dealt with in Bloodworth et al. (1999).

The techniques described below serve to investigate the second and third potential causes of irregular settlement patterns. Firstly, continuum elements, rather than shells, are used to model tunnel linings. Secondly, the effect of applying support pressure to the exposed face at the end of each tunnelling increment is investigated. Finally, the use of a full-length liner, present from the start of the analysis in the soil, is described.

The main preoccupation during examination of these new methods is their efficiency in a large 3D finite element analysis. It is necessary to avoid introducing large numbers of new degrees of freedom since this leads to longer run-times and the requirement for increased memory. It is important to reiterate that the primary aim of this particular research is not to obtain a detailed model of the tunnel itself but of the effect its installation has on the surrounding ground.

This model of tunnelling is concerned only with undrained soil behaviour, since immediate settlements are almost always of greater significance to surface structures than consolidation effects. Aspects such as the placing of grout around tunnel rings and its time-related change in properties are therefore not considered.

3.2 Test problem

The proposed techniques are demonstrated using a simple straight tunnel configuration without a surface structure. In each case, the tunnel is 60m long and has a diameter of 5.0m. The depth to the tunnel axis is 10.0m. For all analyses described in this paper a 1% volume loss is simulated. The finite element mesh used for the analyses contains 9735
nodes and 6476 elements. Only one-half of the problem is modelled due to symmetry.

Excavation of the tunnel is modelled by dividing the elements inside the tunnel into ten groups, one of which is removed during each analysis stage. The calculations described here are based, for simplicity, on the use of a linearly elastic model for the soil, in which stiffness increases with depth. Further work in which the tunnelling procedures are used in conjunction with a non-linear soil model, is currently underway. The soil properties used in these calculations are those given in Augarde et al. (1995). It is well known that linear elastic soil models lead to excessively wide settlement troughs (Ward & Pender, 1981). As a consequence, the magnitude of the predicted settlements are unrealistically small.

The authors do not suggest that linear elasticity is the most appropriate soil model for this problem. The choice of soil model is made to reduce the complexity, and run-times, of the analyses in which the new techniques are demonstrated. The new techniques are to be incorporated into the full model of tunnelling, including the use of non-linear material constitutive models. The analyses described here are the first stages in testing the new methods of tunnel installation modelling.

3.3 Use of continuum elements for tunnel lining

In initial development work (Augarde 1997) shell elements based on Phaal & Calladine (1992) are used to model the tunnel lining. These shell elements are unusual in that they do not possess rotational degrees of freedom. Bending stiffness is provided by a novel overlapping scheme (Augarde et al. 1998). Any potential incompatibility between these elements and the surrounding soil elements, to which they are rigidly attached, can be avoided by using, instead, continuum elements to model the tunnel lining.

To control the ground loss, with continuum elements forming the tunnel lining, loads are applied to the nodes in the tunnel lining elements to produce a specified uniform shrinkage (determined from the ground loss specified) were the liner to be unconnected to the soil. The load vector for each element, \( f_c \), is obtained from the element stiffness matrix, \( K_e \), by,

\[
f_c = K_e d_e
\]

where, \( d_e \) is a vector of element node displacements, compatible with the specified lining element shrinkage. The vector, \( d_n \), is comprised of individual nodal displacement vectors, \( d_n \), which are each determined as follows. In Figure 1, the required radial displacement is \( \delta_n \) and \( N(x,z,y) \) is a node on a lining element, having a position vector, \( p \). The vector \( t_i \) locates the start of the tunnel axis, while \( t \) gives the tunnel axis direction in space. The nodal displacement vector \( d_n \) can be found from,

\[
d_n = \delta_n \frac{n}{|n|}
\]

where,

\[
n = t + \left[ \frac{t \cdot (p - t)}{t \cdot t} \right] t - p
\]

3.4 Face support

In early development work at Oxford, no attempt was made to model support pressures applied to the tunnel face. It is thought, however, that the addition of face support will lead to an improved modelling procedure.

Face support is provided in the numerical model by the application of loads to the nodes at the tunnel face, for each tunnelling increment. Their determination is linked to the calculation of the nodal loads to model excavation.
The vector of excavation loads, $f_{exc}$, for the mesh comprising the new, excavated, volume is,

$$f_{exc} = \int_{V_1} N^T \gamma dV - \int_{V_1} B^T \sigma_0 dV$$  \hspace{1cm} (4)

where $\sigma_0$ are the current stresses in the elements, $N$ is the matrix of element shape functions, $B$ is the strain-displacement matrix and $V_1$ is the new volume after excavation (Augarde et al. 1995). To provide face support, the components of $\sigma_0$ normal to the excavated face are preserved during excavation by their omission from the second term in Equation 4. This requires that the face nodes are identified for the current stage. Their elemental contributions to this term are ignored in calculating the excavation loading.

Figures 2a and 2b show the elements ahead of the tunnel face at an intermediate stage of the analysis. Those shown shaded will form the lining in the next stage of the analysis. With full face support present, there is a small amount of movement in the tunnelling direction (maximum 9.3 mm). Where no face support is provided (Fig. 2a) some face loss is clearly evident. The displacement of the central node on the tunnelling face (towards the excavation) is, in this case, 14 mm. Otherwise, apart from a slight increase in vertical movement of the lining elements in the subsequent stage, there is little difference between the soil movements around the tunnel. The movement of the top face of the subsequent stage of lining elements is thought to be responsible for additional heave at the surface in face supported analyses, as will be seen in the next Section.

3.5 Incremental change of properties

In the shell element liner model (Augarde et al. 1998), the lining is installed by activating groups of elements at each increment of tunnelling. A new technique is described below for the lining of continuum elements.

The initial mesh consists entirely of soil elements.
At each increment of tunnelling, the material properties of elements representing the tunnel lining are changed from soil to concrete. An advantage of this method is that it is not necessary to add elements for the lining after the tunnel has been excavated, thereby reducing the total number of elements used in the analysis. Linear elasticity is assumed for tunnel lining and properties are chosen to represent concrete. The implementation of this procedure with an elasto-plastic material model for the lining would appear to be difficult. In this case, a yield check would be necessary at the point where properties are changed, and it is not clear as to the procedure for determining the state of the material in the event of yielding. This is not regarded as a problem for this research, however, since a model of the lining, in which detailed non-linear behaviour is considered, is not required.

A plot of the deformed liner alone, after 5 (out of 10 in total) stages, of an analysis using this technique, is shown in Figure 3. Irregularities are evident in the shrunken liner at previous tunnelling increment faces. The difference in nodal displacement between peaks and troughs is about 4 mm. The reason for the discontinuities at the joints is that the material properties in the lining are not constant throughout the lining elements during the analysis, but change as the tunnel is excavated. At the tunnel face, the liner in the excavated part consists of concrete while the lining elements ahead have the same material properties as the surrounding soil. Shrinking the concrete tunnel lining results in deformations in the lining ahead of the tunnel face, as shown in Figure 2. In the subsequent stage of the analysis, the properties of this already deformed lining are changed from soil to concrete, and the lining is shrunk again. This procedure results in increased deformations at the liner joints.

Despite these non-uniform tunnel liner displacements, this method gives relatively smooth surface settlement profiles, as can be seen in Figure 4 (without face support) and Figure 5 (with face support). Heaves ahead of the tunnel face are evident, but these are likely to diminish with the use of an elasto-plastic soil model, due to the reduction in the significance of unloading. Figure 6 shows surface settlement profiles above the tunnel centreline for stage 5 of all analyses. The increased heave ahead of the excavated tunnel, due to face support is even clearer, although settlements above excavated zones appear relatively similar, between analyses with and without face support.

3.6 A continuous tube of elements
As a further numerical experiment, an analysis was carried out in which complete concrete liner was preinstalled in the finite element mesh, so no change of properties is necessary during the process of modelling tunnel installation. This eliminates the problem of the uneven tunnel liner as described in Section 3.5 since the lining elements do not change properties during the analysis. A pre-installed tunnel liner, however, departs significantly from processes adopted in practice. Testing this rather unusual approach, however, was thought to be a useful exercise.

A plot of the surface settlements without face support is shown in Figure 7. While the maximum settlement indicated for the continuous tube analysis is roughly one-half that in the earlier analyses, the volumes of the surface settlement troughs are comparable, and are consistent with the 1% ground loss specified for the tunnel. It appears that the continuous tube provides additional local support to the
ground around the tunnel, thus reducing local settlements.

As shown in Figure 6, and as might be expected, face support in this case has a negligible effect on the deformations on the surface since the excavation occurs within a preinstalled tunnel lining. The settlements above the tunnel heading are comparable to those from the analyses where the liner was installed incrementally. Behind the tunnel face, however, the settlements reduce rather unrealistically. This effect is thought to be associated with the longitudinal stiffness of the continuous tunnel lining.

4 SUMMARY

Two procedures are described for the simulation of tunnel construction in a three-dimensional finite element model. The processes that occur during the construction of a real tunnel are highly complex, and a simplified approach is adopted to model liner installation, ground loss and the application of face support.

The use of continuum elements to model tunnel construction is described, in which installation of the tunnel liner is modelled incrementally. This approach led to minor irregularities in liner deformations, although these did not significantly affect the surface settlement contours. An alternative “continuous tube” approach was also investigated but this was found to give unrealistic reductions in settlements behind the tunnelling face. The first approach is clearly to be preferred.

Analyses involving the use of this first approach for modelling tunnel installation, in conjunction with non-linear soil and masonry models, are currently underway.

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REFERENCES


A Model for the Variable Stiffness of Undrained Clay

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ABSTRACT: In this paper a theoretical model for the undrained behaviour of clays is presented. The model takes into account the non-linear behaviour of soil at small strains, and also includes effects such as hysteresis and dependence of stiffness on recent stress history. The model uses multiple yield surfaces within the framework of work-hardening plasticity theory. The quality of fit to experimental data depends on the number of yield surfaces used. The model allows predictions of soil behaviour to be made for any undrained stress or strain paths, and can be readily incorporated within finite element analysis software. The capabilities of the model are illustrated by means of examples of analyses of single elements of soil, and the model has been used for three-dimensional analysis of tunnel installation.

1 INTRODUCTION

One of the most important recent themes in soil mechanics has been the growing appreciation of the influence of the strain amplitude on soil stiffness. It is now well-known that the range of strains for which soils can be regarded as truly elastic (i.e. they exhibit fully recoverable behaviour) is extremely small. As strain amplitude increases the apparent “stiffness” decreases. The term “stiffness” is used here in a somewhat poorly-defined manner, since its meaning is ambiguous if the soil is no longer truly elastic.

The appreciation of the influence of strain on soil stiffness was led by experimentalists, who observed much higher stiffnesses in small-strain dynamic tests than in the unload-reload loops of static tests (e.g. Hardin and Drnevich, 1972), which were usually carried out over a larger strain range. Measurements of shear stiffness over a wide range of strain amplitudes rapidly followed, and the characteristic “S”-shaped curve for the variation of stiffness with log(strain) is now common knowledge, see Fig. 1(a). On a linear scale of strain (Fig. 1(b)) the stiffness reduction typically follows a simpler pattern, although this plot is now rarely used. Importantly it has been established that the higher stiffness at low strain amplitude is associated not with the higher strain rates in dynamic tests, but is primarily due to the strain amplitude itself.

Side-by-side with this knowledge is the relationship between damping ratio and strain amplitude in cyclic tests, which shows a typical pattern as in Fig. 1(c). A direct consequence of this curve is that truly elastic behaviour (no damping) occurs only at very small strain. In terms of the stress-strain curve, both the stiffness and damping relationships are consistent with the observation that, at all except very small strains, an unload-reload loop shows hysteresis, Fig. 1(d). The openness of the hysteresis loop increases with strain amplitude.

A final experimental observation, which proves to be closely linked to the above results, is that, for a soil in a given stress state, then the tangential stiffness observed for a subsequent stress path depends on the immediate past history of the soil. Stress paths which represent a continuation of the immediate past stress path result in the lowest stiffness, and those which involve a complete reversal of direction result in the highest stiffness. Intermediate values are observed for stress paths that represent a sudden change of direction from the immediate past history.

The current state-of-the-art is that there is a much wider appreciation of the problems caused by the above knowledge, than there is knowledge of what to do about them.

At a simple level, the above observations mean that, in carrying out any analysis, an engineer should take into account the expected strain amplitude in selecting the moduli used. Most straightforward displacement analyses use linear elasticity theory, which is unable to capture the above effects, and so engineers must also be
Figure 1: Outline diagrams of shear stress-strain behaviour of typical soils

aware that their analyses will be inadequate in some ways.

At a more detailed level, a theoretical model is required for soil behaviour that is capable of describing the above effects, and yet remains sufficiently simple so that the engineer can understand the concepts on which it is based. It must be capable of being implemented in a reasonably straightforward manner. The theoreticians lag far behind the experimentalists in providing such a model: a simple standard model for the variation of soil stiffness still does not exist. The reason is that, whilst the behaviour described above is easy to describe qualitatively, it is very difficult to encode within a self-consistent theoretical model.

There are a variety of possible approaches. We first exclude linear elastic theory, because it cannot model the change in stiffness, and non-linear elasticity, because it cannot model the damping. Conventional plasticity theories, with a single yield surface, must also be eliminated because they result in a sudden, rather than gradual, change in stiffness. The most promising approaches are both, however, based on plasticity theory, and are (a) use of multiple yield surfaces (usually nested) and (b) the concept of an inner yield surface and an outer ‘bounding’ surface. Other approaches have been adopted, which involve semi-empirical changes to the soil stiffness according to some pre-determined sets of rules. These models are likely to have little long-term application, and must be seen as short-term approaches, to be used only until a more comprehensive method is developed.

The bounding surface models offer an economy in that they do not require the multiplicity of surfaces used in the multiple surface models, but this is at the expense of two drawbacks: (a) they often require the choice of a number of somewhat arbitrary functions, often without obvious physical interpretation and (b) they are usually incapable of describing the effects of the immediate past history as described above.

Multiple surface models are therefore adopted here as the most promising approach. They have, however, some disadvantages. Firstly the multiple surfaces result in a large number of material parameters to be specified. It will be seen below how this disadvantage can be side-stepped. Secondly the multiple surfaces result in a considerable amount of computation: this is a reducing problem as faster computers become available. Thirdly many multiple surface models are inherently complex, and practitioners are therefore reluctant to use them. The purpose of this paper is to present a simple multiple surface model to demonstrate the value of this approach.

It will be found that a multiple surface model can describe all the features of soil behaviour given above. The model described here is limited to undrained behaviour of clays, and is not (at least in its present form) capable of modelling softening behaviour.

The complexity of even the simplest of multiple surface models is such that (for all except very trivial problems) computer analysis is necessary. This usually involves use of finite element analysis, and so the emphasis in the following sections is on the development of a model that can be readily incorporated into a non-linear finite element program. Different solution strategies are adopted in different programs, so the results are presented in as general a form as possible. The loss of simple hand calculations is an inevitable penalty that must be paid for the benefit of improved modelling of soil behaviour.

2 MODELS DEVELOPMENT

2.1 Undrained analysis

The stress-strain behaviour of soil is of course correctly described in terms of effective stresses, and the development of a straightforward multiple surface effective stress model is a long-term goal. For many problems involving the deformation of clay, however, the soil may be idealised as undrained, and for these problems a total stress analysis may be used. This simplification results in some loss of modelling capability, in particular many effects due to the coupling between shear strain and pore pressure are lost. This drawback can be partially offset by careful selection of parameters for the total stress analysis, recognising that they will depend on the stress history prior to the undrained phase (and in particular the overconsolidation ratio). The Author is aware that in some senses the use of total rather than effective stress analysis (e.g. Stallebras and Taylor, 1997) repre-
sents a step backwards from present knowledge of soil modelling, but considers that the benefits of the appreciation of the effects of strain on soil stiffness outweigh this temporary disadvantage.

2.2 Volumetric and Deviatoric Behaviour

The following derivations are carried out in terms of Cartesian tensors, with summation being implied over a repeated index.

It is convenient to consider the stresses and strains as separated into volumetric and deviatoric components, and define the deviators of stress and strain:

\[ s_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \]  
\[ e_{ij} = \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij} \]

where \( \delta_{ij} \) is the Kronecker delta (unit tensor), \( \delta_{ij} = 1 \) for \( i = j \), \( \delta_{ij} = 0 \) for \( i \neq j \).

2.3 Volumetric Behaviour

In this model the volumetric behaviour is always considered as purely elastic:

\[ \sigma_{kk} = 3K \varepsilon_{kk} \]  
\[ e_{ij} = \varepsilon_{ij} + \varepsilon_{ij}^p \]  

where \( \varepsilon_{ij}^p \) occurs for all changes of stress, and is given by eq. (4) (with \( \varepsilon_{ij}^c \) instead of \( \varepsilon_{ij} \), while plastic strain \( \varepsilon_{ij}^p \) only occurs if the stress point lies on the yield surface, which is given by the expression:

\[ f = s_{ij}s_{ij} - 2c^2 = 0 \]

where the constant \( c \) specifies the size of the yield surface (it is equal to the undrained strength in simple shear, and is related to the undrained strength in triaxial compression \( s_u \) by \( c = 2s_u / \sqrt{3} \)). The ratios between the plastic strains are most conveniently expressed in terms of a plastic potential, which is also a surface in stress space. The strains are obtained from the derivative of the plastic potential \( g \):

\[ \dot{\varepsilon}_{ij}^p = \lambda \frac{\partial g}{\partial s_{ij}} \]  
where \( \lambda \) is a scalar multiplier, which is at present undetermined. There are good theoretical reasons for adopting an “associated flow rule”, i.e., \( g \) is identical to \( f \). This is used here, so that on differentiation the plastic strains are given by:

\[ \dot{\varepsilon}_{ij}^p = 2\lambda s_{ij} \]

To complete the model the differential of the yield surface during yielding is also required:

\[ f = \frac{\partial f}{\partial s_{ij}} \dot{s}_{ij} = 2s_{ij} \dot{s}_{ij} = 0 \]

The above equations are sufficient to define the model, but they are not in a convenient form for computation. After some manipulation it is possible to determine \( \lambda \):

\[ \lambda = \frac{1}{4c^2} s_{ij} \dot{s}_{ij} \]  
\[ \dot{s}_{ij} = 2G \left( \delta_{ik} \delta_{jl} - \frac{1}{2c^2} s_{ij} s_{kl} \right) \varepsilon_{kl} \]

which specifies the incremental stiffness matrix. Since this matrix is singular (because the material is perfectly plastic) it cannot be inverted to give a compliance matrix.
2.5 Strain Hardening

Yielding of soil is usually a gradual process, so an elastic-perfectly plastic model is not suitable for the study of displacements under working loads, before failure is reached. For this purpose, a strain hardening theory of plasticity is necessary. The first development of the above model is therefore to introduce strain hardening. The hardening considered here is “kinematic hardening”, i.e. translation of the yield surface as plastic strain occurs (as opposed to “isotropic hardening”, in which the surface expands).

The position of the centre of the yield surface is now allowed to move. This position is specified as \( s_{ij}^0 \), and for convenience we define \( s_{ij}^r = s_{ij} - s_{ij}^0 \). The yield surface and plastic potential become:

\[
f = g = s_{ij}^r s_{ij}^r - 2c^2 = 0
\]

(12)

The differential of the yield surface becomes:

\[
\dot{f} = r \frac{\partial f}{\partial s_{ij}^r} \dot{s}_{ij}^r = 2s_{ij}^r \dot{s}_{ij}^r = 0
\]

(13)

Because the model now involves strain hardening, the expression for the plastic strains takes a slightly different form:

\[
\dot{\varepsilon}_{ij}^p = \lambda \frac{\partial g}{\partial s_{kl}^r} \dot{s}_{kl} = 4\lambda s_{ij}^r \dot{s}_{kl} s_{kl}
\]

(14)

where \( \lambda \) is a scalar multiplier, which is not yet determined.

The final part of the model is a strain hardening law, which determines how much the yield surface moves as plastic straining takes place. In this model we will adopt the simplest possible approach, a linear hardening law:

\[
\dot{s}_{ij}^r = 2h \dot{\varepsilon}_{ij}^p
\]

(15)

where \( h \) is a constant.

The model is then more complete at this stage, but is again in an inconvenient form for computation. The above equations can be manipulated to give:

\[
\lambda = \frac{1}{16c^2 h}
\]

(16)

which is substituted into the original equations to give an incremental stress-strain relationship in a form which gives the compliance matrix:

\[
2G \dot{\varepsilon}_{ij} = \left( \delta_{ik} \delta_{jl} + \frac{G}{2c^2 h} s_{ij}^r \dot{s}_{kl}^r s_{kl} \right) \dot{s}_{kl}
\]

(17)

This equation can be inverted explicitly to give the stiffness matrix:

\[
\frac{\dot{s}_{ij}}{2G} = \left( \delta_{ik} \delta_{jl} - \frac{G}{2c^2 (G + h)} s_{ij}^r s_{kl} \right) \dot{s}_{kl}
\]

(18)

Note that for \( h = 0 \) the stiffness matrix is the same as for the von Mises model, but the compliance matrix cannot be obtained for that case.

The above relationships provide the stress-strain law for the material. During computation it is also necessary to update the values of \( s_{ij}^r \) using the following equation, which can easily be derived:

\[
\dot{s}_{ij}^r = \frac{1}{2c^2} s_{ij}^r s_{kl} \dot{s}_{kl}
\]

(19)

2.6 Multiple Yield Surfaces

The above model is still not sufficient to describe the changes of soil stiffness realistically: a single yield surface results in a single abrupt change in stiffness. A continuous variation of stiffness can be approximated by a model with several yield surfaces, each as described above. By choosing different sizes and strain hardening parameters for different surfaces, an approximation can be made to the continuous variation of stiffness with strain. As larger numbers of yield surfaces are used, better approximations can be achieved. The positions of the centres of the yield surfaces represent a memory of the past history of deformation of the sample, and describe stress-induced anisotropy. The soil may start as isotropic (all the surfaces centred on the origin in deviatoric stress space), but as the sample is strained they move from the origin, and any subsequent stress probes would indicate the soil to be apparently anisotropic.

How are multiple yield surfaces combined? First the strain is divided into the elastic strain and a series of plastic strains, each associated with one of the yield surfaces:

\[
e_{ij} = e_{ij}^e + \sum_{\alpha=1}^{n} e_{ij}^{p\alpha}
\]

(20)

Each of the \( n \) yield surfaces (which are also plastic potentials) is specified in terms of its size \( e_{\alpha} \), and the coordinates of its centre \( s_{ij}^{r\alpha} \) (defining also \( s_{ij}^{p\alpha} = s_{ij} - s_{ij}^{r\alpha} \)):

\[
f_{\alpha} = g_{\alpha} = s_{ij}^{r\alpha} s_{ij}^{r\alpha} - 2c_{\alpha}^2 = 0
\]

(21)

from which differentials of the yield surfaces like equ.
The evolution rules for the movement of the centres of each of the yield loci are again needed, and these take exactly the same form as equ. (19).

2.7 Revisiting the von Mises surface

The model involves summing the effects of several strain-hardening yield surfaces. Unless an outer perfectly plastic surface is also included, then the model does not predict a finite soil strength. The von Mises surface must therefore be reintroduced. Because of the form of equ. (24), this cannot be done simply by setting \( h_\alpha = 0 \) for the largest surface. Using the superscript (or subscript) \( \alpha \) to denote the strain hardening surfaces, and variables without a superscript (or subscript) for the outer perfectly plastic surface, the plastic strains are now written:

\[
e_{ij} = e_{ij}^p + \sum_{\alpha=1}^{n} e_{ij}^{\rho \alpha} + e_{ij}^{\Gamma}
\]

The analysis proceeds exactly as before, except that the solution for \( \Lambda \) is now:

\[
4c_2^2 \Lambda = s_{ij} \dot{e}_{ij} - \sum_{\alpha=1}^{n} \frac{1}{4c_2 h_\alpha} s_{ij}^\rho s_{kl} \dot{s}_{kl}
\]

The stress-strain relationship can be derived as:

\[
\begin{align*}
\left( \delta_{ip} \delta_{jq} + \sum_{\alpha=1}^{n} \frac{G}{2c_2 h_\alpha} \left( s_{ij}^\rho - \frac{1}{2c_2} s_{ij} s_{mn} s_{nm} \right) s_{pq} \right) s_{pq} \\
= 2G \left( \delta_{ik} \delta_{jl} - \frac{1}{2c_2} s_{ij} \dot{s}_{kl} \right) \epsilon_{kl}
\end{align*}
\]

(27)

This equation does not give directly either a stiffness or compliance matrix. The matrix on the right hand side is singular, and so cannot be inverted, but the matrix on the left hand side is non-singular. It can be inverted numerically and multiplied into the right hand side to give the stiffness matrix. The compliance matrix cannot be obtained because the outermost yield surface is perfectly plastic.

The above provide all the necessary equations to form the stiffness matrices, and incremental updating procedures for the multiple yield surface model. It is worth reviewing how these equations are used. At each stage in the calculation a check is made as to whether the stress point lies on any of the yield surfaces. There are several possibilities:

(a) If the stress point touches no surface then the stiffness is purely elastic, equ. (4).

(b) If the stress point touches one strain hardening surface, then the stiffness is as given by equ. (18).

(c) If the stress point touches several strain hardening surfaces (but not the outer surface) then the stiffness is given by numerical inversion of equ. (24), with the summation only carried out over the surfaces touched by the stress point (case (b) can also be treated by this procedure).

(d) If the stress point touches strain hardening surfaces and also the outer surface, the stiffness is obtained from equ. (27), with the summation again being over the surfaces touched.

(e) There is a theoretically possible, but in practice very rare, case where only the outer surface is reached, and the stiffness is given by equ. (11) (although case (d) can also be used).

3 Selection of Parameters

One of the major disadvantages of multiple surface models is that they require specification of a large number of material parameters. For the above model with \( n \) surfaces there are \( 2n + 1 \) material parameters: \( (K, G, c \) and \( n - 1 \) pairs of \( c_\alpha \) and \( h_\alpha \)). Strictly this number can be reduced to \( n + 2 \), because the sizes of the intermediate strain hardening surfaces can be arbitrarily pre-determined as fixed fractions of the size of the outer surface, and the hardening parameters then chosen to fit the required stress-strain curve. It is most convenient to use non-dimensional parameters \( c_\alpha' = c_\alpha / c \) and \( h_\alpha' = h_\alpha / G \). The values of \( c_\alpha' \) are regarded as fixed, and the \( h_\alpha' \) are chosen to fit the stress-strain curve. When this form is used, it is found that, because the “S”-shaped curve of the shear modulus against strain amplitude tends to be similar.
Table 1: Values of non-dimensional constants

<table>
<thead>
<tr>
<th>Surface</th>
<th>( c'_\alpha )</th>
<th>( g'_\alpha )</th>
<th>( h'_\alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.9</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>0.04</td>
<td>0.75</td>
<td>4.5</td>
</tr>
<tr>
<td>3</td>
<td>0.06</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>0.1</td>
<td>0.3</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>0.15</td>
<td>0.2</td>
<td>0.6</td>
</tr>
<tr>
<td>6</td>
<td>0.2</td>
<td>0.15</td>
<td>0.6</td>
</tr>
<tr>
<td>7</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
</tr>
<tr>
<td>8</td>
<td>0.5</td>
<td>0.05</td>
<td>0.1</td>
</tr>
<tr>
<td>9</td>
<td>0.7</td>
<td>0.025</td>
<td>0.05</td>
</tr>
</tbody>
</table>

in shape for many different soils, a typical set of \( h'_\alpha \) values is able to model many different soils.

The process of parameter selection is made easier by recognizing that the tangential stiffness depends indirectly on the \( h'_\alpha \) values. It is more straightforward to specify the variation of tangent stiffness, and treat the \( h'_\alpha \) values as derived quantities. This is done in the following way. Consider for simplicity an initially isotropic soil, which is strained monotonically with a straight path in strain space (e.g., as in a triaxial compression test). Define the following invariants of stress and strain:

\[
\tau = \sqrt{\frac{1}{2} s_{ij} s_{ij}}
\]

\[
\gamma = \sqrt{2 e_{ij} e_{ij}}
\]

The tangential “shear stiffness” \( G_t = d\tau/d\gamma \) after the first \( m \) yield surfaces have become active (i.e., when \( c_m < \tau < c_{m+1} \)) is given by:

\[
G_t = G g'_m = \frac{G}{1 + \sum_{a=1}^{m} h'_a}
\]

from which the following relationship may be derived:

\[
h'_\alpha = \frac{g'_\alpha - g'_\alpha}{g'_{\alpha-1} - g'_\alpha}
\]

where \( g'_0 = 1 \) is also defined.

A series of \( g'_\alpha \) values may therefore be specified, from which the \( h'_\alpha \) may be derived. For instance, for a model with 10 surfaces (9 inner surfaces and one outer surface) the set of values shown in Table 1 has been shown to give very satisfactory results. The values have been deliberately chosen as round numbers.

Using the non-dimensional numbers in Table 1, the remaining constants to be chosen are the undrained strength in simple shear \( c = 2s_u/\sqrt{3} \) (where \( s_u \) is the undrained strength in triaxial compression), the shear modulus at very small strain, usefully expressed in terms of the rigidity index and the undrained strength: \( G = G_u = I_s s_u \), and the bulk modulus, which for undrained analysis is chosen to be a large factor times the shear modulus. This is often achieved by specifying a Poisson’s ratio close to 0.5 in the expression

\[
K = \frac{2(1 + \nu)}{3(1 - 2\nu)}
\]

For \( \nu = 0.49 \), which is frequently used in finite element analysis, this gives \( K = 50G \).

The selection of \( c, G \) and \( K \) therefore involves no more parameter selection than for a simple linear elastic von Mises plastic model. Indeed it proves to be more straightforward, since \( G \) is unambiguously chosen as the shear modulus at very small strain. In contrast, when using a model with a single shear modulus the engineer has to choose some compromise value of \( G \) based on guesswork about the probable strain range to be encountered in the analysis.

The fitting of the non-dimensional parameters in Table 1 deserves some further comment. There are a variety of different ways of expressing the stress-strain curve for a soil. As well as defining the tangent modulus one can use the secant shear modulus which is then defined as \( G_s = \tau/\gamma \). It follows that the secant and tangent moduli are related by the expression \( G_t = d(\gamma G_s)/d\gamma \). There are a number of equivalent ways of showing the stress-strain curve pictorially. Most obvious is the simple stress-strain curve (\( \gamma, \tau \)). Plots of stiffness against logarithm of shear strain amplitude (\( \ln(\gamma), G_s \)) or (\( \ln(\gamma), G_t \)) are also frequently used. Less common, but also useful, are plots of stiffness against stress, either (\( \tau, G_s \)) or (\( \tau, G_t \)). In fitting parameters it has been found convenient to use the plot of \( G_s \) against \( \tau \). The relationship between this and the more conventional plot is shown in Figure 2.

4 EXAMPLE PROBLEMS

The equations derived above can be easily implemented in a numerical analysis. A program has been written in FORTRAN 90 for the analyses below.
Fig. 3 shows the behaviour of the model for a load-unload-reload test. A stiff response is observed when the load is reduced below the maximum previous value, and the stiffness falls again when the load is increased beyond this past maximum value. Hysteresis effects are reconstructed.

The multiple yield surface model can successfully reproduce the effect of stress changes in relation to past history of stress paths. Fig. 4 shows the stress-strain curves for three identical strain paths applied to samples with different prior load histories. Path A, with a complete reversal in direction of stress path, shows the highest initial stiffness. Path C is for a sample in which there is no change in the direction of the stress path and shows the lowest initial stiffness. Path B, which involves a change in stress path through a right angle in stress space shows an intermediate response. The results are simply explained in terms of the multiple yield surface model. Immediately after a change in direction, the stress point lies within a truly elastic region and exhibits a high stiffness. As the point moves in stress space, it activates the yield surfaces in turn, reducing the stiffness. A path that is a continuation of the previous stress path already involves activated yield surfaces. Paths which involve a sharp corner initially cause less movement of the surfaces and result in a stiffer response, but as the centres of the yield surfaces gradually line up behind the stress point the stiffness again reduces.

The stress history is encoded in the positions of the yield surface. Figure 5 for instance illustrates the surfaces for a six-surface model in which the stress path is first from the origin to point A, and then back to the origin. The larger circles have all been displaced so that they pass through point A, whilst the small ones have first been moved in that direction, and then back so that they pass through the origin.

5 OBSERVATIONS ABOUT THE STRUCTURE OF THE MODEL

It is useful to note two special cases of the equations defining the yield surfaces in this model. As the surface becomes infinitesimally small \((c_\alpha \to 0)\) the equations reduce to those of linear elasticity. As the hardening modulus approaches zero \((h_\alpha \to 0)\) the equations approach those of perfect plasticity. Inter-
mediate cases provide a simple interpolation between these two extremes of behaviour.

This observation suggests that a similar procedure could be used to generalise other single-surface plasticity modes. A form for the intermediate surfaces needs to be chosen that reduces to the relevant elastic behaviour at one extreme and the plastic behaviour at the other. This approach will be pursued in future.

Multiple surface models have been used in which the yield surfaces are strictly “nested”, i.e. they do not overlap. Indeed it is sometimes stated that there is some theoretical requirement that this must be the case. No such requirement exists: the nesting of surfaces is adopted in some models as a matter of convenience, but can result in significant complexities. In the present model, surfaces are allowed to intersect, although they do so rather rarely. Fig. 6 illustrates an instance for a model with three surfaces. If a circular stress path is followed just within the second surface, the centre of the smallest yield surface follows approximately the same path as the stress point, with the yield surface “dragged” behind. The inner surfaces thus intersect, and the elastic region (shaded in Fig. 6) is bounded by segments of the two surfaces. Thus it is possible (but rare) to have a stress path which reaches a larger yield surface before a smaller one.

The model has been implemented in a finite element code and used for both 2-D and 3-D analyses, principally tunnelling problems (Chow, 1998, Augearde, 1997, Liu, 1997, Houlsby et al., 1999). A concern that the use of multiple yield surface models in large finite element analyses would greatly increase computation time has proven to be unfounded. In most 3-D analyses, most time is spent in equation solution, and that the constitutive model has little impact on computation time. The time is slightly sensitive to the size of the smallest yield surface, since in the numerical updating of the stress conditions the strain increment is broken into steps resulting in elastic stress changes that are small compared to this surface. In practice this does not prove to be an obstacle.

6 CONCLUSIONS

Expressions have been presented for the stiffness and compliance matrices for model for the undrained behaviour of clay using multiple yield surfaces. The model allows realistic fitting of observed features of soil behaviour such as small strain non-linearity; hysteresis, and the dependence of stiffness on past stress history. The model can be implemented in a numerical analysis, and has been used in large 3-D finite element analyses.

7 ACKNOWLEDGEMENTS

The initial incorporation of the multiple surface model into a finite element program, and preliminary analyses of the tunnelling problem, were carried out by Lily Chow. Assistance with the current implementation of the model has been provided by Charles Augearde.

8 REFERENCES

Transferring a non-linear finite element code to the Oxford Supercomputer, Oscar

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Abstract

A complex three-dimensional finite element model has been developed to study the effects of tunnel construction on adjacent structures. The model uses an in-house finite element code in which complex simulations and non-linear material models have been developed. This paper describes the transfer of this code to a large parallel computer, and the improvements in performance that resulted from the move. The examination of the code, which was necessary for the transfer, also led to improvements in the serial version.

1 Introduction

Numerical modelling of geotechnical engineering problems has been carried out in the Civil Engineering Research Group at Oxford since 1980. Finite element analysis is performed using the in-house finite element program OXFEM, written in Fortran 90 and running on Unix workstations. A particular area of interest of the group, since 1992, has been the modelling of soft ground tunnelling and the interaction between the effects of tunnel construction and surface structures. A complex model, including simulation of tunnel construction and a surface structure has been developed (Augarde [1], Liu [2]) and is currently being improved and validated against field data (Augarde et al. [3], Bloodworth & Houlby [4]).

A significant aspect of this research is the use of three-dimensional analysis. Two-dimensional simulations of tunnel-building interaction are only satisfactory for very simple geometries. A second feature is the use of non-linear material models. An overconsolidated clay deposit is modelled using a weakly non-linear, elasto-plastic formulation. Masonry, for building facades, is modelled using a strongly non-linear elastic-no-tension formulation. The combination of large numbers of degrees of freedom, and the use of an incremental solution technique, to deal with the non-linearities present, leads to large analyses, in terms of memory and run-times. The recent acquisition of a supercomputer at Oxford, with a limited number of users, has allowed more complex analyses to be undertaken.

This paper describes the transfer of the finite element code to the new parallel computer. The paper begins with a detailed background to the current research into tunnelling, to demonstrate the need for three-dimensional modelling. Some results are presented of analyses on workstations and the limitations are highlighted. This is followed by a description of the Oxford Supercomputer, OSCAR. The transfer of the existing finite element code, OXFEM, to the parallel computer is detailed and the improvements in performance are discussed. Apart from making more complex analyses feasible, the process of transfer has highlighted some improvements of the serial code, which have also been implemented.

2 Numerical modelling of tunnelling-induced settlement damage to structures

Tunnelling is a popular solution for the expansion of infrastructure in urban areas, since the impact of the final scheme on the environment is usually small compared to surface alternatives. The locations of many major cities are low-lying or coastal, where the near-surface geology typically involves a significant depth of un lithified deposits, leading to soft ground tunnelling conditions. In these conditions it is necessary to provide permanent ground support to the faces of a tunnel in the form of a lining.

![Diagram of tunnel settlement](image)

Trough parameters: $i$ = distance from centreline to point of inflexion; $S_{max}$ = maximum settlement

Figure 1: Section through circular tunnel driven beneath a building showing assumed "greenfield" settlement trough.
The installation of a lining involves the excavation of a void of a larger diameter than the finished tunnel. An annular void is therefore created, between the lining and the excavated faces, into which the surrounding soil is able to move. In addition, soil is able to deform and flow towards the advancing tunnel face. In soft ground, the effect of these movements will be seen at the surface as settlements, which may have significant and potentially damaging effects on surface structures of historical or national importance. Much effort and expense has therefore been devoted to limiting ground movements in tunnelling schemes. Expensive accommodation works, such as compensation grouting, have been used, on schemes such as the Jubilee Line Extension in London, to prevent tunnelling settlements affecting particular structures.

The prediction of the magnitude of tunnelling settlements is relatively straightforward at “greenfield” sites, through empirical rules calibrated against past field data. The transverse settlement trough is assumed, in this case, to be an inverted Gaussian distribution [5], as indicated in Figure 1. There is, however, little guidance on the effect of a surface structure above the tunnel on a settlement trough. Consequently, little is known of the potential damage to a structure due to tunnelling-induced settlement, where this interaction occurs.

### 2.1 Damage prediction methods

Current practice usually relies on the approaches of Burland and Wroth [6] or Boscardin and Cording [7]. Both assume the settlement of a building to follow a profile predicted for a “greenfield” site (Fig. 1). Measures of damage based on the maximum tensile strain in the building are then used. The building is idealised as a linear elastic deep beam. This approach, neglecting the possible effects of the building weight and stiffness, is usually conservative and provides little indication of the localised damage seen in real structures affected by settlements.

Improvements to the approach outlined above have been sought, in recent years, to produce reliable predictive tools that include soil-structure interaction. Two-dimensional finite element modelling is used by Potts and Addenbrooke [8] to generate modification factors to apply to the “greenfield” settlement trough to allow for building stiffness. This approach, while giving a useful indication of some of the interaction effects, is limited in its application to problems which may be reasonably represented in two-dimensions, and where the building weight is not a significant factor. This study also provides no information on localised building damage.

### 2.2 A three-dimensional finite element model of tunnelling.

A research programme has been in progress since 1992 at Oxford to develop and use an improved numerical model of tunnelling for the prediction of settlement damage. This research has concentrated on masonry buildings located above overconsolidated clay soil [9].

The approach taken in this research is new, mainly because the problem is modelled in three dimensions rather than two. This is judged essential to represent adequately the geometry of a tunnel constructed beneath a building. Three-dimensional modelling allows the unambiguous representation of the true orientation of the building relative to the tunnel, and allows the incremental advance of the tunnel heading to be simulated.

![Diagram of stress in principal stress space](image1)

**Figure 2:** A plot in principal stress space of nested yield surfaces after a stress path from the origin to A and back.

![Diagram of cracked elastic no-tension material](image2)

(a) Cracked elastic no-tension material

![Diagram of uniaxial behaviour](image3)

(b) Uniaxial behaviour

**Figure 3:** Elastic no-tension material model for masonry
The numerical model developed incorporates procedures for modelling excavation of the tunnel volume and lining installation [1]. The primary source of surface settlements, the volume loss due to the over-excavation of the real tunnel (as described above), is simulated by shrinking the lining. The lining is modelled with faceted shell elements [10, 11], where bending stiffness arises from a novel overlapping scheme, rather than through shape functions involving rotational terms, as in conventional shell elements.

The choice of material models for the ground and structure is vital for the accuracy of the final solution. The types of non-linearities that occur in the problem have been identified. Overconsolidated clay, in the undrained condition, to approximate London Clay, exhibits a gradual change in stiffness at small strains. This is modelled by a nested-surface work-hardening plasticity model described in detail by Houlby [12]. This type of constitutive model is able to capture the stress-strain history of an overconsolidated soil, now recognised to be of crucial importance in the determination of its subsequent behaviour. This is accomplished with nested von Mises surfaces as shown in Figure 2.

Building masonry on the other hand is a brittle material, experiencing an abrupt change of state in tension, moving from an uncracked elastic state to a cracked state with almost complete loss of stiffness normal to the crack direction. A material model has been implemented which models cracking and subsequent crack reclosure [2]. The stress-strain behaviour of this model is illustrated in Figure 3.

A final innovative feature of this model is a numerical scheme to "tie" a two-dimensional finite element mesh to one in three-dimensions [2, 13]. This is used to attach two-dimensional meshes, representing building facades, onto a three-dimensional mesh of the ground in which tunnel excavation is simulated. Openings in the facades may be represented either as holes in the mesh or as regions of reduced stiffness [4].

The complete model was demonstrated by Augarde [1] for a simple tunnelling scheme, involving a masonry building at a skewed angle to a tunnel. Figure 4 shows some details of this analysis and the meshes used for the ground and the building. Currently, work is underway to collect field data of buildings subjected to tunnelling settlement and to conduct verification analyses with the model [4]. A detail of the finite element mesh of one of these analyses is shown in Figure 5. In this case, a shaft is excavated close to the corner of a masonry church building. The structure is rectangular in plan with a number of openings in the facades. The predicted settlements on the surface of the combined model of ground, shaft and building are shown in Figure 6. The effect of the building in distorting the shape of the settlement bowl around the shaft is evident from this plot.
3 Serial analysis

3.1 Computational procedures

The use of the model of tunnelling described above, is complex and includes considerable pre- and post-processing stages. Mesh generation is carried out using the commercial analysis package I-DEAS. Unstructured meshing is used for the soil mesh, with partitioning to provide blocks of elements for excavation simulation. Unstructured meshing enables greater control over local element lengths where more detail is required, around the tunnel and the building footprint.

A variety of Unix workstations have been used to conduct this research, prior to and since the arrival of the Oxford Supercomputer. At present, the fastest serial platform available to the Group is a Sun Microsystems Ultra 2, having two 300 MHZ processors and 512Mb of RAM.

The solution procedure adopted in the finite element program OXFEM is a highly optimised implementation of the Frontal method. While this method has proved adequate for this research to date, iterative solution techniques are also under investigation, prompted by the considerable debate, at present, as to the viability of iterative solution techniques with geotechnical material models [14]. These techniques, however, appear to provide the most promising way to conduct the very large analyses planned for the future.

3.2 Performance

The demonstration analyses, such as that shown in Figure 4, require many degrees of freedom, even with relatively coarse meshes. The mix of non-linearities, described above, leads to a large number of incremental load steps. The combination of these two features leads to very long run-times. The verification analyses [4], for comparison to field data, use even larger and more complex models than shown in Figure 4. This is necessary to model adequately the geometry and level of detail of a practical site. For models with over 30,000 degrees of freedom, and up to 500 load steps to model a tunnel advance, run-times of one to two weeks are required on the fastest serial platform. One cause of these excessive run-times is the need to use memory well in excess of the RAM during execution.

Figure 7 shows the ground mesh for a recent verification analyses. A model with 40,000 degrees of freedom is necessary to model the construction of an underground railway tunnel beneath the Mansion House in London (construction completed in 1991). The viability of this analysis on the serial platform is dubious since run-time for a single load step, of which there are 500 in the whole analysis, is 1.5 hours.
4 The OSCAR Supercomputer

4.1 Background and history

The Oxford Supercomputing Centre (OSC) was established in 1998, with a mission to promote multi-disciplinary research in the application of high performance computing, particularly in parallel analysis techniques. Funding for the centre came from an approximately £1.4m grant under the UK Higher Education Funding Council (HEFCE) Joint Research Equipment Initiative, with additional support from Silicon Graphics Inc. (SGI). The main resource is the Oxford Supercomputer, OSCAR commissioned in June 1998.

Nineteen research groups at Oxford, covering a wide range of disciplines in science and engineering have access to the supercomputer. As well as civil engineering, research on OSCAR is in other branches of engineering science and in biochemistry, bioinformatics, biophysics, computing, earth sciences, materials, physics, physical and theoretical chemistry and physiology. The research groups involved share similar interests in large-scale, often mesh-based, numerical modelling.

The OSC is funded for a three-year period initially, from April 1998. In addition to maintaining and managing the system, including running a batch queue system for maximum utilisation of the resource, the OSC provides training and advice on parallel computing methods, including arranging outside speaker meetings, and facilitates contact and dissemination of knowledge between the member research groups.

OSCAR is an SGI Cray Origin 2000 supercomputer, similar in appearance to Figure 8, currently having 84 No. R10000 MIPS processors (upgradable to 128), 21GB of RAM and 256GB of disc storage. A group may make use of any number of processors at one time, with the CPU and memory usage charged to a notional account. The processors are physically arranged in pairs with RAM and access to disk space, to form modules, which in turn communicate via the CrayLink interconnect (Fig. 9).

The peak processor speed is 195Mhz, 780 MIPS or 390 MFLOPS (2 integer plus 2 floating point execute and one load/store per cycle). The peak computing power is 32 GFLOPS. The architecture is CC-NUMA (cache coherent, non-uniform memory architecture). This means that the memory, although physically distributed on the nodes of the machines, behaves as one large block of shared memory. Access times to memory vary from 1 cycle to 100 cycles, depending on whether the data is stored in the local cache of the processor or in the memory or cache of a processor in a remote module.

4.2 Parallel programming paradigms

The FORTRAN 77, FORTRAN 90, C and C++ programming languages are available on OSCAR via its SGI MIPSpro compilers. Programs may be compiled and run in serial form, but the preference is for OSCAR to be used for parallel programming, for which a number of models are available.

The first is automatic parallelisation, in which the compiler parallelises the source code without further user intervention. This may be suitable for simple programs but is less efficient for more complex code and is not often used.

Shared memory parallelisation uses directives inserted by the user in the source code, which appear as comments to a serial compiler. These directives instruct the compiler where to compile the code for parallel execution. OSCAR supports the OpenMP standard for such directives. This parallelisation method takes full advantage of the shared memory architecture; the user is not concerned about how the memory is allocated across different processors, nor how and when communication between processors occurs. It also has the advantage that the same source code may be compiled and run on a serial machine. This feature has practical advantages for program development with this project, since serial running continues side-by-side with the use of OSCAR. Extensions to the OpenMP directive set allow some user intervention over memory allocation if desired.
The most interventionist form of parallelisation is termed explicit distributed memory parallel programming. Each processor remains associated with its own local memory during program execution. The programming paradigms BSP (Bulk Synchronous Processing) and MPI (Message Passing Interface) [15], both of which are supported on OSCAR, enable the user to control the transfer of data between processors during program execution. A greater level of understanding of the system architecture is required; in particular the times taken for transfers between different parts of memory. The approach may utilise the parallel resource in the most efficient manner, but the same source code may not be compiled and run on a serial machine, so the overhead in program development is greater.

5 Transferring the serial FE code to OSCAR

To exploit this new computing resource, it was first necessary to move the OXFEM finite element program to OSCAR, and adapt it to run in parallel. OpenMP parallel directives for shared memory programming were used since this appeared to be a simple procedure that maintained a single code for use on serial or parallel platforms.

Parallelising an existing serial program first involves identifying the sections of the code that can be parallelised while maintaining correct functioning. The serial code is then optimised. Ideally, only then should parallel directives be inserted, and the parallel code then optimised for maximum speed-up and efficiency, which are defined as follows:

- Speed-up, \( S_p = \frac{T_1}{T_p} \)
  where \( T_1 \) is the run-time on a single processor and \( T_p \) the run-time on \( p \) processors.

- Efficiency, \( E_p = \frac{S_p}{p} \times 100\% \)

5.1 Profiling

The first stage of the move to OSCAR involved examining the serial code. A profiling program, speedshop was used for this purpose. This program provides a number of utilities for performance tuning. Different ‘experiments’ may be run, giving information, for example, on the CPU time spent in each subroutine in the program or, alternatively, the number of cache misses and floating point exceptions.

The CPU timings for the individual subroutines in the original serial version of OXFEM, running a medium-sized analysis are given in Figure 10. This Figure is taken from speedshop output and gives the CPU seconds spent in the subroutine named in the final column. The preceding columns give the time in seconds and the percentage overall. Listing is in descending order. The output indicates that 98.3% of the analysis time is spent exclusively in the subroutine frontl. (This routine implements the Frontal solution method, as described above). The profiling therefore quickly indicated where effort should be put into optimisation and parallelisation.

By breaking a subroutine down into smaller subroutines it is possible to concentrate on the sections of code where most analysis time is spent. In the case of the frontl subroutine attention focussed on the few lines which carry out the elimination of the degrees of freedom from the Front matrix. (Once all elements that contribute element stiffness terms to a degree of freedom in the mesh have been assembled, the coefficients in the structure stiffness matrix for that term may be eliminated. Operations then continue on a much smaller Front matrix of active degrees of freedom). As an initial experiment, OpenMP parallel directives were placed around these lines. Figure 11 shows a code fragment including the OpenMP directives.

<table>
<thead>
<tr>
<th>Function, procedure (dso: file, line)</th>
</tr>
</thead>
<tbody>
<tr>
<td>frontl (oxfem_seq: frontl.f90, 1)</td>
</tr>
<tr>
<td>ms_mult_d (oxfem_seq: matrix.f90, 853)</td>
</tr>
<tr>
<td>input (oxfem_seq: input.f90, 1)</td>
</tr>
<tr>
<td>force_shell (oxfem_seq: force_shell_sub.f90, 1)</td>
</tr>
<tr>
<td>report_alloc Realm (oxfem_seq: alloc_report.f90, 42)</td>
</tr>
<tr>
<td>m_mult_d (oxfem_seq: matrix.f90, 557)</td>
</tr>
<tr>
<td>ms_mult_lw (oxfem_seq: matrix.f90, 809)</td>
</tr>
<tr>
<td>initil (oxfem_seq: initil.f90, 1)</td>
</tr>
<tr>
<td>mincws (oxfem_seq: matrix.f90, 125)</td>
</tr>
<tr>
<td>mdet (oxfem_seq: matrix.f90, 42)</td>
</tr>
<tr>
<td>codes (oxfem_seq: codes.f90, 1)</td>
</tr>
<tr>
<td>g6029 (oxfem_seq: gauss.f90, 2194)</td>
</tr>
<tr>
<td>bshell (oxfem_seq: bshell.f90, 1)</td>
</tr>
<tr>
<td>mincwm (oxfem_seq: matrix.f90, 108)</td>
</tr>
</tbody>
</table>

Figure 10: CPU times obtained from speedshop experiment on unoptimised serial code
5.2 Initial parallel code results

The results obtained for speed-up and efficiency, following this relatively crude attempt to parallelise the code are shown in Figures 12(a) and 12(b) for between 2 and 8 processors. Two results sets are given in Figure 12(a): one in which $T_i$, used in the determination of the speed-up, $s_p$, is the run-time for a single processor on OSCAR and the second where $T_i$ is the run-time on the fastest serial platform. A maximum speed-up of just over four times over the serial platform was obtained, with the optimum arrangement being 5 processors. It is clear from these plots that a single processor on OSCAR is much faster than the serial platform. This is ascribed to the more advanced compiler optimisation available on OSCAR.

Figure 12(b) shows that, although a speed-up was obtained on OSCAR, the actual efficiency was well below optimal, and decreased rapidly with increasing numbers of processors used. This indicated that there were probably still significant improvements to be gained. These results were, however, gratifying given the minimal effort involved.

5.3 A second stage of optimisation

The code fragment in Figure 11, the heart of the Frontal solver, shows that elimination across the row of the Frontal matrix is split into two loops, one each side of the pivot. It is possible to bind these two loops into one, to give improved parallel performance.

In addition, the addresses of array storage in FORTRAN 90 run column by column rather than row by row. It is, therefore, quicker to access consecutive array entries if operations proceed down columns, rather than along rows. By storing the information in a transpose of the Front matrix, and then interchanging the loops in the code shown in Figure 11, a significant speed up of around 3 was obtained on the serial machine, and around 7 on OSCAR with 8 processors.

5.4 Variables and scheduling

Further optimisation of the parallel sections of code is possible by considering the declaration of variables used in the loops, and the method by which the calculation load was shared between the processors, the latter being termed scheduling.

The OpenMP standard allows the declaration of variables occurring in a parallel region of code as either PRIVATE, meaning that each processor keeps its own copy of the variable, or SHARED; the latter being the default. When a particular processor accesses a shared variable, a lock is put on address in memory to prevent it being accessed or updated by another. Other processors must wait until the lock is released before using the variable, which has a time penalty. Often the compiler can determine automatically whether the variable should logically be PRIVATE (for example the loop indices I and J in the code fragment in Figure 11). In optimising the OXFEM code, it was apparent, however, that it was necessary to explicitly declare the factor MULT in Figure 10 as PRIVATE for greater efficiency.

A number of types of scheduling are available in OpenMP. In static scheduling, each processor is given an equal number of iterations of the parallelised ‘DO’ loop to execute. In dynamic scheduling, a processor is given a smaller parcel of the total calculation load, and on completion of each, requests a new parcel. Dynamic scheduling is often more efficient when the amount of calculation load varies between loop increments, but incurs an overhead each time a processor requests new work.

The elimination carried out in the frontl subroutine, where attention has been focused, involves the same number of floating-point operations per loop. The loop-counter changes between eliminations but this happens outside the parallel region. Static scheduling is therefore most appropriate and likely to yield improvements in performance in this case.

Figure 13 shows the same section of the frontl subroutine following implementation of the variable declarations and

```c
RPIV = 1.0_dp / FRONT(LL,LL)
!$OMP PARALLEL DO
    do L = 1, LL - 1
        MULT = FRONT(L,LL)*RPIV
        FRONT(L,1:LWFRON) = FRONT(L,1:LWFRON) - FRONT(LL,1:LWFRON)*MULT
        GLOAD(L) = GLOAD(L) - GLOAD(LL)*MULT
    end do
!$OMP END PARALLEL DO
!$OMP PARALLEL DO
    do L = LL + 1, LWFRON
        MULT = FRONT(L,LL)*RPIV
        FRONT(L,1:LWFRON) = FRONT(L,1:LWFRON) - FRONT(LL,1:LWFRON)*MULT
        GLOAD(L) = GLOAD(L) - GLOAD(LL)*MULT
    end do
!$OMP END PARALLEL DO
```

Figure 11: First stage parallelisation of Frontal solve routine
Figures show that the time taken per step has been reduced from about 20 minutes to less than 4 minutes (with 8 processors). The importance of this is that a full analysis with 500 steps becomes viable; finishing in about 33 hours, as compared to nearly 1 week before these relatively simple changes were added.

A slight reduction in wall clock time is obtained by increasing to 16 or 32 processors, as Figure 14 indicates, but the efficiency reduces. All experience with OSCAR and the analyses described here has shown this problem to be most suitable for between 8 and 16 processors. The reasons for this are thought to be the nature of the algorithms employed in OXFEM, although further work is necessary to be certain of this. Storage of the Frontal matrix in cache local to the processor, using the OpenMP directives appears to be a profitable next step to take in optimisation.

## 6 Future work

The development of the finite element code described here is a constant process. Porting the code to OSCAR is one aspect of this work. The next logical step in parallelisation of the code is the use of MPI or BSP distributed memory paradigms. Since this approach is likely to need a full re-write of the code, it is thought that the extra investment required in learning these new techniques is unlikely to be balanced by proportional gains in speed-ups and efficiency or comparable to those found in this first stage of parallelisation. This is probably due to the relative simplicity of the algorithms used here.

As indicated above, future development of OXFEM, to cope with non-linear analyses with over 50,000 degrees of freedom, is likely to include investigation of iterative solution techniques, which require less memory during execution [16, 17]. Multi-frontal methods are also available [18] for implementation and use on OSCAR. The use of OXFEM for soil/structure interaction problems other than tunnelling is also likely and this will, doubtless prompt the development of new simulation techniques within the finite element code.

```
RPIV = 1.0_dp / FRONT_T(LL,LL)
GLOAD_LL = GLOAD(LL)
ELIM_COLUMN = FRONT_T(1:LFRON, LL)
!$OMP PARALLEL PRIVATE(MULT, J, L)
!$OMP& SHARED(LFRON, FRONT_T, ELIM_COLUMN, RPIV, GLOAD, GLOAD_LL, LL)
!$OMP DO SCHEDULE(STATIC)
do L = 1, LFRON
   MULT = FRONT_T(LL,L)*RPIV
   do J = 1, LFRON
      FRONT_T(J,L) = FRONT_T(J,L) - ELIM_COLUMN(J) * MULT
   end do
   GLOAD(L) = GLOAD(L) - GLOAD_LL*MULT
end do
!$OMP END DO
!$OMP END PARALLEL
```

Figure 13: Second stage parallelisation of Frontal solver routine
7 Concluding Remarks

The move across to the OSCAR Supercomputer has made feasible the analysis of significantly larger 3-D non-linear finite element models of tunnelling. These models are being used to develop improved predictive methods for the effects of tunnelling settlement damage on surface structures.

The process of optimising the code for parallel execution has also stimulated improvements to the serial version of the code. The move to parallel analyses was relatively straightforward because of the algorithm used and the choice of parallel paradigm. Profiling reveals that most of the computational effort, in this finite element code, is concentrated in a few lines. Once identified, various additions in this area of the code, using OpenMP directives have led to major reductions in the run-times.

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