NON-LINEAR DYNAMIC ANALYSIS OF OFFSHORE JACK-UP UNITS

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ABSTRACT

A two-dimensional finite element program for the non-linear dynamic analysis of offshore jack-up units under storm loading is described. The program aims to incorporate consistent and reasonable levels of approximation of all the major system parameters; this is in contrast to many previous approaches, which have tended to model some aspects of the problem in great detail while adopting a very simplified approach to others. Accurate modelling of the jack-up legs is achieved using an Eulerian formulation of beam-column theory. The complex non-linear behaviour of the spudcan footings is represented by a work-hardening plasticity model. Several options for calculating wave kinematics are available, including Stokes’ fifth order wave theory. The resulting non-linear equations are solved in the time domain using implicit integration algorithms. Results for some representative test cases are presented.

Keywords: jack-up units; spudcan foundations; non-linear dynamic analysis; beam-column theory; soil plasticity; wave loading.
1. INTRODUCTION

Mobile jack-up rigs are widely used in offshore oil and gas exploration, and increasingly in other applications, especially in marginal fields. A typical jackup comprises a hull supported by three K-lattice legs, each resting on a large inverted conical footing known as a spudcan. Figure 1. During installation, the rig is positioned with the footings resting on the seabed and the hull is raised a few metres above the water surface. The unit is then preloaded by pumping seawater into the hull, thus pushing the spudcans into the seabed. Once the desired penetration has been achieved, the preload is removed and the hull raised to its operating height.

Jack-ups were originally designed for shallow waters, but there is growing demand for their use in deeper waters and harsher environments. Under these circumstances, accurate analysis of jack-up behaviour, avoiding excessive conservatism, becomes increasingly important. There are several areas of uncertainty in the analysis of a jack-up:

(i) **The importance of dynamic effects**: jack-ups are flexible structures with natural periods of the same order as the predominant wave periods for many seas. In spite of this, they have traditionally been analyzed using quasi-static methods. Such an approach will not be acceptable for jack-ups operating in extreme environments.

(ii) **Structural modelling of the legs**: it is uneconomic in a dynamic analysis to model the lattice elements explicitly, so most analysts make use of equivalent beam models. The sophistication of these models varies widely, from simple linear beam theory to fully geometrically non-linear formulations, accounting for $P$-$\Delta$ and Euler effects.

(iii) **The degree of base fixity provided by the spudcan**: experimental research (e.g. Martin, 1994) has shown that the behaviour of spudcan footings under combined loads is complex and non-linear. In spite of this nearly all analysts use simple pinned supports to represent the foundations.

(iv) **The nature of the wave loading**: ocean waves are random in nature, but in order to provide engineering solutions the use of regular wave theories is common. There is a wide range of such theories, ranging from simple linear Airy theory to high order formulations and stream function theory.

In the past, many jack-up analyses have been very simple, often quasi-static. When more sophisticated analyses have been attempted, they have tended to go into great detail on one or two aspects (most commonly the structure and/or the wave loading) while retaining a very crude model of another aspect (particularly the foundations). The program presented here, entitled JAKUP, takes a more balanced approach by using reasonably sophisticated non-linear models for all of the major features of the problem. Because the model is fully non-linear, the solution is carried out in the time domain. The following sections detail the principal characteristics of the structural model, foundation model, wave loading and numerical solution techniques used. Lastly, some typical results are presented.
2. STRUCTURAL MODEL

2.1 Stiffness matrix

The hull of a jack-up is relatively stiff compared to the legs, so the structural model concentrates on the accurate description of the load-deformation characteristics of the legs. The triangular lattice legs are modelled by equivalent beam elements. A stiffness matrix based on simple linear beam theory is inadequate for these elements for several reasons:

- the presence of axial loads reduces the flexural stiffness of the legs;
- $P$-$\Delta$ effects in the legs are significant;
- shear can induce significant additional leg deformations.

The stiffness matrix used here is therefore derived using an Eulerian beam-column formulation (Oran, 1973) with an optional modification to account for shear deformations (Martin, 1994). This is developed in incremental form, so that the structural response is path-dependent. This formulation has two main advantages. Firstly, it allows arbitrarily large chord rotations because rigid-body rotations are uncoupled from local member deformations. Secondly, it is computationally efficient, since accurate solutions to large deformation problems can be obtained using relatively few beam-column elements.

Figure 2 shows a beam-column element of length $L$, cross-sectional area $A$, second moment of area $I$ and Young's modulus $E$, subjected to an axial load $Q$ and end moments $M_1$ and $M_2$, with corresponding deformations $\delta$, $\theta_1$ and $\theta_2$. Ignoring shear effects, the forces and deformations are related by:

\[
M_1 = \frac{EI}{L} (c_1 \theta_1 + c_2 \theta_2) \quad (1)
\]

\[
M_2 = \frac{EI}{L} (c_2 \theta_1 + c_1 \theta_2) \quad (2)
\]

\[
Q = EA (\delta / L - c_b) \quad (3)
\]

where the coefficients $c_1$ and $c_2$ and the variable $c_b$, which accounts for the change in length due to bowing, are functions of the ratio $q = Q/Q_E$, where $Q_E$ is the Euler buckling load. For a compression element ($q > 0$):

\[
c_1 = \frac{\phi \sin \phi - \phi \cos \phi}{2(1 - \cos \phi) - \phi \sin \phi} \quad , \quad c_2 = \frac{\phi (\phi - \sin \phi)}{2(1 - \cos \phi) - \phi \sin \phi} \quad , \quad \phi^2 = \pi^2 q \quad (4)
\]
and for a tension element \((q < 0)\):

\[
c_1 = \frac{\psi (\psi \cosh \psi - \sinh \psi)}{2(1 - \cosh \psi) + \psi \sinh \psi}, \quad c_2 = \frac{\psi (\sinh \psi - \psi)}{2(1 - \cosh \psi) + \psi \sinh \psi}, \quad \psi^2 = -\pi^2 q
\] (5)

With no axial load \((q = 0)\), \(c_1 = 4\), \(c_2 = 2\) and Equations (1) and (2) reduce to those of simple linear bending theory. Lastly, the bowing parameter is given by:

\[
c_b = \frac{(c_1 + c_2)(c_2 - 2)}{8\pi^2 q} (\theta_1 + \theta_2)^2 + \frac{c_2}{8(c_1 + c_2)} (\theta_1 - \theta_2)^2
\] (6)

For a non-linear time-stepping approach, a tangent stiffness matrix \(k_t\) is needed, relating the increments of force and deformation during a timestep. This is found by evaluating:

\[
k_{t,ij} = \frac{\partial S_i}{\partial u_j} + \frac{\partial S_i}{\partial q} \cdot \frac{\partial q}{\partial u_j} \quad i, j = 1, 2, 3
\] (7)

where \(S_1 = M_1, S_2 = M_2, S_3 = QL, u_1 = \theta_1, u_2 = \theta_2, u_3 = \delta/L\). The tangent stiffness matrix equation can therefore be found:

\[
\begin{bmatrix}
M_1 \\
M_2 \\
QL
\end{bmatrix} = \frac{EI}{L} \begin{bmatrix}
c_1 + \frac{G_1^2}{\pi^2 H} & c_2 + \frac{G_1 G_2}{\pi^2 H} & \frac{G_1}{H} \\
\frac{G_1 G_2}{\pi^2 H} & c_2 + \frac{G_2^2}{\pi^2 H} & \frac{G_2}{H} \\
\frac{G_1}{H} & \frac{G_2}{H} & \frac{\pi^2}{H}
\end{bmatrix} \begin{bmatrix}
\theta_1 \\
\theta_2 \\
\delta/L
\end{bmatrix}
\] (8)

where

\[
G_1 = \frac{\partial c_1}{\partial q} \theta_1 + \frac{\partial c_2}{\partial q} \theta_2
\] (9)

\[
G_2 = \frac{\partial c_2}{\partial q} \theta_1 + \frac{\partial c_1}{\partial q} \theta_2
\] (10)
\[ H = \frac{\pi^2 I}{AL^2} + \frac{\partial}{\partial q} \left[ \frac{(c_1 + c_2)(c_2 - 2)}{8\pi^2 q} \right] (\theta_1 + \theta_2)^2 + \frac{\partial}{\partial q} \left[ \frac{c_2}{8(c_1 + c_2)} \right] (\theta_1 - \theta_2)^2 \] (11)

If shear deformations are now also taken into account, their effect is to introduce additional end rotations, such that the total end rotations are now given by:

\[ \bar{\theta}_1 = \theta_1 + \theta_{\text{add}} \quad \bar{\theta}_2 = \theta_2 + \theta_{\text{add}} \quad \theta_{\text{add}} = \frac{M_1 + M_2}{A_s GL} \] (12)

where \( A_s \) is the shear area and \( G \) the shear modulus. The above formulation can be written in terms of these total rotations by modifying the expressions for the coefficients \( c_1 \) and \( c_2 \) and the bowing function \( c_b \) (Martin 1994). For example, Equation (1) can be written as:

\[ M_1 = \frac{EI}{L} (\bar{c}_1 \bar{\theta}_1 + \bar{c}_2 \bar{\theta}_2) \] (13)

where

\[ \bar{c}_1 = \frac{12c_1 + \beta (c_1^2 - c_2^2)}{12 + 2\beta (c_1 + c_2)} \quad \bar{c}_2 = \frac{12c_2 + \beta (c_2^2 - c_1^2)}{12 + 2\beta (c_1 + c_2)} \quad \beta = \frac{12EI}{A_s GL^2} \] (14)

The structural model does not at present include the significant non-linearities which arise from the details of the jack housings, and which are often modelled using bilinear or "gapping" elements.

2.2 Mass matrix

A consistent mass matrix is used, derived using shape functions based on simple linear beam theory. It would be possible to derive a time-varying mass matrix, including corrections to account for axial load and shear effects, but the increase in accuracy would be small, since the mass matrix is derived using only the shape functions themselves, and not their differentials. Additional mass terms due to water particle motions are included in the wave loading formulation.

2.3 Damping matrix

Structural damping is modelled using the Rayleigh approach (see e.g. Clough and Penzien 1975). The damping matrix is taken as a linear combination of the mass matrix and the initial elastic stiffness matrix:
\[ C = a_0 M + a_1 K \]  \hspace{1cm} (15)

from which it follows that the damping of mode \( r \) is related to its frequency by:

\[ \xi_r = \frac{1}{2} \left( \frac{a_0}{\omega_r} + a_1 \omega_r \right) \]  \hspace{1cm} (16)

Using Equation (16), the coefficients \( a_0 \) and \( a_1 \) can be chosen so as to set the damping to a desired value at two particular frequencies. Responses at frequencies outside this range will then be artificially heavily damped; this is useful as it allows unwanted higher mode responses to be filtered out from a problem which is dominated by the relatively low frequency surge and sway modes. Hydrodynamic damping effects are included in the wave loading formulation.

3. FOUNDATION MODEL

Most jack-up analyses use simple pinned supports to model the soil-structure interface (Reardon 1986, Freize et al 1995). However, in reality the foundation behaviour is considerably more complex. The footing has finite stiffnesses in the vertical, horizontal and rotational directions, cross-coupling exists between the horizontal and rotational degrees of freedom, and non-linearities may occur at large deformations. In the program JAKUP, therefore, options are included allowing for fully pinned or fixed behaviour, or for a more realistic footing model known as Model B, which includes cross-coupling and plasticity. The Model B formulation was developed by Martin (1994), based on an extensive programme of laboratory testing in clays.

3.1 Linear behaviour in Model B

Figure 3 shows the vertical, horizontal and rotational degrees of freedom for a typical spudcan footing. Extensive finite element work has shown that, for clays, the linear elastic force-displacement relationship has the form:

\[
\begin{pmatrix}
\Delta V \\
\Delta H \\
\Delta M/R
\end{pmatrix} =
\begin{bmatrix}
K_1 & 0 & 0 \\
0 & K_2 & K_4 \\
0 & K_4 & K_3
\end{bmatrix}
\begin{pmatrix}
\Delta z \\
\Delta h \\
R \Delta \theta
\end{pmatrix}
\]  \hspace{1cm} (17)

where the coefficients \( K_i \) depend on the embedment depth \( z_p \), and are taken from tabulated values given by Bell (1991). The coefficient \( K_4 \) represents the cross-coupling between the horizontal and rotational degrees of freedom.
3.2 The Model B work hardening plasticity model

The footing plasticity model has three essential components: (i) a yield surface definition in \( V: H: M/R \) space, (ii) a hardening law, describing how the size of the yield surface varies with embedment depth, and (iii) a flow rule, defining the relative magnitudes of the plastic displacement vectors during an elasto-plastic increment.

The yield surface is defined by the function:

\[
f(V, H, M/R) = \left( \frac{H}{H_0} \right)^2 + \left( \frac{M/R}{M_0/R} \right)^2 - 2 \left[ e_1 + e_2 \left( \frac{V}{V_0} \right) \left( \frac{V}{V_0} - 1 \right) \right] \left( \frac{H}{H_0} \right) \left( \frac{M/R}{M_0/R} \right) \
- \left[ \frac{(\beta_1 + \beta_2)^{\beta_1} + \beta_2^{\beta_2}}{\beta_1 \beta_2} \left( \frac{V}{V_0} \right)^{2\beta_1} \left( 1 - \frac{V}{V_0} \right)^{2\beta_2} \right] = 0 \quad (18)
\]

where \( V_0 \), \( H_0 \) and \( M_0 \) are the pure vertical, horizontal and moment load capacities respectively. This function is chosen as a best fit to experimental data. It forms a cigar-shaped surface in \( V: H: M/R \) space (Figure 4), but with rounded ends, so that the normal vector to the surface is uniquely defined at all points. A section through the surface is elliptical on any plane \( V = \text{constant} \) and approximately parabolic on any plane \( M/R = \text{constant} \). From the physical tests the best-fit parameters were found to be:

\[ H_0/V_0 = 0.127, \quad M_0/RV_0 = 0.166, \quad \beta_1 = 0.764, \quad \beta_2 = 0.882, \quad e_1 = 0.518, \quad e_2 = 1.18. \]

Using these values, the size of the yield surface is defined solely in terms of the pure vertical load capacity, \( V_0 \). The variation of \( V_0 \) with embedment depth \( z^p \) therefore constitutes a hardening law for the yield surface, and is defined using a bearing capacity factor approach.

Lastly, it is necessary to define a flow rule. The assumption of associated flow implies that the displacement increments are normal to the yield surface when the displacements are plotted coincident with the \( V: H: M/R \) axis system. It was found that associated flow modelled well the horizontal and rotational components but overestimated the vertical plastic footing displacement. This was therefore modified by the inclusion of a reduction factor \( \zeta \), giving:

\[
\begin{bmatrix}
\Delta z \\
\Delta h \\
R \Delta \theta
\end{bmatrix}
= u^p
\begin{bmatrix}
\zeta \frac{\partial f/\partial V}{\partial f/\partial H} \\
\frac{\partial f/\partial (M/R)}{\partial f/\partial H}
\end{bmatrix}
\]

where \( u^p \) is the magnitude of the plastic displacement increment.
4. ENVIRONMENTAL LOADING MODELS

Environmental loads on offshore structures are caused by waves, current and wind. The latter two vary slowly compared to the natural periods of jackup oscillations (SNAME, 1993) and are treated as steady phenomena. Waves, however, occur at periods likely to excite significant dynamic response. Their time-dependent properties must therefore be modelled realistically.

4.1 Wave kinematics

Exact solutions of the equations of wave kinematics are difficult to achieve since they require the application of boundary conditions at the free surface, the exact form of which is itself unknown until the equations have been solved. A wide variety of approximate solutions have therefore been developed, and three alternative approaches have been adopted in JAKUP: linear Airy theory, linear Airy theory with Wheeler stretching, and Stokes’ fifth order theory. These are outlined briefly in this section; the coordinate system and notation used are defined in Figure 5.

The simplest of the three approaches is the linear theory of Airy (1845), in which \( \eta \), the \( z \)-coordinate of the free surface, is given by:

\[
\eta(x,t) = \frac{H}{2} \cos \alpha
\]  

(20)

where

\[
\alpha = 2\pi \left( \frac{x}{L} - \frac{t}{T} \right)
\]  

(21)

This leads to the following expressions for horizontal water particle velocity and acceleration:

\[
u(x,z,t) = \frac{\pi H}{T} \frac{\cosh k(z + d)}{\sinh kd} \cos \alpha
\]  

(22)

\[
\ddot{u}(x,z,t) = \frac{2\pi^2 H}{T^2} \frac{\cosh k(z + d)}{\sinh kd} \sin \alpha
\]  

(23)

where \( k \) is the wave number, given by:

\[
k = \frac{4\pi^2}{g T^2 \tanh (2\pi d/L)}
\]  

(24)
Linear Airy theory is based on the simplifying assumptions that the wave heights are small compared to both the wavelength and the depth, and that the normal to the free surface is vertical. Its use is therefore restricted to waves of low height (SNAME, 1993). The theory is not valid above the mean water level, but is nevertheless often extrapolated into this region. This results in an overprediction of wave kinematics in the crest, though in deep waters the difference in velocity prediction between linear and higher order theories may be small.

A number of empirical corrections are available to linear theory, of which one of the most widely used is Wheeler stretching. This involves a simple alteration to Equations (22) and (23), with the depth decay term \( \cosh k(z + d) \) replaced by \( \cosh \left( \frac{k(z + d)}{1 + \eta/d} \right) \). This has the effect of lowering crest velocities and raising trough velocities.

In addition to these comparatively simple approaches, one higher order method is also implemented. Stokes' finite order wave theories (Stokes, 1847) are derived by substituting Taylor series approximations for the variables in the free surface boundary conditions; the order of solution depends on the number of Taylor series terms included. The use of fifth order solutions (Skjelbreia and Hendrickson 1960, Bartrop and Adams 1991) is now quite common. These take the general form:

\[
\eta(x, t) = \sum_{n=1}^{5} A_n \cos nk(x - \bar{c}t) \tag{25}
\]

\[
u(x, z, t) = \sum_{n=1}^{5} B_n \cosh nk(z + d) \cos nk(x - \bar{c}t) \tag{26} \]

\[
u(x, z, t) = \sum_{n=1}^{5} C_n \cosh nk(z + d) \sin nk(x - \bar{c}t) \tag{27} \]

where \( \bar{c} \) is the wave celerity and the coefficients \( A_n, B_n \) and \( C_n \) are functions of \( k, \bar{c} \) and \( d/L \). Stokes' fifth order wave theory is suitable for modelling a substantial range of wave heights in deep water, but breaks down in shallow water.

The modelling of the wave kinematics is considered the least satisfactory aspect of the analysis. Work is in progress on the incorporation of random sea states through the use of the New Wave approach (Tromans et al, 1991).
4.2 Wave forces

Having established the wave particle kinematics, the resulting loads on the jackup legs are determined using the extended Morison equation, which takes account of relative motions between the fluid and the leg:

\[ F(x,z,t) = \frac{1}{2} C_d \rho D (u - v) |u - v| + C_m \rho A \dot{u} - (C_m - 1) \rho A \dot{v} \] (28)

Here \( C_d \) and \( C_m \) are the damping and inertia coefficients, \( D \) the member diameter, \( A \) its cross-sectional area, \( \rho \) the fluid density and \( v \) the structural velocity of the leg. If there is current present in addition to waves, then \( u \) is the sum of the wave and current velocities.

5. DYNAMIC ANALYSIS

Two dynamic analysis options are available in JAKUP: the well-known Newmark \( \beta = \frac{1}{4} \) method (Newmark, 1959), in which accelerations are essentially assumed to be constant over a timestep, and the linear acceleration method (Clough and Penzien, 1975), in which the acceleration is taken to vary linearly during a timestep. The advantages of these approaches are that they are both single-step methods, they do not suffer from overshoot, and they are relatively efficient for problems involving non-diagonal mass and damping matrices.

The choice between the two methods involves considerations of stability and accuracy. The Newmark \( \beta = \frac{1}{4} \) method is unconditionally stable, so that the choice of timestep is governed by accuracy requirements. The linear acceleration method is conditionally stable, requiring a short timestep to ensure stability. However, so long as it is operating within its stability limit, it is more accurate than the Newmark method, which can introduce significant period elongation. The linear acceleration method is therefore preferred in analyses where all modal responses are required to a high degree of accuracy. When response in the higher modes is not required to a very high degree of accuracy, the Newmark \( \beta = \frac{1}{4} \) method is used with an appropriate choice of timestep.

6. SAMPLE RESULTS

In this section some typical results from the program JAKUP are presented and discussed. The analyses performed all relate to the simple, plane idealisation of a representative jackup rig shown in Figure 6. The stiffness matrix formulation for the rig includes shear effects. All the results shown are from dynamic analyses of the rig under combined wind, wave and current loading, with the wave kinematics modelled using Stokes' fifth order theory and numerical integration performed by the Newmark \( \beta = \frac{1}{4} \) method. The footing model and wave characteristics vary between the analyses.

Figure 7(a) shows the time history of lateral hull displacement for a rig with pinned feet. The approximate fundamental natural period of non-linear oscillations for this rig is 8.3 s (i.e. a natural frequency of 0.12 Hz). The rig is subjected to a wave of height \( H = 13 \text{ m} \) and period
$T = 17 \text{ s}$ (frequency 0.059 Hz). The response shows oscillations at two predominant frequencies about a mean lateral displacement of approximately 0.4 m.

The dual frequency nature of the displacement response can be even more clearly seen by plotting its spectrum, Figure 7(b), and can be explained by reference to the spectrum of the hydrodynamic loading, Figure 7(c). This shows that the combination of wave and current velocities in Morison's equation gives a loading function with most of its energy concentrated at the wave frequency (0.059 Hz), but also with a significant second harmonic (0.118 Hz). In this particular loadcase, the second harmonic is very close to the rig natural frequency, so that a large resonant response occurs at that frequency.

Figure 8 shows two displacement time histories for the same rig but with Model B footings, resulting in a fundamental natural period of 6.6 s (natural frequency 0.15 Hz). Figure 8(a) shows the response to a wave of height 25 m and period 16 s (frequency 0.063 Hz). It can be seen that the hull displacement quickly reaches a steady state and does not continue to increase with each cycle of loading. It can be concluded that, for this particular case, yielding at the foundation occurs in a stable manner, with the failure envelope expanding due to the increase in vertical footing penetration.

This can be contrasted with the time history shown in Figure 8(b), in which the wave height has been increased to 30 m, with all other parameters unchanged. Within the timescale shown the footing does not reach a steady state and can be considered an unstable foundation for this wave condition.

The results of an extensive parametric study using the model described here are given by Williams et al (1997).

7. CONCLUSIONS

Several of the analytical methods used in the study of offshore structures have been brought together to create the advanced plane frame analysis program JAKUP. An important aspect of the program is that each element of the analysis has been developed to approximately the same level of rigour.

The feasibility of incorporating a work-hardening plasticity model for soil-structure interaction into a dynamic analysis has been demonstrated. This approach results in a more realistic prediction of soil-structure interaction than analyses assuming pinned or fixed footings. One of the major advantages of such a model is that a direct indication of yield at the foundation is given.
REFERENCES


NOTATION

$A$  cross-sectional area
$A_s$  shear area
$C$  damping matrix
$C_d$  drag coefficient
$C_m$  inertia coefficient
$ar{c}$  wave celerity
$D$  diameter
$d$  mean water depth
$E$  Young's modulus
$G$  shear modulus
$H$  horizontal load on footing, or wave height
$h$  horizontal footing displacement
$I$  second moment of area
$K$  initial elastic stiffness matrix
$k_t$  tangent stiffness matrix
$k$  wave number
$L$  beam element length, or wavelength
$M$  mass matrix
$M$  moment on footing
$M_1, M_2$  end moments in beam-column element
$Q$  axial load in beam-column element
$Q_E$  Euler buckling load
$q$  $Q/Q_E$
$R$  radius of spudcan footing
$T$  wave period
$u$  water particle velocity
$V$  vertical load on footing
$\nu$  structural velocity
$z$  vertical footing displacement
$\varepsilon^p$  footing plastic penetration
$\delta$  axial displacement of beam-column element
$\eta$  vertical coordinate of wave surface
$\theta$  footing rotation
$\theta_1, \theta_2$  end rotations in beam-column element
$\xi$  damping ratio
$\rho$  water density
FIGURE 1. A typical three legged jack-up unit
FIGURE 2. End forces and displacements in a beam-column element

FIGURE 3. Forces and displacements on a spudcan footing
FIGURE 4. Cigar-shaped yield surface

FIGURE 5. Regular wave coordinate system
Single leg $EI = 3.0 \times 10^6 \text{ MNm}^2$  Single leg $AE = 120.0 \times 10^3 \text{ MN}$
Deck mass = $16.1 \times 10^6 \text{ kg}$  Single leg mass = $1.93 \times 10^6 \text{ kg}$

**FIGURE 6.** Representative jack-up model
(a) Hull displacement time history

(b) Amplitude spectrum for hull displacement

(c) Amplitude spectrum for hydrodynamic load

FIGURE 7. Response of a model with pinned feet to a wave with height 13m, period 17s
FIGURE 8. Hull displacement time histories for model with Model B footings subjected to a wave with period 16s.

(a) Wave height of 25m

(b) Wave height of 30m