ANALYSIS OF MEMBRANE ACTION IN REINFORCED UNPAVED ROADS

by

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Report No. OUEL: 2035/94

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Abstract
Polymer grid or geotextile reinforcement may be used to improve the performance of reinforced fill layers placed on soft ground. This paper is concerned with the mechanics and design of reinforced unpaved roads built over soft clay which is a particular application of this reinforced soil technique. A discussion is given of the mechanics of reinforced unpaved roads for the case of a single application of a plane strain, monotonic, load and the design procedures that are currently available for this type of structure are reviewed. A new analytical design model is proposed. This new model is based on a membrane reinforcement mechanism and is appropriate for cases where large surface deformations are acceptable. Results obtained using this new model are shown to compare well with data obtained from previously published laboratory tests. The use of a finite element method to study this type of structure is described and the results of finite element analysis are used to discuss the accuracy of the proposed analytical model.

Key words: Soil reinforcement, unpaved roads, membrane, finite elements, reinforcement mechanisms, foundations.

Introduction
The structural performance of fill layers compacted over soft clay may often be substantially improved by placing a layer of geotextile or geogrid reinforcement on the surface of the clay prior to placement of the fill (Fig. 1). This reinforced soil technique is routinely used for a variety of applications including working platforms, parking areas and low cost unpaved haul roads. The various mechanisms of reinforcement that act in structures of this sort, however, are complex and not fully understood. Current design methods, particularly those that are based on assumptions about the mechanics of individual reinforcement mechanisms, are therefore considered to be not wholly satisfactory.

In many applications of this type of construction, the surface of the fill layer is subjected to repeated loading caused, for example, by passing vehicles. It is generally found in practice
that the deformations occurring in the reinforced unpaved road tend to increase in magnitude as the number of load applications increases. The mechanism of this repeated loading degradation is not fully understood, even for the more conventional case of unreinforced fill layers on soft clay. In view of this, many of the current design methods for reinforced fill layers are based on an approach in which the structure is analysed for the case of a single load application with an empirical correlation used to deal with the effects of repeated loading.

Previous research on the analysis and design of reinforced unpaved roads has generally followed one of two broad approaches. One of these approaches is to use an analytical model to represent one, or more, of the reinforcement mechanisms that may be assumed to act within the system. This approach relies on the use of important assumptions about the nature of the mechanisms concerned and may result in significant modelling errors if these assumptions are incorrect or inaccurate. The models developed using this approach generally require the specification of parameters relating to the assumed reinforcement mechanisms; appropriate values for these parameters may be difficult to determine in advance for a particular structure. Typical examples of design procedures developed using this general approach are the membrane analysis models proposed by Giroud and Noiray (1981), Sellmeijer et al. (1982) and Bourdeau (1989), and the small deformation models proposed by Houlsby et al. (1989), Milligan et al. (1989), Houlsby and Jewell (1990) and Sellmeijer (1990).

The second of these two broad approaches is to use a numerical method to formulate, and solve, the compatibility, equilibrium and constitutive equations for the complete system. This type of procedure is generally based on the use of a suitable finite element method. Although this class of calculation is often complex and lengthy to perform, this general approach has the important advantage that it is necessary only to specify the geometry of the structure and values of the appropriate material parameters. No assumptions are necessary about the detailed nature of the reinforcement mechanisms and appropriate material properties may be determined using suitable laboratory tests or empirical correlations. Typical analyses of this sort are described by Zeevaert (1980), Poran (1985), Burd, (1986), Burd and Houlsby (1989), Burd and Brocklehurst (1991) and Brocklehurst (1993). It is important to note that in order to model successfully the large deformation mechanisms that are known to act in this type of system, it is necessary to base the finite element analysis on a suitable large deformation
formulation. This adds a certain degree of complexity to the finite element equations and solution procedures, but is a feature that is available in several of the finite element programs that are currently available.

Although finite element analysis techniques are becoming more accessible to practising engineers, practical design calculations are likely to continue to be based on the use of relatively simple analytical models of behaviour. Finite element solutions will continue to have significant value, however, in investigating the mechanisms that operate in reinforced soil systems of this type and in checking proposed design models.

The purpose of this paper is to discuss the reinforcement mechanisms that act in reinforced unpaved roads when a single monotonic load is applied. A new analytical model of a reinforced unpaved road deforming in plane strain is also proposed. This model is based on the analysis of a reinforcement mechanism that requires large geometry changes to occur to become mobilised. Design procedures based on this model are therefore appropriate for cases where significant deformations are acceptable as, for example, might be the case in a temporary or low cost haul road. Although no attempt is made to develop a theory suitable for the case of repeated loading, this proposed model may be readily combined with existing empirical approaches (for example the procedures described by Giroud and Noiray (1981) and De Groot (1986)) for the design of roads subjected to repetitive loads.

**Mechanisms of reinforcement in reinforced unpaved roads**

It is thought that the structural mechanisms that act in reinforced unpaved roads may be conveniently divided into two broad groups. One of these groups (referred to here as the group of 'small displacement mechanisms') does not require large geometric changes to occur within the structure in order to be mobilised. Reinforcement mechanisms in the second group, (described here as 'large displacement mechanisms') do, however, require significant structural displacements in order to become active. In addition to these two groups of mechanisms, geotextiles and geogrids may provide a useful separation function; this is particularly the case when the subgrade is very soft. This function is especially important in cases where the loading is repeated since it reduces the problem of subgrade degradation caused by the punching of individual fill particles into the clay.

One of the small displacement mechanisms that has been identified by previous workers is associated with the shear stresses that are induced at the base of the fill layer by a load
applied to the fill surface. In an unreinforced unpaved road, the fill layer transmits shear stresses as well as normal stresses to the clay subgrade; these shear stresses tend to reduce the vertical bearing capacity of the subgrade. If a layer of reinforcement is placed at the base of the fill, however, then a proportion of these shear stresses is sustained directly by the reinforcement. This generates tensile forces in the reinforcement and a corresponding reduction in the outward acting shear stresses applied to the subgrade surface. The amount by which the shear stresses transmitted to the subgrade are reduced is thought to depend on the relative stiffnesses of the reinforcement and the surrounding soil. It has also been suggested by previous workers that the presence of reinforcement may improve the load-spread characteristics of the fill layer and that this may constitute another small displacement mechanism. There appears to be little evidence to support this suggestion, however.

The second broad group of mechanisms require large deformations to occur before they become active. It is generally accepted that the most important of these large deformation mechanisms is associated with the membrane action of the reinforcement. This membrane mechanism is associated with the curvatures that develop within the reinforcement when the magnitude of the deformations becomes large. When tensile forces are present in the reinforcement and coincident with appreciable reinforcement curvature, then the normal stresses acting on each side of the reinforcement will be unequal. This effect tends to reduce the normal stresses transmitted to the subgrade immediately underneath the load with a consequential improvement in the capacity of the road. The reinforcement curvatures may also tend to provide additional vertical stresses to the surface of the clay in the area immediately outside of the loaded area. This has the effect of increasing the bearing capacity of the clay in the area beneath the load thus providing further improvement.

**Design methods**

One of the earliest design methods for reinforced unpaved roads in which an attempt was made to develop a detailed model of the interaction between the reinforcement and the soil was proposed by Giroud and Noiray (1981). Although this model is based on assumptions that simplify the behaviour of the system, it has been most successful in stimulating interest in design methods for this type of structure and is extensively cited in later work. This design model, which is based on the analysis of a membrane reinforcement mechanism, deals with the interaction that occurs between two wheel loads and makes the implicit assumption that
the clay subgrade behaves in a rigid-perfectly plastic manner. The reinforcement is assumed to be a linearly elastic sheet of material placed at the base of the fill.

The Giroud and Noiray method, in common with most of the other proposed analytical models of this type of structure, makes use of a simple load-spread mechanism for the fill in which load applied at the surface is assumed to be uniformly distributed over an area at the fill base defined by a load-spread angle, $\beta$ (Fig. 2). In this type of approach, the value of $\beta$ needs to be specified at the start of the analysis. It is unfortunately the case that the result of any prediction made using this type of load-spread model is sensitive to the value of $\beta$ that is adopted and yet it is often unclear how an appropriate value of $\beta$ should be chosen for a particular application. It is clearly to be expected that an appropriate value of $\beta$ will depend on the quality of the fill itself. There is some evidence to suggest (for example the results presented by Meyerhof (1974), the laboratory studies described by Love (1984) and Love et al. (1987) and finite element studies discussed by Brocklehurst (1993)), however, that the strength of the clay may also have an important influence on the mechanism of load-spread within the fill layer with the magnitude of $\beta$ tending to be greater for soft than firm clays.

In the Giroud and Noiray (1981) design method, the shape of the reinforcement is approximated by three parabolas as shown in Fig. 3 where the points of zero vertical displacement (points X and Y) correspond to the edges of the loaded area at the base of the fill. The displacement of the wheel at the fill surface is assumed to be equal to the displacement of the reinforcement immediately beneath the wheel centre-line, $\Delta$. The mean reinforcement strain, corresponding to a particular value of $\Delta$, is obtained from geometrical considerations by making certain assumptions about the fixity of the reinforcement at points X and Y. Finally, the contribution of the reinforcement force to the strength of the system is assessed by considering the equilibrium of the portion of reinforcement immediately beneath the wheels. The assumptions made about reinforcement fixity in this method lead to a model that may predict an excessively stiff response (Burd 1986), particularly for the case of stiff reinforcement and large rut depths.

Several other analyses of membrane mechanisms of reinforcement have been proposed. Sowers et al. (1982), for example, suggest a method of analysis for a single wheel that is in many respects similar to that of Giroud and Noiray (1981). Sellmeijer et al. (1982) propose an alternative approach in which a set of normal stresses acting at the reinforcement interface is assumed and membrane equations formulated and solved to obtain the corresponding
deformed shape. The reinforcement boundary condition adopted in this case is zero horizontal displacement at the edge of the road. While this boundary condition may be appropriate if the reinforcement is indeed pinned at the road edge, it may in other cases lead to a spurious predicted dependence of performance on the width of the structure.

The model of membrane action described by Bourdeau (1989) represents an important development in the analysis of this type of system. In this model, an assumed set of stresses is applied to the reinforcement and a solution is obtained to the appropriate membrane equations. This approach differs from that of Sellmeijer et al. (1982) in the important respect that the soil is assumed to apply shear stresses to the reinforcement in addition to normal stresses. This leads to the possibility of the use of a compatibility condition in which the change of reinforcement length associated with gross geometry changes is equated to the reinforcement extension caused by the assumed shear stresses acting at the soil-reinforcement interface. This compatibility condition avoids the need to choose an arbitrary boundary condition for the reinforcement tension and therefore removes one of the main difficulties of the earlier methods. This particular model, however, is developed specifically for the case of an elastic subgrade and might, for example, be suitable to model the behaviour of reinforced fill layers on peat. The assumptions adopted in this model lead to a relatively complex set of equations that require the use of a numerical method in order to obtain a solution.

Few authors have proposed design models based on small displacement mechanisms of behaviour. One model of this sort, however, is described by Houlshby et al. (1989) and Milligan et al. (1989) for a plane strain analysis and by Houlshby and Jewell (1990) for an axisymmetric geometry. The broad assumptions on which this model is based are illustrated in Fig. 4 for the case of a single footing applied to the fill surface. If a load is applied to the footing, then the corresponding vertical stresses, $\sigma_v$, at the surface of the subgrade are assumed to be distributed according to a conventional load-spread mechanism. The fill immediately beneath the footing is assumed to be in a state of active failure and the fill immediately adjacent to the footing is assumed to be in a passive failure state. Equilibrium considerations are used to evaluate the value of the resulting shear stress, $\tau_f$, that is developed at the base of the fill. In the unreinforced case (Fig. 4(a)), the magnitude of $\sigma_v$ necessary to cause failure is calculated using plasticity solutions in which the tendency of the outward acting shear stress to reduce the subgrade bearing capacity is included. In the reinforced case (Fig. 4(b)), the shear stresses at the base of the fill are assumed to be sustained by the
reinforcement and the shear stresses applied to the clay surface are assumed to be zero. It is possible in this case to estimate the value of reinforcement force, $T_s$, acting on the footing centre-line by considering horizontal equilibrium of the reinforcement.

**A model of membrane behaviour**

An attempt is described below to develop a new model for membrane action in reinforced unpaved roads; the model is intended to be straightforward to use and based on realistic assumptions about the mechanics of the system. The starting point for the analysis is the assumption of a set of assumed stresses acting on the reinforcement. A set of membrane equations are then solved to obtain the deformed reinforcement shape. A compatibility condition, similar to that proposed by Bourdeau (1989), is used to obtain the width of reinforcement for which the tension is non-zero. The proposed analysis differs from that described by Bourdeau (1989) in that a conventional load-spread model is adopted for the fill and a rigid-cohesive (rather than an elastic) model is used for the subgrade. These assumptions lead to a mathematical model that is relatively straightforward to use in design.

Consider a footing, of width $2B$, acting on the surface of a plane strain reinforced unpaved road as shown in Fig. 5(a). A sheet of thin elastic reinforcement of total width $2L_r$ is placed symmetrically about the footing centre-line at the base of the fill. The vertical stresses associated with the load, $P$, applied to the footing are assumed to be spread uniformly over a width $2B'$ at the base of the fill where the value of $B'$ is calculated using a suitable load-spread model. The value of $B'$ is assumed to be independent of footing displacement. A uniform normal reaction, $N_s s_u$, where $N_s$ is an appropriate bearing capacity factor and $s_u$ is the plane strain undrained shear strength of the clay subgrade, is applied by the clay to the base of the reinforcement for $0 < x < b$ in addition to the vertical stress required to balance the self-weight of the fill (Fig. 5(b)). Note that this analysis is concerned with the additional stresses acting at the reinforcement-soil interface caused by the footing load. For small displacements this approach is exact since the effect of the fill self-weight on the clay bearing capacity is neutral. Although this strictly ceases to be the case for large values of footing displacement, it will be assumed that the self-weight of the fill has a negligible influence on both the bearing capacity of the subgrade and the normal stresses tending to cause failure.
At small values of footing displacement (i.e. when the membrane mechanism is not active) the value of the applied load, $P_o$, is given by

\[ P_o = 2B'N_s\]  

It is shown later that this membrane model may be conveniently cast in a form in which the load-spread behaviour of the fill is specified by an appropriate value of $P_o$. This allows users of the analysis complete freedom in the choice of model used to describe the small displacement performance of the system.

It is assumed that relative slip occurs between the soil and the reinforcement for $b < x < x_o$ and that over this region a uniform shear stress, $\tau_r$, acts on the reinforcement (Fig. 5(b)) where

\[ \tau_r = \alpha s_u + \gamma D\tan\phi_r \]  

The term $\alpha s_u$ represents the adhesion at the reinforcement-clay interface where $\alpha$ is a constant in the range $0 < \alpha < 1$. The parameter $\phi_r$ is the angle of friction between the fill and the reinforcement. The variable $x_o$ is an important feature of the analysis and defines the ‘active half-width’ of the reinforcement, a feature of the system previously identified by Bourdeau (1989). It is assumed that the fill is sufficiently stiff for the vertical displacement of the reinforcement on the footing centre-line, $\Delta$, to be equal to the footing displacement. This assumption is shown by Palmeira and Cunha (1993) to be consistent with a range of published experimental and numerical results.

An element of the reinforcement of length $ds$ subjected to a net normal stress $\sigma$ and shear stress $\tau$ is shown in Fig. 6. Resolving forces parallel and perpendicular to the reinforcement gives

\[ \tau = -\frac{dT}{ds} \]  

\[ \sigma = T\frac{d\theta}{ds} \]  

where $T$ is the reinforcement tension and $\theta$ is the local inclination of the reinforcement to the horizontal.
The value of the normal stress, $\sigma_u$, applied to the top surface of the reinforcement (see Fig. 5(b)) is assumed to be constant for $0 < x < B'$. The load applied to the footing, $P$, is related to the value of $\sigma_u$ by

$$\text{[5]} \quad P = 2 \int \sigma_u \cos \theta \, ds$$

where the integral is taken over the portion of the reinforcement for $0 < x < B'$. Since $ds \cos \theta = dx$, [5] may be integrated to give

$$\text{[6]} \quad P = 2B' \sigma_u$$

If the assumption is made that the bearing stress in the clay is applied in a direction normal to the reinforcement, then a similar analysis may be used to show that

$$\text{[7]} \quad P = 2N_c s_u b$$

Equation [3] may be integrated over the range $b < x < x_o$ to give the distribution of reinforcement tension

$$\text{[8]} \quad T(x) = T_o \frac{(x_o - x)}{(x_o - b)} \quad \begin{cases} 0 < x < b \\ b < x < x_o \end{cases}$$

where

$$\text{[9]} \quad T_o = \tau_o (x_o - b)$$

The deformed shape of the reinforcement for a specified value of footing displacement may be determined as described below. If the slope of the reinforcement is small then [4] may be approximated to give

$$\text{[10]} \quad \sigma = T \frac{d^2y}{dx^2}$$

where $y$ is the vertical displacement of the reinforcement. For $0 < x < B'$, (i.e. the region
immediately beneath the footing), [10] may be written as

\[ T_o \frac{d^2 y}{dx^2} = \frac{P}{2B'} - N_c s_u \]

This equation may be integrated to give

\[ y = \left( \frac{P - 2N_c s_u B'}{4B'T_o} \right) x^2 - \Delta \]

where the slope of the reinforcement is set to zero for \( x = 0 \). For the portion of reinforcement for \( B' < x < b \), [10] may be used to derive the expression:

\[ T_o \frac{d^2 y}{dx^2} = -N_c s_u \]

This equation may be integrated, choosing constants of integration to ensure continuity of slope and displacement at \( x = B' \), to give

\[ y = -\frac{N_c s_u x^2}{2T_o} + \frac{P_x}{2T_o} - \left( \frac{PB'}{4T_o} + \Delta \right) \]

It is now assumed that at \( x = b \) the vertical displacement of the clay surface is zero in which case, from [14], the displacement on the footing centre-line is shown to be

\[ \Delta = \frac{P}{8N_c s_u T_o} (P - P_o) \]

where \( P_o \) is given by [1]. The final stage in the analysis is to find the value of \( x_o \) by equating the change of reinforcement width associated with the change of geometry and the width increase compatible with the strains associated with the assumed shear stresses. Two possible cases exist, the 'fully-reinforced' case where the tension falls to zero within the available width of the reinforcement and the 'under-reinforced' case where the value of \( x_o \) is set equal to the half-width of the reinforcement, \( L_r \). The length of reinforcement for
$0 < x < x_o$, calculated from the reinforcement strains, is

$$L^* = x_o + \int \frac{T(x)}{J} \, ds$$

where $J$ is the reinforcement stiffness, $T(x)$, the reinforcement tension, is given by [8] and the integral is evaluated over the range $0 < x < x_o$. If the slope is assumed to be small, then $ds = dx$ and [16] may be integrated to give

$$L^* = x_o + \frac{T_o}{2J} (b + x_o)$$

The length of reinforcement for $0 < x < x_o$ consistent with the deformed shape is

$$L^* = \int_0^x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

If the slope is again assumed to be small then this equation may be approximated with little error to give the expression

$$L^* = \int_0^x 1 + \frac{1}{2} \left(\frac{dy}{dx}\right)^2 \, dx$$

which is evaluated to give

$$L^* = x_o + \frac{2\Delta^2}{3b}$$

Combining [15], [17] and [20], and using [9] to substitute for $T_o$ gives the quadratic
expression

$$\left(\frac{x_o}{b}\right)^4 - 2\left(\frac{x_o}{b}\right)^3 + 2\left(\frac{x_o}{b}\right)^2 - \left(1 + \frac{2J[P - P_o]^3}{3P^3\tau_r}\right) = 0$$

Equations [9] and [15] may be combined to give the expression

$$\Delta = \frac{P(P - P_o)}{8N_c S_u \tau_r (x_o - b)}$$

In order to obtain the value of $x_o$ for use in [22], it is necessary to find the appropriate root of [21]. A range of numerical procedures are available for the solution of equations of this type and these are described in many of the standard texts. If the solution for $x_o$ obtained from [21] exceeds the half-width of the reinforcement, $L_r$, then the system is under-reinforced and the value of $x_o$ in [22] should be set to $L_r$.

It is possible to recast the equations in a different way in order to obtain a value for $\Delta$ directly. For the fully-reinforced case, the various relevant equations may be combined to show that

$$\left(\frac{\Delta}{b}\right)^4 - \frac{3(P - P_o)}{8J}\left(\frac{\Delta}{b}\right) - \frac{3N_c S_u [P - P_o]^2}{32JP \tau_r} = 0$$

When the value of $\Delta$ has been obtained from [23], the corresponding value of $x_o$ may be calculated from a rearranged form of [22]

$$\frac{x_o}{b} = \frac{P - P_o + 4\tau_r \Delta}{4\tau_r \Delta}$$

If the value of $x_o$ calculated in this way is found to exceed $L_r$, then the value of $\Delta$ should be re-calculated using [22] taking $x_o$ equal to $L_r$.

**Example analysis**

As an example of the application of this model, consider a reinforced road consisting of fill of unit weight 20 kPa/m and depth 0.5 m placed on a uniform clay subgrade of shear strength 40 kPa. Reinforcement of stiffness 500 kN/m and half-width 2 m for which the coefficients $\alpha$ and $\phi_r$ may be assumed to be 0.9 and 20° respectively is placed at the base of the fill. A
value of load-spread angle, $\beta$, of $\tan^{-1}0.5$ is assumed. If a footing of half-width 0.25m is placed on the surface of the fill then the appropriate value of $P_o$ calculated from [1] is 206 kN/m, taking $N_c = \pi + 2$. If a load of 240 kN/m is applied to the footing then from [7] the corresponding value of $b$ is 0.583 m.

In this case, [23] is found to have two real and two complex roots

$$\left( \frac{\Delta}{b} \right) = 0.341, \quad -0.0906 \pm 0.282i, \quad -0.160$$

where $i = \sqrt{-1}$. Since $\Delta$ must be both real and positive, the appropriate solution is $\frac{\Delta}{b} = 0.341$ from which $\Delta$ is calculated to be 0.199 m. The active half-width of the reinforcement, $x_o$, is found from [24] to be 1.22 m which is less than the half-width of reinforcement provided and so the system is fully-reinforced.

Comparison with model tests

A detailed set of monotonic loading model tests carried out on plane strain reinforced unpaved roads is described by Love (1984) and Love et al. (1987). A comparison between the results of these model tests and predictions made using the proposed analytical model has been carried out in order to assess the accuracy of the assumptions made in the proposed model. This comparison is described below.

The model tests were carried out at one-quarter geometric scale using a consolidated kaolin subgrade and a compacted granular fill; the test arrangement is shown in Fig. 7. The reinforcement used in the tests was a scaled down version of a proprietary geogrid. Three different nominal clay strengths (6 kPa, 9 kPa and 13 kPa corresponding to prototype values of 24 kPa, 36 kPa and 42 kPa respectively) and three values of fill thickness (50 mm, 75 mm and 100 mm) were adopted. In order to proceed with this comparison it is necessary to choose appropriate values of $P_o$ (or, alternatively, $\beta$) for each of the tests. In the absence of a detailed model of load-spread within the fill, values of $P_o$ used in the back-analysis were taken from data given by Houlsby et al. (1989); these data were obtained by visual inspection of the experimental footing pressure-footing displacement curves. Values of subgrade shear strength measured in each of the tests are also listed by Houlsby et al. (1989) and adopted in these comparative calculations.
<table>
<thead>
<tr>
<th>Analytical prediction ref.</th>
<th>D (mm)</th>
<th>$s_u$ (kPa)</th>
<th>$P_o$ (kN/m)</th>
<th>$\beta$</th>
<th>Model test ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>50</td>
<td>5.9</td>
<td>3.68</td>
<td>25°</td>
<td>m1</td>
</tr>
<tr>
<td>a2</td>
<td>50</td>
<td>8.9</td>
<td>5.63</td>
<td>26°</td>
<td>m2</td>
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<tr>
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<td>15.2</td>
<td>8.33</td>
<td>18°</td>
<td>m3</td>
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<tr>
<td>a4</td>
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<td>5.8</td>
<td>5.63</td>
<td>30°</td>
<td>m4</td>
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<td>100</td>
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<td>7.35</td>
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<td>m5</td>
</tr>
<tr>
<td>a6</td>
<td>100</td>
<td>13.5</td>
<td>9.53</td>
<td>17°</td>
<td>m6</td>
</tr>
</tbody>
</table>

$\alpha = 0.5$, $J = 28$ kN/m, $\phi_s = 20^\circ$, $B = 37.5$ mm, $\gamma = 19$ kPa/m

TABLE 1. Parameters used for analytical predictions

Six of the tests described by Love et al. (1987) are selected for comparison with the proposed model; the fill thicknesses, subgrade strengths and values of $P_o$ appropriate for these tests are listed in Table 1. Also shown in Table 1 are the values of $\beta$ corresponding to the specified values of $P_o$. It is interesting to note the marked variability in $\beta$, in particular the tendency for $\beta$ to reduce in value as the clay strength is increased. Values of $\alpha$ of 0.5 and $\phi_s$ of 20° are thought to be appropriate to model the soil-reinforcement interface in the analysis.

Comparison between the footing pressure - footing displacement responses obtained from the model tests (indicated by the reference codes m1 to m6) and the proposed analytical model (indicated by the reference codes a1 to a6) is shown in Fig. 8. At small values of footing displacement the analytical model predicts an excessively stiff response; this is to be expected since the model does not include the effects of elastic soil deformation. At larger displacements, however, the analytical model is seen to provide a good fit with the test data.

Measurements of reinforcement tension were not made during these laboratory tests and so direct comparison cannot be made with values calculated using the proposed model. It is desirable, however, to determine whether the nature of the reinforcement tensions and soil-reinforcement interface stresses calculated using the proposed analytical model are realistic.
In order to obtain a suitable set of independent data for comparison with the proposed analytical model, a finite element back-analysis of one of the model tests (test reference m1) was carried out. Details of this finite element calculation are given below.

This finite element model used for this analysis is based on the Updated Lagrangian formulation described by Burd (1986) and Burd and Houlsby (1989). The mesh, shown in Fig. 9, consists of a total of 970 six-noded triangles to model the soil and 33 three-noded line elements (Burd and Houlsby 1986) to model the reinforcement. A set of six-noded interface elements is placed above, and below, the reinforcement layer in order to model the characteristics of the soil-reinforcement interface. The interface element formulation (Burd and Brocklehurst 1991) is based on a development of an approach originally proposed by Goodman et al. (1968) in which modifications are included to allow elastic-cohesive and elastic-frictional interfaces to be modelled. This interface element formulation allows large rotations, but not large relative slip, to occur.

The fill material is modelled using an elastic - perfectly frictional model (Burd et al. 1989) with a Young modulus of 4000 kPa, Poisson’s ratio of 0.35 and plane strain friction and dilation angles of 36° and 22° respectively. This value of dilation angle is adopted in preference to a lower value that might be more representative of real material behaviour in order to improve the stability of the analysis. This approach is thought to be justified on the basis that the dilation characteristics of the fill are not expected to have an important influence on the predicted performance. An elastic - perfectly cohesive constitutive model is used for the clay subgrade in which a Poisson’s ratio of 0.49 and a rigidity index \( \frac{G}{s} \) of 31 are adopted. The reinforcement is modelled as a linearly elastic-no compression material with a stiffness of 28 kN/m. These data are all taken from a discussion given by Burd (1986) of these model tests. The interface elements placed above the reinforcement are assigned an angle of friction, \( \phi_r \), of 20° and a dilation angle of zero. The interface elements below the reinforcement are assigned a shear strength of 2.95 kPa, corresponding to a value of \( \alpha \) of 0.5. For this interface element formulation it is also necessary to specify values for the elastic shear and normal stiffness of the interface. While these parameters do not have any clear physical significance it is important to choose their values carefully. If the magnitude of these parameters is too small then this can result in significant additional shear and normal displacements within the interface. If the stiffness values are set too high, however, then the finite element equations can become ill-conditioned. In these finite element calculations the
interface properties were selected to ensure that the stiffness of each row of interface elements was comparable to a layer of clay subgrade of thickness 0.5mm.

The finite element analysis was carried out by applying vertical prescribed displacements of 30mm and zero horizontal prescribed displacement to the nodes at the base of the footing. It was necessary to limit the applied displacement to 30mm in the finite element calculation in order to avoid excessive element distortions in the neighbourhood of the footing. The specified footing displacements were applied in equal increments in 300 steps using a Modified Euler solution algorithm. The footing load at the end of each calculation increment was computed by summing the vertical forces acting at each node at the base of the footing, and reinforcement tensions and interface stresses were evaluated by sampling the stresses at the Gauss points of the appropriate elements. A comparison between the footing pressure-footing displacement response calculated using the finite element model and the results obtained from the relevant model test is shown in Fig. 10. Also shown for comparison is the result of the analytical prediction reference a1. The finite element results are seen to provide a reasonable fit with the experimental data. Although the match is not exact it is thought that the numerical solution is sufficiently accurate for the computed values of reinforcement tension and soil-reinforcement interface stress to be used with some confidence to indicate the nature and magnitude of the stresses that were generated in the model test.

The calculated reinforcement tensions at footing displacements of 15mm and 30mm are compared with the analytical predictions in Fig. 11. It may be observed from the finite element results that as the footing displacement increases then the width of reinforcement for which the tensions are non-zero also increases. This feature is matched well by the analytical model. The magnitude of reinforcement tension calculated using the analytical model also agrees well with the finite element results except in the region immediately below the footing where the numerical results are significantly larger. This discrepancy may be explained in terms of shear stresses that are developed at the base of the fill layer as described below.

The shear stresses induced above, and below, the reinforcement obtained from the finite element analysis at a footing displacement of 30 mm are plotted in Fig. 12. The individual Gauss point stresses obtained from the finite element analysis showed a certain degree of scatter; the data was therefore smoothed by averaging the stresses over the three Gauss points in each interface element. These data indicate that the shear stresses induced below the reinforcement are approximately zero in the region immediately beneath the footing and then
increase to 2.95 kPa (the shear strength of the interface) as the distance from the footing centre-line increases. These stresses then decay to a small value at the point where the reinforcement force falls to zero. This behaviour is in good agreement with the assumptions made in the analytical model. It is particularly worthy of note that the tendency of the reinforcement to reduce the magnitude of the shear stresses applied to the clay immediately beneath the footing is clearly evident in the numerical results. The shear stresses applied to the top of the reinforcement show a peak at a value of \( x \) of about 0.05 m. These shear stresses are not included in the analytical model and are the major cause of the discrepancy in the predictions of reinforcement tension in the area below the footing, as illustrated in Fig.11. Although it may be possible to include these additional shear stresses in the analytical model this would add considerably to its complexity and is not attempted here. In particular, it is thought that the shear stresses developed at the base of the fill may depend on the stiffness of the reinforcement, with the magnitude of the stresses tending to increase with increasing reinforcement stiffness (Burd and Brocklehurst 1990).

The normal stresses acting above and below the reinforcement at the end of the finite element calculation are shown in Fig. 13. The general magnitude of the stresses above the reinforcement in the region of the footing is substantially greater than that of the stresses beneath the reinforcement. This is a clear indication of the membrane action of the reinforcement. It may also be observed from this figure that the normal stresses at the base of the fill differ markedly from the simple distribution adopted in the analytical model; in particular these stresses show a marked peak in the area immediately below the footing edge. It is evident that a simple load-spread model does not provide an accurate reflection of these numerical results; it is not clear, however, how this simple model could be improved. The computed normal stresses in the clay however are approximately constant in the region beneath the footing; this agrees well with the assumptions made in the analytical model.

**Conclusions**

Current design methods for reinforced granular layers on soft clay are generally based on relatively simple analytical models of behaviour. Whilst these models are convenient for design they do not always reflect accurately the reinforcement mechanisms that act in this type of system. Finite element methods may be used to perform detailed analyses of this type of structure; these calculations tend to be expensive and time-consuming to perform, however,
and not appropriate for routine design. It is suggested that the design of this type of reinforced soil structure will continue to be based on the use of simple analytical models of behaviour.

An analytical model of a plane strain membrane mechanism of reinforcement is described in the paper. This model is intended for use in routine design calculations and leads to equations that are relatively simple to manipulate. The model is shown to provide a reasonable fit with previous model test data and also the results of finite element analysis although some features of performance, for example elastic soil deformations and shear stresses developed at the base of the fill immediately beneath the load, are not modelled well.

Finite element methods remain the most appropriate theoretical approach to obtaining high quality information about the behaviour of reinforced unpaved roads although these calculations tend to be difficult and expensive to perform. The finite element analysis described in this paper illustrates well the tendency of the reinforcement to reduce the magnitude of the shear stresses transmitted to the subgrade and also the membrane mechanism that becomes increasingly important at large deformations.

Acknowledgements

The results and ideas presented in this paper were developed as part of a program of research at Oxford University on reinforced unpaved roads. The author gratefully acknowledges many useful and stimulating conversations during the course of this research with Dr C.J. Brocklehurst, Professor G.T. Houlbsy, Dr. R.A. Jewell and Dr. G.W.E. Milligan.

References


List of symbols

\( b \)  
half-width of loaded area on subgrade surface

\( B \)  
footing half-width

\( B' \)  
half-width of loaded area at base of fill

\( D \)  
fill thickness

\( ds \)  
element of reinforcement length

\( G \)  
shear modulus

\( J \)  
reinforcement stiffness

\( L_r \)  
reinforcement half-width

\( N_c \)  
bearing capacity factor

\( P \)  
footing load per unit length

\( P_o \)  
footing load per unit length at zero displacement

\( s_u \)  
plane strain undrained shear strength

\( T(x) \)  
reinforcement tension

\( T_o \)  
reinforcement tension acting beneath footing centre-line

\( x \)  
distance from footing centre-line

\( x_o \)  
active half-width of reinforcement

\( y \)  
vertical displacement of reinforcement

\( \alpha \)  
adhesion factor

\( \beta \)  
load-spread angle

\( \Delta \)  
maximum vertical reinforcement displacement

\( \gamma \)  
unit weight of fill

\( \sigma \)  
net normal stress applied to reinforcement

\( \sigma_v \)  
vertical stress at base of fill

\( \tau \)  
net shear stress applied to reinforcement

\( \tau_r \)  
shear stress developed at the base of the fill

\( \tau_r^* \)  
assumed shear stress applied to the reinforcement

\( \phi_r \)  
fill-reinforcement interface friction angle

\( \theta \)  
inclination of reinforcement to horizontal
Fig. 1. Reinforced unpaved road (after Houlsby et al. 1989).

Fig. 2. Load spread within the fill layer.
Fig. 3. Deformed shape of reinforcement (after Giroud and Noiray 1981).

Fig. 4. Shear stress mechanism (after Houlsby et al. 1989).
5(a). Deformed shape of reinforcement

5(b). Assumed stresses

Fig. 5. Assumed stresses acting on reinforcement and corresponding deformed shape.
Fig. 6. Equilibrium of a reinforcement element.

Fig. 7. Model test arrangement (after Love 1984).
Fig. 8. Comparisons between model test results and predictions from proposed model.
Fig. 9. Finite element mesh.
Fig. 10. Comparison between footing load-footing displacement responses.
Fig. 11. Reinforcement tension computed from finite element analysis and from proposed model.
Fig. 12. Shear stresses acting on reinforcement.

Fig. 13. Normal stresses acting on reinforcement.