INTERPRETATION OF SHEAR MODULI FROM CONE-PRESSUREMETER TESTS IN SAND

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The interpretation of the "elastic" shear modulus examined from cone-pressuremeter tests is discussed, in order to assess the influence of installation disturbance and other factors on the measurements of soil stiffness. In particular, an attempt is made to demonstrate that the cone-pressuremeter test can give reliable values of unload-reload shear modulus with properly programmed unload-reload loops. The typical shape of unload-reload loops is closely examined, and two procedures to calculate the shear modulus are proposed. A comparison is made between the shear moduli calculated from measurements made with strain arms and the shear moduli calculated from volume change measurements. A method is presented that allows the relevant stress level acting around the pressuremeter during the test to be taken into account when interpreting measurements of the shear modulus. The values of the moduli obtained from the cone-pressuremeter were found to be comparable with those obtained from the self-boring pressuremeter.

1 - Introduction

The cone-pressuremeter is a new site investigation device, conceived as an alternative approach to pressuremeter testing, in which a pressuremeter probe is incorporated behind a standard 15 cm² cone penetrometer (Withers et al, 1986). The major difference from other categories of pressuremeter tests lies in the method of installation of the instrument into the ground. This new device is pushed into the ground and the pressuremeter test is carried out in soil which has been displaced by the penetration of the cone. Since one of the most common uses of the pressuremeter test in sands is for the evaluation of soil moduli (Wroth, 1982), this paper examines possible effects of installation disturbance on the measurements of soil stiffness, and the ability of the cone-pressuremeter to yield shear moduli measurements which are comparable with those obtained from other pressuremeters.

In the interpretation of the shear modulus measured by pressuremeter tests, it is assumed that the pressuremeter is a long cylindrical cavity expanding radially under plane strain conditions in the axial direction. From this, if a pressuremeter is expanded in a linear isotropic elastic
material the relationship between the change in cavity effective stress $\psi$ and the change in cavity
hoop strain $\varepsilon$ is well known, so that the shear modulus $G$ can be calculated from the slope of
the pressure-expansion curve as:

$$G = \frac{1}{2} \frac{d\psi}{d\varepsilon}$$

[1]

which implies that the soil surrounding the probe is only subjected to shear, and the test measures
the "elastic" shear modulus of soil $G$ (Wroth, 1975; Wroth et al, 1979). The strains imposed by
the cone-pressuremeter to the surrounding soil are large, and Hencky's logarithmic strain definition

$$\varepsilon = \ln\left(\frac{R}{R_o}\right)$$

[2]

was chosen as it simplifies the mathematical treatment of cavity expansion theory in large strain
(Houlsby and Withers, 1988).

Ideally, three different shear moduli can be computed from a pressuremeter test: the initial
tangent modulus $G_t$, the unload-reload modulus $G_u$, and the reload-unload modulus $G_r$. Use of
a shear modulus defined over some percentage of the ultimate or yield strength ($G_{50}$, $G_{50}$ etc)
as often used in the interpretation of laboratory tests is not within the scope of the present work.

The initial tangent shear modulus $G_t$ is estimated from the initial slope of the pressure-expansion
curve. In sands the almost inevitable disturbance during installation makes $G_t$ values extremely
unreliable, even for the self-boring pressuremeter. For cone-pressuremeter tests expansion
initially occurs in a soil that has already experienced plastic deformations due to cone penetration,
and therefore $G_t$ measurements are not expected to yield values which relate closely to the elastic
properties of the soil.

An alternative approach was proposed by Hughes (1982) and Wroth (1982), in which the elastic
shear modulus is measured by performing small unload-reload loops during the test. This
approach implies that any unload of the expanding cavity brings the stress state of surrounding
soil to a point below the currently expanded yield surface into a zone where strains are small
and to a large extent reversible. In carrying out small unload-reload loops it is necessary not to
exceed the elastic limit of the soil during the unloading phase. Wroth (1982) suggested that the
magnitude of the change in cavity effective stress during elastic unload should not exceed:
The typical shear strain amplitude of loops in the cone-pressuremeter test is of about 0.1% to 0.2%, which for the 10 cm² device is equivalent to a variation in radius $\Delta R$ of approximately 0.02 mm to 0.04 mm. For the 5 cm² device $\Delta R$ can be as small as 0.01 mm. For each loop approximately 100 readings were recorded, so that cavity strain between two consecutive readings was of the order of 0.001% ($\Delta R = 2 \times 10^{-4} \text{mm}$), which is of the order of the resolution of the strain-arm measuring system. In a few tests some occasional interference on the electronics was observed, and a few points deviated from the overall pattern of the loop (as illustrated in Figure 2.b). The resolution of the volume measuring system was of the order of 0.02%. At the scale of Figure 3 it is possible to observe that, for such resolution, small steps are noticed in cavity strain values recorded from volume measurements, reducing the accuracy of $G$ values calculated from these loops.

The typical shape of the unload-reload loops shown in Figures 2 and 3 can illustrate some important aspects to be considered in the evaluation of the shear modulus. Up to shear strain amplitudes of about 0.1% to 0.2% a non-linear hysteretic loop was formed, even if the reduction in pressure was not sufficient to cause failure in extension (see equation 3). This hysteretic behaviour has been previously noted in laboratory and field tests (Hardin and Drnevich, 1972; Iwasaki et al, 1978; Hughes, 1982; Bellotti et al, 1989). Hysteresis in some field tests has been thought to be a result of the excess of pore pressure generated during inflation and not dissipated before start of the loop, and also due to creep strains that occur during membrane expansion (Hughes, 1982). Since tests were carried out in a calibration chamber on dry sand, the accumulation of strains at constant cavity pressure was not associated with pore pressure dissipation. The possibility of creep cannot be totally eliminated, but the tests were carried out at very slow rates of shearing.

The sensitivity of pressure-expansion curves to strain rates to creep deformation has been recently discussed by Withers et al (1989). The amount of creep deformation is known to increase as cavity pressure increases. To reduce the influence of creep deformation on small unload-reload loops a test procedure has been suggested by Withers et al (1989), in which cavity stress is held constant until an acceptably low rate of creep strain has been reached (about 0.1%/min) and then the loop is performed. An alternative approach applicable to self-boring pressuremeter tests was suggested by Bellotti et al (1989), in which an unload-reload loop is performed without any holding period. A hysteretic loop is then observed and the slope is calculated by drawing a line between the two apices of the loop.
In the present work a test procedure was established in which the membrane was inflated at very slow rates to keep creep strains to a minimum. An inflation and deflation rate of about 0.4%/min was imposed in the tests. Before carrying out an unload-reload loop, the cavity pressure was held constant until the cavity strain rate gradually reduced to a very small value. A strain controlled loop was then performed, at a strain rate of about 0.025%/min. The time for carrying out a loop was about 10 minutes and for a complete pressure-expansion curve was about 2.5 hours. There was no intention to adopt a test procedure which would be appropriate for commercial tests, where constraints of time impose faster inflation rates, but to establish a procedure which would ideally minimise creep strain effects on pressuremeter testing data.

To allow an accurate determination of shear modulus the experimental data was processed through a FORTRAN program designed to operate on IBM compatible micro-computers. The software calculates the pressuremeter pressure-expansion curve and the shear modulus from individual unload-reload loops, introducing the necessary corrections for membrane stiffness and system compliance. The programme first searches for the two apices of the loop. First the lowest cavity pressure measured in the loop is established (point A in Figure 1). Once point A is known, the program computes the cavity strains of the points in the vicinity of A and comparisons are made until the intersection of the unloading and reloading segments is found (point B in Figure 1). The slope of the loop and the corresponding shear modulus G are than easily calculated.

In the cone-pressuremeter tests carried out in the calibration chamber, membrane expansion was monitored by both strain gauged arms and volume changes. For each loop two values of G were then computed - the shear modulus calculated from strain arm measurements $G'$ and the shear modulus calculated from volume change measurements $G''$.

The unload-reload loops showed a non-linear hysteretic behaviour even for a test procedure in which creep strains were allowed to occur before the loop was performed and no pore pressure was generated during inflation (see Figures 2 and 3). Under these conditions, two procedures to calculate the slope of the loops were examined:

a) to draw a single line between the two apices of the loop, and from the slope calculate the shear modulus $G_{ap}$.

b) to perform a simple linear regression analysis based on the least square fit of all points included between the two apices of loop, and from the slope calculate the shear modulus $G_{eq}$.
As a result, four shear moduli were computed from each unload-reload loop (or reload-unload loop): apices modulus values \( G_{ap}^* \) (arms) and \( G_{ap}^* \) (volume) and least square fit modulus values \( G_{sq}^* \) (arms) and \( G_{sq}^* \) (volume).

3.1 - Calibrations and Corrections

Calibrations are an essential operation required to obtain the corrected pressure-expansion curve, and they can have an appreciable influence on the measurement of the unload-reload shear modulus. In stiff soils, the strain amplitude observed during an unload-reload loop is very small and inaccuracies in the strain arm measuring system or in the volume measuring system can result in large errors. To account for small volume or radius changes due to system compliance very careful calibrations were carried out.¹

The effect of pressuremeter compliance on the measured shear modulus has already been recognized (Mair and Wood, 1987; Fahey and Jewell, 1990). Fahey and Jewell (1990) suggested a calibration procedure to determine the compliance of the measuring system. It involves expanding the pressuremeter inside a thick-walled steel cylinder until the membrane comes in contact with the inside wall of the cylinder, and then increasing the pressuremeter pressure in a manner which mimics the procedure adopted in tests. A series of compliance tests was performed with the cone-pressuremeter testing device. The results of one such calibration are shown in Figure 4, in which cavity pressure is plotted against cavity strain calculated from both strain arm measurements and volume change measurements. Several unload-reload loops were performed to estimate the "system shear modulus" \( G_{\text{system}} \) at different levels of cavity pressure. The compliance of the volume measuring system is significantly higher than that observed in the strain arm measuring system, but in both cases the stiffness of the system increases as cavity pressure increases. In both cases the relationship between shear modulus \( G_{\text{system}} \) and cavity pressure at the beginning of unloading \( \psi_c \) was well fitted by an exponential expression:

\[
G_{\text{system}}/p_a = \exp(A + (\psi_c/p_a)B)
\]

with \( A = 6.460, B = 0.254 \) and a correlation coefficient squared \( r^2 = 89.67\% \) for strain arm measurements; and \( A = 6.300, B = 0.142 \) and a correlation coefficient squared \( r^2 = 88.40\% \) for volume measurements. In equation [4] shear modulus and stresses have been normalised by

¹ The compliance of the strain arm system means any compression of the rubber membrane, together with the actual compliance of the strain arms. The compliance of the volume measuring system is understood as line and control unit flexibility.
atmospheric pressure in order to make the expression dimensionless. Other expressions could as well have been chosen to curve fit this correction, but the above expression was found convenient. By making use of equation [4] to determine $G_{\text{system}}$, the effect of compliance (i.e. the stiffness of the system) on the measured soil shear modulus can be expressed as:

$$\frac{1}{G_{\text{corrected}}} = \frac{1}{G_{\text{measured}}} - \frac{1}{G_{\text{system}}} \quad [5]$$

The magnitude of the correction due to compliance has a considerable influence on shear moduli, especially for volume change measurements, and was applied to test data with the specific purpose of comparing shear moduli calculated from the volume measuring system to shear moduli calculated from the strain arm measuring system. In a few loops, the measured soil shear modulus was roughly of the same order of magnitude as the system shear modulus, leading to unacceptably large correction factors. In such cases the volume measuring system was considered inadequate to measure the unload-reload shear modulus of the soil and the corrected values of $G$ were not included in further correlations. One consolation is that the compliance observed in the volume measuring system, as well as the strain arm measuring system, is not as significant as that observed for probes currently used in engineering practice (as reported by Fahey and Jewell, 1990).

Small corrections to account for system compliance have an appreciable influence on the modulus obtained from unload-reload loops. To make a best estimate of the shear modulus another minor correction has to be applied to account for the finite length of the pressuremeter probe. This problem has been investigated by Houlsby and Carter (1990) using two dimensional finite element analysis. Volume changes and central deflections of the membrane were computed for probes of different length to diameter ratios. As far as elastic theory is concerned, corrections to the measured shear modulus were given to compensate for the constraints imposed by the end of the membrane. The correction factors obtained for a 10:1 length to diameter probe were applied to the cone-pressuremeter data. The shear moduli calculated from measurements of the central deflection of the membrane were corrected by multiplying by a factor of 0.997, whereas the shear moduli calculated from volume change measurements were corrected by a factor of 0.907.

A block diagram is presented in Figure 5 to illustrate the correction procedure adopted to obtain the best possible estimate of the unload-reload shear modulus. The inflation pressure recorded during the test is first corrected by subtracting the "membrane correction", which is undertaken by inflating the pressuremeter in air to obtain the membrane stiffness (e.g. Mair and Wood,
1987). The corrected pressure-expansion curve is then used for calculating the shear moduli $G_{sp}$ and $G_{sq}$. As previously discussed, two other corrections are still necessary to account for compliance effects and for the finite length of the pressuremeter probe. Table 1 is presented in order to quantify the magnitude of these corrections, in which the values of $G_{sp}$ were corrected to account for compliance effects and then further corrected to account for the finite length of the probe. The values of $G_{sp}$ (before corrections) and $G_{sp}^{c}$ (after overall corrections) were used to calculate the correction factors $G_{sp}^{c}/G_{sp}$. The average correction, as well as the maximum and minimum corrections recorded on a single loop, are presented in the table. The magnitude of the correction to account for compliance of the equipment has serious implications for the measured shear modulus obtained from unload-reload loops and the need for calibrations is emphasized. The adoption of corrected factors to account for the finite length of the pressuremeter probe is recommended, specially if volume measurements are to be used.

Tables 2 to 4 summarize the values of shear modulus calculated from each unload-reload loop carried out using the cone-pressuremeter testing device, as well as the relevant data associated to each loop. Table 2 presents test results obtained with the 15 cm$^2$ device. In this series of tests the shear moduli measured from volume changes were not recorded, because the pressuremeter was inflated by nitrogen gas. Table 3 shows results obtained with the 10 cm$^2$ cone-pressuremeter prototype. Finally Table 4 presents tests carried out with the 5 cm$^2$ prototype, in which the shear modulus was computed from volume changes only (due to limitations in diameter the instrument has no strain gauged arms).

4 - Interpretation of shear modulus

The interpretation of the unload-reload (or reload-unload) shear modulus consisted of three parts. In the first, the two procedures to calculate the slope of the loops are examined and values of $G_{sp}$ and $G_{sq}$ are compared. Secondly, comparisons between the shear moduli calculated from strain arm measurements and the shear moduli calculated from volume change measurements are presented. In the third part, a method that allows the shear modulus to be normalised with respect to stress level is discussed.

4.1 - Apices modulus $G_{sp}$ against least square fit modulus $G_{sq}$

Examples of unload-reload loops from cone-pressuremeter tests were presented earlier in this paper in Figures 2 and 3. Although shear strain amplitudes of about 0.1% to 0.2% were recorded, the loops have shown a non-linear hysteretic behaviour even for a test procedure in which creep
strains were allowed to occur before the loop was performed. To assess the influence of non-linearity of a loop on the measured shear modulus two procedures were examined to calculate the slope of the loops, and values of $G_{sp}$ and $G_{sq}$ were determined.

Note that if sand behaves as a perfectly elastic-plastic material then the unload-reload loop will be a straight line of gradient $2G$, and identical values of $G_{sp}$ and $G_{sq}$ would be obtained. In practice a non-linear hysteretic unload-reload loop is formed. A procedure for calculating the slope from the two apices of the loop is then recommended (e.g. Bellotti et al., 1989), hypothesizing that a line drawn between the two apices corresponds approximately to the rotational axis of symmetry of the loop. If this simplification is corrected any difference between $G_{sp}$ and $G_{sq}$ values will be negligible, and the choice of procedures would have no effect on the evaluation of the shear modulus.

A direct comparison between $G_{sp}$ and $G_{sq}$ obtained from strain arm measurements is shown in Figure 6, whereas the same comparison obtained from volume change measurements is shown in Figure 7. Although some scatter is observed, $G_{sq}$ values are generally smaller than $G_{sp}$ values. In average the ratio $G_{sq}/G_{sp}$ is of about 6.80 for strain arm measurements and of about 0.70 for volume change measurements. The ratio $G_{sq}/G_{sp}$ is apparently not dependent on density, stress level or pressuremeter strain at the start of unloading.

The pressuremeter has been designed primarily to measure the shear modulus of the soil. High precision is required in the measurements of strains for a reliable evaluation of the shear modulus. However, as seen above a simple change in the definition used for $G$ leads to average typical changes of 20% to 30% on $G$ values calculated from small unload-reload loops. The understanding of the differences between $G_{sp}$ and $G_{sq}$ can only be addressed in a qualitative way. The unloading and reloading segments are non-linear and also not symmetrical. A close observation of recorded unload-reload loops from cone-pressuremeter tests show that generally the unloading remains sensibly linear, whereas reloading exhibits a markedly non-linear behaviour (this pattern is clearly illustrated in Figure 2). The shear modulus $G_{sp}$ is dominated by the slope of the unloading part, whereas $G_{sq}$ is influenced by measurements recorded in both unloading and reloading. Since reloading is highly non-linear $G_{sp}$ values are usually stiffer than $G_{sq}$ values.

The relationship between $G_{sp}$ and $G_{sq}$ ($G_{sq}/G_{sp}$ from 0.7 to 0.8) was observed for tests carried out with the $15 \text{ cm}^2$, $10 \text{ cm}^2$ and $5 \text{ cm}^2$ cone-pressuremeter testing devices in Leighton Buzzard sand, from strain arm measurements and volume change measurements. Loops were carried out under controlled conditions, in which the change in cavity effective stress did not exceeded the
elastic limit of the sand during the unloading phase (variations in cavity effective stress were usually less than half the maximum value suggested by equation 3). Therefore, the ratio \( G_{sp}/G_{sq} \) is not associated with large plastic deformations or mechanical features of the displacements sensors.

Shear moduli \( G_{sp} \) and \( G_{sq} \) were calculated from unload-reload loops, in which creep strains were allowed to occur before the loop was carried out. The effect of creep deformations is known to increase the non-linear hysteretic behaviour of loops, but its influence on the measured shear modulus (and therefore the ratio \( G_{sq}/G_{sp} \)) is still to be investigated. Since a complete explanation of the effect of creep strains on the measured shear modulus is still needed, any test procedure should be designed to allow creep displacements to occur, before the unload-reload loop is carried out. The evaluation of the shear modulus from unload-reload loops should be made using the two procedures described in this paper (to calculate \( G_{sp} \) and \( G_{sq} \)). The use of the ratio \( G_{sq}/G_{sp} \) can provide additional information on the accuracy of the strain-measuring system, as it is sensitive to occasional interferences on the electronics and to anomalous behaviour of the mechanical displacement sensors.

4.2 - Strain arm measurements against volume change measurements

On tests carried out with the 15 cm\(^2\) prototype, membrane expansion was monitored by the central deflection of the membrane measured by strain gauged arms, whereas with the 5 cm\(^2\) prototype membrane expansion was monitored only by measurements of volume change (due to limitations in diameter the instrument has no strain gauged arms). On tests carried out with the 10 cm\(^2\) prototype, membrane expansion was monitored by both strain arms and volume changes, which allows a direct correlation between strain arm measurements and volume change measurements. Cavity strain was calculated using equation [2], in which the radius R was taken from the central deflection of the membrane measured by strain gauged arms and was calculated from volume change measurements assuming that the membrane expands as a right cylinder. Adopting Hencky's strain definition, the shear modulus was calculated from the slope of unload-reload loops using equation [1].

Comparisons between the shear moduli calculated from strain arm measurements and the shear moduli calculated from volume change measurements are plotted in Figures 8 and 9 for tests carried out with the 10 cm\(^2\) prototype. A reasonably good agreement can be seen between the two moduli from both comparisons - \( G^*_{sp} \) against \( G^*_{sq} \) and \( G^*_{sp} \) against \( G^*_{sq} \). The ratio \( G^*/G^* \) falls
within a range of 0.5 to 2.0, suggesting that volume change measurements can produce G values of the same order of magnitude as G values calculated from measurements of the central deflection of the membrane (taken from strain gauged arms).

The compliance problem in the cone-pressuremeter measuring system has serious implications on the plots shown in Figures 8 and 9. As previously discussed, such a problem can result in corrections of more than 20% of the measured shear modulus. Since the compliance of the volume measuring system is significantly higher than that observed in the arm measuring system (see Figure 4), the ratio of \( G' / G'' \) is also largely affected by these corrections. It is believed (but cannot easily be proven) that the scatter in Figures 8 and 9 is principally due to variability in the measurements using volume, since the resolution of this system was smaller, and the corrections larger.

4.3 - Influence of stress level

Three unload-reload loops and one reload-unload loop were carried out in a typical pressure expansion-compression curve performed with the cone-pressuremeter in the calibration chamber. The first loop was carried out at \( \varepsilon \) of about 5.0%, to avoid the initial stages of the pressure-expansion curve, in which the pattern of mean effective stresses at different radii is probably influenced by cone penetration (unexpectedly high values of G obtained at early stages of test (\( \varepsilon = 2\% \)) were reported by Withers et al (1989)).

The correct interpretation and application of the measured shear modulus should account for the relevant stress level and strain amplitude acting around the pressuremeter during the test. At very early test stages an elastic deforming zone is formed around the pressuremeter and the mean effective stress remains unchanged. When the soil surrounding the probe reaches failure, the mean effective stress increases as a function of the effective radial stress. An increase in the modulus is observed as the position of the loop moves further from the start of the test, which reflects an increase in mean effective stress in plastic zones, when no excess pore pressure is observed (Wroth, 1982; Fahey and Randolph, 1984; Robertson and Hughes, 1986; Bellotti et al, 1989).

However, the mean effective stresses in the plastic zone decreases with increasing radius from the probe, becoming equal to the in situ stress at some distance from the expanding cavity (Fahey, 1980; Robertson and Hughes, 1986; Yu, 1990). Robertson and Hughes (1986) suggested that the unload-reload loop response is dominated by the maximum stresses generated at the soil-instrument interface, so that G values should be normalised by the maximum mean effective
stresses. On the basis of this assumption, the mean effective stresses around the pressuremeter cavity can be taken as either the mean octahedral effective stress \( p' = \frac{1}{3}(\sigma'_3 + \sigma' + \sigma'_0) \) (Robertson, 1982) or the mean plane strain effective stress \( s' = \frac{1}{2}(\sigma'_3 + \sigma'_0) \) (Fahey and Randolph, 1985; Bellotti et al., 1989; Yu, 1990).

For the interpretation of the cone-pressuremeter data the following method is proposed. The sand surrounding the pressuremeter is assumed to be deformed under conditions of axial symmetry and plane strain. The mean effective stress is calculated at the soil-instrument interface at the start of the unload-reload loop, hypothesizing that during the loop the soil behaves as a perfect elastic material, so that the hoop stress decreases by the same amount as the radial stress increases and therefore the mean effective stress remains constant. A simplified assumption of an elastic perfectly plastic material is made to define sand behaviour, in which a constant stress ratio is observed during failure. The ratio of the principal stress can then be expressed in cylindrical coordinates

\[
\sigma'_0 = \sigma'_r \left( \frac{1 - S}{1 + S} \right) \tag{6}
\]

where \( S = \sin \phi'_p \). A problem arises when computing the intermediate principal stress \( \sigma'_3 \), since there is no unique solution to determine \( \sigma'_3 \) from \( \sigma'_r \) and \( \sigma'_0 \). However, for a case in which a fully associated flow rule is used (i.e. \( \phi' = \nu \)) a relatively simple expression can be obtained from Matsuoka’s plastic model (Matsuoka, 1976). Burd (1986) gives the equation, in which \( \sigma'_3 \) can be expressed as

\[
\sigma'_3 = (\sigma'_r \sigma'_0)^\frac{1}{2} \tag{7}
\]

assuming no plastic deformations in the vertical direction. This simple expression has been empirically validated by results of plane strain tests in sand in an independent stress control apparatus (Green and Bishop, 1969; Bishop, 1971; Green, 1971). Combining equations [6] and [7] the average mean effective stress at the edge of the expanding cavity at the start of the unload-reload loop can be given by

\[
p' = \frac{\sigma'_r}{3} \left[ 1 + \frac{1 - S}{1 + S} + \left( \frac{1 - S}{1 + S} \right)^\frac{1}{2} \right] \tag{8}
\]
The expression relating mean effective stress at the face of the pressuremeter probe to effective radial stress is dependent upon the friction angle. To quantify this dependency, equation [8] can be written as:

\[ p' = \alpha \sigma' \]  \hspace{1cm} [9]

where \( \alpha = 0.61 \) for \( \phi'_p = 34^\circ \) (loose sand), \( \alpha = 0.54 \) for \( \phi'_p = 43^\circ \) (medium sand) and \( \alpha = 0.50 \) for \( \phi'_p = 49^\circ \) (dense sand). For dense sand the value of \( \alpha = 0.50 \) is the same as that proposed by Robertson and Hughes (1986), which is based on empirical evidence taken from a large number of field data.

During the initial phase of cavity contraction the whole of the soil behaves elastically, with elastic unloading of the previously plastic region. The elastic phase ends when:

\[ \frac{\Delta \sigma'_r}{\sigma'_r} = \frac{2S}{1 + S} \]  \hspace{1cm} [10]

where \( \Delta \sigma'_r \) is the reduction in cavity pressure. In the elastic region there are no volumetric strains and no change in mean effective stress. Reverse plasticity then occurs, where stress are related by:

\[ \sigma'_0 = \sigma'_r \left( \frac{1 + S}{1 - S} \right) \]  \hspace{1cm} [11]

so that the mean effective stress can be expressed as:

\[ p' = \frac{\sigma'_r}{3} \left[ 1 + \frac{1 + S}{1 - S} \left( \frac{1 + S}{1 - S} \right)^{\frac{1}{2}} \right] \]  \hspace{1cm} [12]

For granular materials the relationship between the elastic moduli and the mean effective stress can be expressed in a similar way to that suggested by Janbu (1963):

\[ \frac{G}{p_a} = K_G (p'/p_a)^\nu \]  \hspace{1cm} [13]

where \( K_G \) = modulus number
Shear Modulus

\[ n = \text{modulus exponent} \]
\[ p_a = \text{atmospheric pressure} \]
\[ p' = \text{mean effective stress} \]

This approach has been used by several workers (Duncan and Chang, 1970; Lade and Duncan, 1975; Wroth et al, 1979; Bellotti et al, 1986; Robertson and Hughes, 1986). The value of the exponent \( n \) is typically in the range 0.4 to 0.8 (Wroth et al, 1979; Bruzzi et al, 1986; Bellotti et al, 1986), and is dependent on the strain amplitude over which the modulus is measured (Wroth et al, 1979). In order to determine the modulus number \( K_G \) and the modulus exponent \( n \) from cone-pressuremeter data for Leighton Buzzard, the values of \( \log G \) have been plotted against \( \log p' \) for loose, medium and dense sand, as shown in Figures 10, 11 and 12 respectively. The examples illustrate the behaviour of the shear moduli \( G_{ap} \) obtained from measurements of the central deflection of the membrane by strain gauged arms. A remarkably similar pattern was also obtained when plotting the corrected square fit moduli \( G_{eq} \) from strain arm measurements.

Although scatter is evident, a regression analysis was performed to obtain the best relationship between shear moduli and mean effective stresses for each of the three densities tested. The modulus values \( K_G \) and \( n \) calculated from each plot are given in Table 5. Despite the scatter, relationships between \( G \) and \( p' \) produced consistent values, in which the modulus exponent \( n \) increases with increasing density (i.e. decreasing strain amplitude) in the range of 0.4 to 0.8, and the modulus number \( K_G \) remains approximately constant. For Leighton Buzzard sand the modulus number is about \( K_G = 480 \), whereas the modulus exponent varies with density approximately as:

\[ n = 0.28 + 0.59R_d \]  \[14\]

The scatter observed in Figures 10 to 12 is thought to be principally a result of the influence of shear strain amplitude on shear modulus. To examined this problem, the shear modulus measured from each unload-reload loop \( G(\text{measured}) \) was normalised by the shear modulus calculated at the same mean stress by equation [13], \( G(\text{calculated}) \), using the values of \( K_G \) and \( n \) presented in Table 5. The shear strain amplitude recorded from each loop \( \gamma(\text{measured}) \) was normalised by the average shear strain amplitude \( \gamma(\text{average}) \), calculated by averaging the shear strain amplitude of all loops carried out at the same density. Results are presented in Figures 13 and 14, in which the ratio \( G(\text{measured})/G(\text{calculated}) \) is plotted against the ratio \( \gamma(\text{measured})/\gamma(\text{average}) \). As a general trend, ratios of \( \gamma(\text{measured})/\gamma(\text{average}) < 1 \) are observed for \( G(\text{measured})/G(\text{calculated}) > 1 \), i.e. the measured shear modulus is greater than the value calculated by the regression analysis whenever the shear strain amplitude recorded in the loop is smaller than the average amplitude.
calculated from all loops. Ratios of $\gamma$(measured)/$\gamma$(average) > 1 are in turn observed for G(measured)/G(calculated) < 1. The pattern observed in the figures confirms the assumption that scatter on the relationship between shear moduli and mean effective stresses is a result of variations on the shear strain amplitude recorded in unload-reload loops. Note that Figures 13 and 14 have shown results obtained from loose and medium sands respectively. For dense sand fewer loops were performed and the relationship between shear moduli and strain amplitude is not as clear as that obtained for loose and medium densities.

Equations [13] and [14] can be combined to express the dependence of the shear moduli on stress level and relative density, as presented in Figure 15. A review of Tables 2 to 4 shows that the shear modulus values were measured at mean effective stresses from approximately 150 kPa to 1500 kPa. In this range the shear moduli increase with increasing mean effective stress and increasing density. As the mean effective stress increases, the effect of relative density is more significant.

The evaluation of the experimental coefficients $K_o$ and $n$ obtained from the relationship between log G and log $p'$ from volume change measurements did not produce totally consistent values. The scatter observed in the relationship can be due to

a) lack of accuracy on the volume measurement system, since a resolution of only 0.02% has been achieved on the tests.

b) the excessively large correction factors applied to the measured shear moduli to account for the compliance of the system.

Improvements on both accuracy and compliance should be made to allow the variation of moduli with mean stress to be quantified.

Finally, Figure 16 shows a comparison between the shear modulus measured from unload-reload loops from both the cone-pressuremeter and the self-boring pressuremeter (SBP) tests in dense sand. SBP tests have been carried out at Cambridge University on Leighton Buzzard sand in a calibration chamber similar to the Oxford chamber, as reported by Fahey (1980). Values of apices moduli $G_{ap}$ obtained from strain arm measurements were compared with the appropriate mean effective stress, according to the method proposed in this work. The moduli from cone-pressuremeter compared well with values obtained from the self-boring test. It is suggested that the scatter in results was essentially caused by the impossibility of correcting some of the
G values to account for compliance effects, since calibrations were not available for the SBP probe and the 15 cm² cone-pressuremeter device. The generally good agreement observed in the figure tends to confirm

a) the observation that the shear moduli obtained from unload-reload loops is not very sensitive to the initial disturbance due to the installation process (Hughes, 1982; Wroth, 1982; Hughes and Robertson, 1985; Bellotti et al, 1989; Withers et al, 1989).

b) the applicability of cylindrical cavity expansion theory to predict the elastic shear modulus of soil from cone-pressuremeter testing data.

A correct interpretation of the measured shear modulus must also account for the relevant shear strain magnitude observed during tests (Kondner, 1963; Wroth et al, 1979; Robertson and Hughes, 1986; Bellotti et al, 1986). Results presented in Figures 13 and 14 clearly illustrate such a dependency. Previous comparisons between the elastic moduli $G_w$ measured from pressuremeter tests and the elastic moduli $G_{\text{max}}$ measured from downhole shear wave velocities and resonant column tests indicate a relationship between $G_w/G_{\text{max}}$ in the range 0.2 to 0.6. (Hughes and Robertson, 1985; Bellotti et al, 1989). This is not surprising considering that the moduli $G_{\text{max}}$ is recorded at shear strain levels of about $1 \times 10^{-4}\%$, whereas the moduli $G_w$ is recorded at shear strain levels of about $1 \times 10^{-1}\%$. Furthermore, the reduction in shear moduli of sands with increasing shear strain amplitude has been extensively studied in laboratory tests (e.g. Iwasaki et al, 1978; Seed et al, 1986, Lo Presti, 1987). Appropriate correction for shear strain magnitudes were given by Hughes and Robertson (1985) and Bellotti et al (1989) to enable the modulus G obtained from pressuremeter tests to be linked to the relevant strain magnitude expected in design problems.

5 - Conclusions

The pressuremeter is a unique method for assessing directly the in situ shear stiffness of soils. Current practice for measuring shear modulus applies small strain elastic theory to small unload-reload loops. The corrected interpretation and application of the modulus measured from cone-pressuremeter tests was investigated.

Calibrations were essential to obtain the corrected pressuremeter pressure-expansion curve, and they had an appreciable influence on the measurement of the shear modulus. A procedure is
recommended to obtain the best possible estimation of the shear modulus from unload-reload loops, which accounts for membrane stiffness, compliance effects and finite length of the pressuremeter probe.

In carrying out small unload-reload loops, care was taken not to exceed the elastic limit of the soil during the unload phase (Wroth, 1982). Nevertheless the unload-reload loops exhibited a non-linear hysteretic behaviour even for a test procedure in which creep strains were allowed to occur before the loop was performed and no excess pore pressure was generated during inflation. To assess the influence of non-linearity of a loop on the measured shear modulus two procedures for calculating the slope of the loops were examined. A procedure of drawing a single line between the two apices of the loop (to calculate $G_{sp}$) produced values systematically greater than a procedure of performing a linear regression analysis based on the least square fit of all points included between the two apices of loop (to calculate $G_{sq}$). Since a simple change in the definition used for $G$ led to average changes of about 20% to 30% on $G$ values calculated from small unload-reload loops, a recommendation was made to calculate both $G_{sp}$ and $G_{sq}$ for every loop. Further comparisons are needed to advance the understanding of these observations to evaluate the influence of sand type and test procedure on the ratio $G_{sq}/G_{sp}$.

Comparisons between the shear moduli calculated from strain arm measurements $G^s$ and the shear moduli calculated from volume change measurements $G^v$ were established for the 10 cm$^2$ cone-pressuremeter prototype. A reasonably good agreement was observed between the two moduli. The ratio $G^v/G^s$ fell within a range of 0.5 to 2.0, suggesting that volume change measurements can produce $G$ values of the same order of magnitude as $G$ values calculated from strain arm measurements.

The application of the measured shear modulus to engineering design must account for the relevant stress and strain acting around the pressuremeter during the test. A method to calculate the mean stress level at the pressuremeter-soil interface was proposed, in which a fully associated flow rule was used to compute the intermediate principal stress from Matsuoka’s plastic model (Matsuoka, 1976). The measured values of shear modulus were related to the mean effective stress by a power expression similar to that suggested by Janbu (1963). Under this framework, the shear moduli obtained with the cone-pressuremeter produced values in good agreement with those obtained from the self-boring pressuremeter. Comparisons were based on calibration tests performed in similar chambers in dense sand.
REFERENCES


Shear Modulus


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Table 1 - Summary of shear moduli corrections
### Table 2 - Shear moduli obtained from unload-reload loops using the 15 cm³ cone-pressuremeter device

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Notes:
1. G = unload-reload shear modulus
2. ψ_c = cavity pressure at start of unloading
3. ε_c = pressuremeter cavity strain at start of unloading (calculated from strain arm measurements)
4. γ_{ur} = average shear strain amplitude of unload-reload loop (calculated from strain arm measurements)
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Table 3 - Shear moduli obtained from unload-reload loops using the 10 cm$^2$ cone-pressuremeter device
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<td>175.1</td>
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</table>

Notes
(1) ψ C = pressuremeter cavity pressure at start of unloading
(2) ε C = pressuremeter cavity strain at start of unloading (calculated from strain arm measurements)
(3) γ UR = average shear strain amplitude of an unload-reload loop (calculated from strain arm measurements)

Table 3 (continued)- Shear moduli obtained from unload-reload loops using the 10 cm² cone-pressuremeter device
<table>
<thead>
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<th>TEST</th>
<th>LOOP</th>
<th>$\varepsilon_C$</th>
<th>$\psi_C$</th>
<th>$\gamma_{UR}$</th>
<th>$G'_{sp}$</th>
<th>$G'_{sq}$</th>
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<td></td>
<td></td>
<td>%</td>
<td>kPa</td>
<td>%</td>
<td>MPa</td>
<td>MPa</td>
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</table>

Notes

1. $\psi_C =$ pressuremeter cavity pressure at start of unloading
2. $\varepsilon_C =$ pressuremeter cavity strain at start of unloading (calculated from volume change measurements)
3. $\gamma_{UR} =$ average shear strain amplitude of an unload reload loop (calculated from volume change measurements)

Table 4 - Shear moduli obtained from unload-reload loops using the 5 cm$^3$ cone-pressuremeter device
<table>
<thead>
<tr>
<th></th>
<th>( G^*_{sp} )</th>
<th></th>
<th>( G^*_{sq} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LOOSE</td>
<td>MEDIUM</td>
<td>DENSE</td>
</tr>
<tr>
<td>( K_G )</td>
<td>682.0</td>
<td>455.8</td>
<td>493.7</td>
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<td>( n )</td>
<td>0.346</td>
<td>0.700</td>
<td>0.790</td>
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<tr>
<td>( r^2 ) (%)</td>
<td>32.0</td>
<td>58.2</td>
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</table>

Table 5 - Modulus number \( K_G \) and modulus exponent \( n \) for Leighton Buzzard sand.
Figure 1 - Example of a pressuremeter pressure-expansion curve
Figure 2 - Expanded plots of unload-reload loops measured by the central deflection of the membrane by strain gauged arms (best and worst examples as far as resolution is concerned are shown in plots (a) and (b) respectively)
Figure 3 - Typical expanded plot of unload-reload loop calculated from volume change measurements

Figure 4 - Compliance tests in a thick-walled steel cylinder
Figure 5 - Block diagram representing the corrections to the measured unload-reload shear modulus
Figure 6 - Comparison between the apices moduli $G_{ap}$ and the least square fit moduli $G_{eq}$ obtained from strain arm measurements

Figure 7 - Comparison between the apices moduli $G_{ap}$ and the least square fit moduli $G_{eq}$ obtained from volume change measurements
Figure 8 - Comparison of the apices moduli $G_{ap}$ calculated from strain arm measurements and from volume change measurements

Figure 9 - Comparison of the least square fit moduli $G_{sq}$ calculated from strain arm measurements and from volume change measurements
Figure 10 - Relationship between shear moduli and mean effective stress for loose sand

Figure 11 - Relationship between shear moduli and mean effective stress for medium sand
Figure 12 - Relationship between shear moduli and mean effective stress for dense sand

Figure 13 - Influence of shear strain amplitude on the shear modulus in loose sand
Figure 14 - Influence of shear strain amplitude on the shear modulus in medium sand

Figure 15 - Variation of shear moduli with mean effective stress and relative density
Figure 16 - Comparison of shear moduli from cone-pressuremeter and self-boring pressuremeter tests