ANALYSIS FOR SOIL REINFORCEMENT WITH BENDING STIFFNESS

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Analysis for soil reinforcement with bending stiffness

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1 Introduction

Soil-structure interaction is a complex aspect of soil mechanics analysis. A solution for the interaction between stiff reinforcement and soil, which is important for soil nailing design, was first outlined by Schlosser (1983). This has provided the basis for the subsequent design guidance on soil nailing given in Mitchell and Villet (1987), and in the soil nailing design manual being developed by Elias and Juran (1988), and discussed by Juran et al (1990).

In a recent paper, Jewell and Pedley (1990) have suggested that there may be an error in the current work concerning the ability of soil nails to support combined axial tension, bending moment and shear force. If this is the case, the matter will be of immediate importance to engineers involved with soil nailing projects. To facilitate a clear and useful discussion, the analysis for the interaction between soil and reinforcement with bending stiffness is reviewed below, and revised equations are presented for design.

The finding is that reinforcement bending stiffness for soil nailing appears only to bring a modest additional improvement to soil stability compared with that due to the axial tension in the reinforcement. The conclusion is that bending stiffness is not the correct focus for the design of soil nailing in routine excavation and steep slope applications, as is being currently suggested.

2 Reinforcement under combined loading

The relation between the combination of forces that a structural member or reinforcement bar can support may be summarised as follows.

The presence of axial force $T$ in a bar reduces the maximum bending moment $M_{max}$ that can be supported

$$M_{max} = fn(T)$$

... (1)

In turn, the maximum shear force $T_e$ that can be developed in a bar depends on the maximum bending moment $M_{max}$.
Analysis for soil reinforcement with bending stiffness

\[ T_c = \text{fn}(M_{max}) \]  \hspace{1cm} \ldots (2)

Thus, there is a direct link between the maximum values of the axial and shear force that a reinforcement bar can support

\[ T_c = \text{fn}(T) \]  \hspace{1cm} \ldots (3)

### 2.1 Axial force and moment

Two limiting combinations of axial force and maximum bending moment, such as described in equation (1), are shown in Figure 1. The fully plastic values \( T_p \) and \( M_p \) apply for a bar in pure tension or pure bending. The inner quadrilateral defines safe loading combinations (below the plastic limit) for reinforcement of any cross-sectional shape, and this is the relation adopted in many structural steel codes, for example BS5950 (1985).

The outer envelope defines the plastic limit for a rectangular bar bending about a principal axis, and is described by the equation, Calladine (1985).

\[ \frac{M_{max}}{M_p} + \left( \frac{T}{T_p} \right)^2 = 1 \]  \hspace{1cm} \ldots (4)

No such simple equation exists for a circular bar, but the load carrying capacity is only slightly greater than that indicated by equation (4). Jewell and Pedley (1990) used equation (4) to define, slightly conservatively, the plastic limit for circular bars in their analytical investigation of the role of reinforcement bending stiffness.

The limiting quadrilateral shown in Figure 1 is the common assumption in structural codes, and has been used to derive the design equations presented in this paper. For absolute values

\[ \frac{M_{max}}{M_p} + \frac{T}{T_p} = 1 \]  \hspace{1cm} \ldots (5)

where the superscript \( p \) indicates a plastic limit.
For the elastic analysis it is consistent to determine the combination of forces which just causes the elastic limit or yield stress $\sigma$, to be reached in the reinforcement bar. Recalling that the limiting elastic moment for a circular bar in pure bending is $M_{\text{lim}}^e = 2\sigma J/D$, the expression for the elastic limit for combined bending and tension is

$$\frac{M_{\text{max}}^e}{M_{\text{lim}}^e} + \frac{T}{T_p} = 1$$

where the superscript $e$ indicates an elastic limit.

For any given tension $T$, the limiting elastic and plastic moments given by equations (5) and (6) are in the same fixed ratio

$$\frac{M_{\text{max}}^e}{M_{\text{max}}^p} = \frac{M_{\text{lim}}^e}{M_p}$$

which equals 0.59 for circular bars.

### 2.2 Shear force and moment

The relation between the maximum shear force $T_c$ and the maximum moment $M_{\text{max}}$, such as described in equation (2), depends on the magnitude and distribution of the lateral loading on the reinforcement bar. This loading can be investigated by elastic or plastic analysis.

#### 2.2.1 Elastic analysis

The elastic analysis for soil nailing was set out by Schlosser (1983), and is described in Mitchell and Villet (1987) and Juran et al (1990). The analysis is essentially an adaption of the closed form elastic solution derived by Hetenyi (1946) for a laterally loaded pile. The following relation between the maximum shear force and moment is found

$$T_c = \frac{4.9M_{\text{max}}^e}{l_i}$$

...(8)
where $l_s$ is the distance between the points on the reinforcement on either side of the shear plane in the soil which experience the maximum moment, Figure 2. The distance $l_s$ in the elastic analysis is defined by the expression

$$l_s = \frac{\pi}{2} l_a = \frac{\pi}{2} \sqrt[4]{\frac{4EI}{K_s D}}$$

...(9)

where $K_s$ is the modulus of subgrade reaction for the soil, and $E$ the Young’s modulus, $I$ the second moment of area and $D$ the diameter of the bar. The distance $l_a$ is the transfer length often referred to in the elastic analysis.

The ratio $l_s/D$ is a useful non-dimensional measure of this distance, and for circular bar, recalling that $I = \pi D^4/64$,

$$\left(\frac{l_s}{D}\right)^4 = \frac{\pi}{4} \sqrt[4]{\frac{\pi E}{K_s D}}$$

...(10)

The maximum bearing stress $(\sigma_b)^\gamma_{\text{max}}$ between the soil and the reinforcement required to achieve the envisaged elastic equilibrium is, Figure 2,

$$(\sigma_b)^\gamma_{\text{max}} = \frac{M^\gamma_{\text{max}}}{0.16l^2_s D} = \frac{15.4M^\gamma_{\text{max}}}{l^2 D}$$

...(11)

It should be noted that the maximum elastic moment $M^\gamma_{\text{max}}$ which can be supported is reduced by any axial tension in the bar, equation (6). In design it will be necessary to use equation (11) to check whether the maximum moment can indeed be supported in the bar without exceeding the maximum allowable bearing stress between the reinforcement and the soil. In cases where the soil bearing stress is the limiting factor equation (11) rather than equation (6) will govern the maximum allowable elastic moment in the reinforcement.

### 2.2.2 Plastic analysis

Jewell and Pedley (1990) derived a plastic analysis for a soil nail under the lateral loading shown in Figure 3. The corresponding equations for this distribution of lateral loading are
\[ T_c = \frac{4M_{\text{max}}}{l_e} \]  
...(12)

and

\[ \left( \frac{l_e}{D} \right)^p = \sqrt{\frac{4\sigma_y}{3\sigma'_b}} \]  
...(13)

where \( \sigma_y \) is the reinforcement yield stress and \( \sigma'_b \) is the limiting bearing stress between the reinforcement and the soil. The ratio \( (l_e/D)^p \) in equation (13) applies when there is no axial force in the reinforcement, and is discussed further in Section 6.

The limiting bearing stress \( (\sigma'_b)_{\text{max}}^p \) between the soil and the reinforcement required to achieve the envisaged plastic equilibrium is

\[ (\sigma'_b)_{\text{max}}^p = \frac{8M_{\text{max}}}{l_e^2 D} \]  
...(14)

An equivalent analysis for the limiting plastic loading shown in Figure 4, a model widely attributed to Hansen and Lundgren (1960), is derived in Appendix A, and also leads to equations (12) to (14).

The difference between the two plastic analyses is in the length \( l_e \) to which the reinforcement must extend beyond the point of maximum moment, Figures 3 and 4. For the first case \( (l_e/l_e) = \sqrt{3}/2 \), and for the second case \( (l_e/l_e) \approx 1/\sqrt{2} \).

The requirement which must be satisfied for the plastic analysis is that the reinforcement length \( L \) on both sides of the shear plane must exceed the minimum \( L \geq l_e + l/2 \), or approximately 1.4\( l_e \) and 1.2\( l_e \) for the two cases respectively.

### 2.2.3 Limiting bearing pressure

A critical parameter in the analysis is the bearing stress \( \sigma'_b \) between the soil and the reinforcement. This stress is analogous to the bearing stress which is mobilised between soil and reinforcement to develop bond. Two bounding solutions for this bearing stress were derived by Jewell et al (1984), and these have been well supported by the test data, Palmeira and Milligan (1989).
The lower, safe estimate recommended for design examines a punching shear failure of the soil around the bar. The appropriate expression applied to a soil nail in a steep slope is, Jewell and Pedley (1990),

$$\frac{\sigma'_b}{\sigma'_v}_{\text{design}} = \frac{(1 + K_a)}{2} \tan(\pi/4 + \phi'/2) \exp((\pi/2 + \phi') \tan \phi')$$  \hspace{1cm} ...(15)

where $K_a$ is the active earth pressure coefficient, and $\sigma'_v$ is the vertical effective stress in the soil.

The corresponding upper estimate is for fully developed plastic shear in the soil around the bar, but this appears almost never to be achieved in practice (see the data in the above references). For soil nails in steep slopes this upper, but unsafe estimate for the bearing stress is given by the expression

$$\frac{\sigma'_b}{\sigma'_v}_{\text{upper}} = \tan^2(\pi/4 + \phi'/2) \exp(\pi \tan \phi') = N_q$$  \hspace{1cm} ...(16)

where $N_q$ is the bearing capacity factor derived by Vesic (1973).

It is of historical note that the range of bearing pressures described by equations (15) and (16) correspond closely to the range for the ultimate lateral stress against a rigid pile suggested by Hansen (1961), Figure 5.

3 Analysis for bars in soil (no grout)

In soil reinforced by a bar carrying an axial force $T$, the maximum shear force $T$, in the reinforcement should be as large as possible to achieve the maximum improvement from the reinforcement bending stiffness. If the axial force $T$ is assumed to be constant over the span $l$, then the maximum plastic moment $M_{\text{max}}^p$ is, from equation (5),

$$M_{\text{max}}^p = M_p \left(1 - \left(\frac{T}{T_p}\right)\right)$$  \hspace{1cm} ...(17)

and the corresponding maximum elastic moment $M_{\text{max}}^e$ is, from equation (6),
\[ M_{\text{max}}^* = M_{\text{lim}}^* \left( 1 - \left( \frac{T}{T_p} \right) \right) \] ...

(18)

While equation (17) always applies for the plastic analysis, the maximum elastic moment \( M_{\text{max}}^* \) may be limited by the available bearing stress between the soil and the reinforcement. The limiting elastic moment governed by the soil bearing stress is, from equation (11),

\[ (M_{\text{max}}^*)_{\text{bearing}} = (\sigma'_{b})_{\text{design}} \frac{l^2D}{15.4} \] ...

(19)

where the design value of the maximum bearing stress \((\sigma'_{b})_{\text{design}}\) is given by equation (15).

Having determined the maximum allowable moment in the bar, the maximum shear force \( T_c \) in the reinforcement may be calculated as

\[ T_c = \frac{CM_{\text{max}}}{l_x} \] ...

(20)

where the constant \( C = 4.9 \) for the elastic analysis, equation (8), and \( C = 4 \) for the plastic analysis, equation (12).

To recapitulate briefly, the maximum moment \( M_{\text{max}} \) to be used in equation (20) for the elastic analysis is governed either by the properties of the reinforcement or by the maximum available soil bearing stress, equations (18) and (19).

By contrast, the properties of the reinforcement always provide the limit to the maximum moment in the plastic analysis (presuming that the reinforcement is of sufficient length, Section 2.2.2). Equation (20) may therefore be expressed more conveniently in terms of the maximum tensile and shear force in the reinforcement. Combining equations (17) and (20), dividing through by \( T_p \) and recalling that \( M_{p}/T_p = 2D/3\pi \) for circular bar, gives

\[ \frac{T_c}{T_p} = \frac{CM_{\text{max}}^p}{l_xT_p} = \frac{0.85}{(l_x/D)^2} \left( 1 - \frac{T}{T_p} \right) \] ...

(21)
3.1 Comparisons for typical soil nailing parameters

The limiting combinations of maximum shear force $T_c$ and maximum axial force $T$ may now be determined for the typical combination of soil nailing parameters summarised in Table 1. The equilibrium for a soil nail at approximately 5m, 10m and 20m depth in a steep-sided excavation is considered.

Table 1. Soil nailing parameters assumed for the analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nail diameter</td>
<td>$D$ 0.025 m</td>
</tr>
<tr>
<td>Yield stress</td>
<td>$\sigma_y$ 200 $10^3$ kN/m$^2$</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>$E$ 210 $10^6$ kN/m$^2$</td>
</tr>
<tr>
<td>Modulus of subgrade reaction</td>
<td>$K_s$ 50 $10^3$ kN/m$^3$</td>
</tr>
<tr>
<td>Soil angle of friction</td>
<td>$\phi'$ 35°</td>
</tr>
</tbody>
</table>

*Derived values:*

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Second moment of area</td>
<td>$I$ 19.2 $10^3$ mm$^4$</td>
</tr>
<tr>
<td>Maximum elastic moment</td>
<td>$M_{e,lim}$ 0.31 kNm</td>
</tr>
<tr>
<td>Plastic moment</td>
<td>$M_p$ 0.52 kNm</td>
</tr>
<tr>
<td>Plastic axial capacity</td>
<td>$T_p$ 98 kN</td>
</tr>
</tbody>
</table>

Table 2 summarises the results from both the elastic and plastic analysis.

The upper part of Table 2 give the ratio $(l/D)$ where it is governed by the properties of the reinforcement bar. The lower part of Table 2 is for the elastic analysis and gives the maximum elastic moment and the corresponding maximum shear force $(T_c/T_p)_{lim}$ limits set by the available soil bearing stress.

The corresponding envelopes of limiting force combinations in the reinforcement are shown in Figures 6 and 7. These were determined following the procedure described earlier using the equations indicated in the Table, and using a non-dimensional form of equation (20).
Table 2. Results for the elastic and plastic analysis for ungrouted nails.

<table>
<thead>
<tr>
<th>UNGROOTED NAILS</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma'$ †</td>
<td>$kN/m^2$</td>
<td>100</td>
<td>200</td>
</tr>
<tr>
<td><strong>Loading geometry set by the reinforcement properties</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(l_D/D)^*$</td>
<td>eqn. (10)</td>
<td></td>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>$(l_D/D)^*$</td>
<td>eqn. (13)</td>
<td></td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td><strong>Limit to elastic capacity set by the soil bearing stress</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\sigma'<em>b)</em>{design}$</td>
<td>$kN/m^2$</td>
<td>eqn. (15)</td>
<td>562</td>
<td>1125</td>
</tr>
<tr>
<td>$(M_{P_{max}})_{bearing}$</td>
<td>$kN.m$</td>
<td>eqn. (19)</td>
<td>.25</td>
<td>.50</td>
</tr>
<tr>
<td>$(T_c/T_p)_\text{lim}$ ‡</td>
<td></td>
<td>eqn. (22)</td>
<td>.024</td>
<td>.048</td>
</tr>
</tbody>
</table>

† At 5m, 10m and 20m depth assuming $\gamma = 20 \text{ kN/m}^2$.
‡ $C = 0.9$ for the elastic analysis.

\[
\frac{T_c}{T_p} = \frac{CM_{max}}{l_D T_p}
\]  

...(22)

Examination of Figures 6 and 7 shows that the magnitude of shear force calculated from the two entirely separate analyses is very similar. At shallow depth, at 5m for example, $(l_D/D)^*$ is approximately equal for the two analyses, Table 2. The fully plastic shear force exceeds only by about 30% the equivalent shear force at the elastic limit, Figures 6 and 7. Higher bearing stress is available at greater depth, and this allows the ratio $(l_D/D)^*$ for the nails to be reduced, thereby increasing the plastic shear force which can be mobilised, Figure 7. However, for excavations up to 20m depth the shear force in the nails calculated from the plastic analysis is still on average no more than a factor 2 greater than the shear force at the elastic limit.

It should also be noted that the maximum allowable bearing stress only marginally effects the elastic capacity of the nails at shallow depth, Figure 6. Use of a constant modulus of subgrade reaction means that $(l_D/D)^*$ is independent of depth, and therefore so is the shear capacity of the nails in the elastic analysis.

Finally, the magnitude of the maximum shear force $T_c$ in the nails should be noted. Compared with the maximum axial force $T_p$, the maximum shear force $T_c$ in the nail amounts to only about 3% for the elastic analysis, or 4% to 8% for the plastic analysis, Figures 6 and 7.
3.2 Conclusions for ungrouted bars

There are two important conclusions to be drawn for ungrouted bars.

1. The geometry of the deformation of the reinforcement (l/D) and the
   limiting envelopes of allowable load determined from the two entirely
   separate elastic and plastic analyses are very similar, Table 2 and Figures
   6 and 7. This gives confidence that the predicted forces and envisaged
   loadings are reasonable.

2. The magnitude of the maximum shear force $T_c$ that can be mobilised in the
   reinforcement is extremely small in comparison with the maximum axial
   force $T_p$ (for all cases $T_p \geq T \geq 0$).

   The implication from this is that there is little additional benefit to be gained
   from the shear force developed because of reinforcement bending stiffness over
   and above the benefit which can be gained from the reinforcement axial force.
   This point is illustrated further in Section 5.

4 Analysis for grouted bars

By far the most common method of installing soil nails is to grout a circular
reinforcement bar into a larger diameter hole. The typical dimensions are steel
bar of diameter $D = 0.025m$ grouted into a hole of diameter $D_g = 0.10m$. Bruce
and Jewell (1986/7) have reviewed current soil nailing construction practice.

One obvious advantage of the larger hole diameter for a grouted nail is that
the surface area available to mobilise bond to develop axial force, and bearing
stress to develop shear force, is greatly increased. The latter mechanism is the
principal concern of this paper, and it is shown below that the increased grouted
nail diameter reduces the ratio (l/D), and hence increases the ratio $T_c/M_{\text{max}}$ for
both the elastic and the plastic analyses.

Mitchell and Villett (1987) clearly stated that

"In the case of grouted bars, it is possible
   to consider the compressive strength of the
   grout provided that the bar is located at the
   centre of the composite bar and grout."

but offered no further advice on whether this advantage could be safely
assumed for design.
If the grout has no tensile capacity it will not affect the axial capacity of the nail above that provided by the reinforcement bar. An accurately centred reinforcement bar can take advantage of the grout capacity in compression to significantly increase the bending stiffness of the soil nail.

Common site practice for installing grouted nails involves positioning the reinforcement bar in the hole using centralisers before fixing it in place with cement grout. The grout is usually introduced by gravity feed or by a low pressure pump.

In this method of installation it is unlikely that the reinforcement bar will be placed centrally because of the practical difficulties involved in centralising a long flexible bar and because of the sag which will develop between widely spaced centralisers. There is also some uncertainty about the consistency, integrity and insitu compressive strength capacity of grout introduced under gravity or low pressure.

The effect of a poorly centralised reinforcement bar is illustrated in Figure 8. The bending stiffness on the side of the potential shear plane where the bottom of the hole is put into tension would be enhanced by the grout which is able to support compressive stress. However, there would be little or no benefit from the grout - for either the bending stiffness $EI$ or the ultimate plastic moment capacity $M_p$ - on the opposite side of the potential shear plane where the bottom of the grouted nail would be put into compression.

In view of the above, and the likely economic implications of guaranteeing an accurately centered and comprehensively grouted bar, it seems unlikely that designers would wish to allow for increased bending stiffness for grouted nails. That the very stability of the soil nailing would depend directly on these construction tolerances if designers were to allow for enhanced bending stiffness would be of concern to the contractor, and would be reflected in the pricing of the soil nailing technique which to be cost effective must be constructed rapidly.

The conclusion drawn is that for routine soil nailing applications no benefit should be assumed to the reinforcement bending stiffness or the plastic moment capacity from the grout. The benefit of the grout in the revised equations presented below is assumed solely to enhance the contact between the reinforcement and the surrounding soil, particularly the contact surface area over which soil bearing stresses can act.
4.1 Revised equations

It is assumed for routine design that a grouted reinforcement bar has the same overall bending stiffness as the bar alone (no benefit from the grout) but that the effective diameter of the soil nail equals the diameter of the hole $D_g$.

4.1.1 Elastic analysis for grouted bars

The influence of the larger diameter $D_g$ is to reduce the ratio $(l_p/D)^\text{'}$. (Note that the steel bar diameter $D$ has still been used to define this ratio). Equation (10) should be modified as follows

$$
\left( \frac{l_p}{D} \right)^\text{'} = \frac{\pi^4}{4} \sqrt{\frac{\pi E}{K_p D_g}}
$$

...(23)

where $D_g$ is the diameter of the grouted hole.

The larger contact area between the soil and the grouted nail also increases the maximum elastic moment which can be generated from the available soil bearing pressure. Equation (19) should be modified as follows

$$
(M_{\text{max}}^e)_{\text{bearing}} = \left(\sigma_b^\prime\right)_{\text{design}} \frac{l_p^2 D_g}{15.4}
$$

...(24)

4.1.2 Plastic analysis for grouted bars

The larger diameter of the grouted nail $D_g$ reduces the ratio $(l_p/D)^\text{'}$ and equation (13) should be modified as follows

$$
\left( \frac{l_p}{D} \right)^\text{'} = \sqrt{\frac{4\sigma_y}{3\sigma_b^\prime D_g}}
$$

...(25)

4.2 Comparison with ungrouted nails

The limiting envelopes of allowable load for the range of cases examined in Section 3.1, can be recalculated to allow for the grout. The analysis is for a typical grouted hole diameter $D_g = 0.10m$, with all the other parameters
unchanged. The results are summarised in Table 3, which may be compared with Table 2, and in Figures 9 and 10, which may be compared with Figures 6 and 7.

Table 3. Results for the elastic and plastic analyses for grouted nails.

<table>
<thead>
<tr>
<th>Loading geometry set by the reinforcement properties</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$(l/D)^*$</td>
<td>eqn. (23)</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$(l/D)^p$</td>
<td>eqn. (25)</td>
<td>11</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Limit to elastic capacity set by the soil bearing stress</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\sigma'<em>p)</em>{\text{design}}$</td>
<td>kN/m$^2$</td>
<td>eqn. (15)</td>
<td>562</td>
</tr>
<tr>
<td>$(M_{\text{max}})_{\text{bearing}}$</td>
<td>kNm</td>
<td>eqn. (24)</td>
<td>0.51</td>
</tr>
<tr>
<td>$(T_s/T_p)_{\text{lim}}$</td>
<td>eqn. (22)</td>
<td>0.68</td>
<td>0.136</td>
</tr>
</tbody>
</table>

† Assuming $\gamma = 20$ kN/m$^2$.
‡ $C = 4.9$ for the elastic analysis.

As shown in Tables 2 and 3, the influence of the larger hole diameter for the grouted nail is to reduce the ratio $(l/D)^*$ by almost 30% for the elastic case, from 21 to 15. The maximum shear force that can be developed is increased correspondingly, Figures 6 and 9. However, even with this increase the maximum shear force $T_s$ that can be developed in the grouted nail at the elastic limit is still only 4% of the maximum axial force $T_p$, Figure 9.

The larger hole diameter for the grouted nail has a much greater influence on the plastic equilibrium, Figures 7 and 10. The ratio $(l/D)^p$ is reduced by a half and the maximum shear force is doubled, equation (21). However, even with this increase the maximum shear force $T_s$ that can be developed at the plastic limit is still less than about 10% of the maximum axial force $T_p$ for nails up to a depth 10m, Figure 10.

4.3 Conclusions for grouted bars

The conclusion to be drawn for grouted soil nails is that the increased contact area between the nail and the soil increases the available bond and axial force, as well as the lateral loading that the soil can exert on the reinforcement, and hence the shear force that can be developed.
However, even with these benefits, the increased magnitude of the maximum shear force $T_s$ that can be mobilised in the reinforcement is still small in comparison with the maximum axial force $T_p$. The ratio of the maximum values of these forces is 4% for the elastic analysis and in the range 7% to 17% for the plastic analysis, for the cases examined.

5 Improved shearing resistance

The implication from Sections 3 and 4 is that reinforcement bending stiffness allows only a rather small additional shear force to be mobilised compared with the axial force capacity of the reinforcement.

The improvement in soil shearing resistance stemming from these forces is illustrated in Figures 12 and 13 for grouted nails (using the forces from Figures 9 and 10) and for the elastic and plastic analyses respectively. The results in Figure 10 are for nails at about 10m depth.

These figures are similar to those presented by Jewell and Pedley (1990) but cover a range of angles of intersection $\theta$ between the nail and the potential shear surface through the soil, Figure 11. The plots in the earlier work were for the most beneficial case for bending stiffness, $\theta = 0^\circ$.

The forces in the reinforcement at the shear plane $(T, T_c)$ modify the force resultants acting on the shear surface through the soil, Figure 11. The gross shearing resistance in the soil alone (without reinforcement) is

$$S = P \tan \phi'$$  \hspace{1cm} (26)

The effect of the reinforcement tensile and shear forces acting in the soil is to improve the shearing resistance by an amount

$$\Delta S = (T \cos \theta - T_c \sin \theta) \tan \phi' + (T \sin \theta + T_c \cos \theta)$$  \hspace{1cm} (27)

To provide a non-dimensional measure of improvement, it is helpful to compare the improvement in shearing resistance for the general case with that for the reinforcement carrying the maximum axial force $T_p$ only, ie

$$\frac{\Delta S}{T_p (\cos \theta \tan \phi' + \sin \theta)} = \frac{T}{T_p} + \frac{T_c}{T_p} \left( \frac{\cos \theta - \sin \theta \tan \phi'}{\cos \theta \tan \phi' + \sin \theta} \right)$$  \hspace{1cm} (28)
and this is what has been plotted in Figures 12 and 13. The lower solid line in
the figures shows the improvement in shearing resistance due to axial
reinforcement force only. The additional improvement above this line is due to
the reinforcement bending stiffness.

The results show that even for the most beneficial orientation of the
reinforcement, $\theta = 0^\circ$, the additional improvement due to bending stiffness is
relatively small - for both the elastic and the plastic analyses. Even then, this
small benefit is sensitive to the reinforcement orientation, and at the typical
orientations $\theta = 15^\circ$ to $45^\circ$ for soil nailing the influence of bending stiffness is
reduced to what might well be described as a second order effect for the typical
cases considered here, Figures 12 and 13.

Compared with the improvement due to axial force represented in Figures 12
and 13, bending stiffness can offer additional improvement but (1) it is small,
and (2) it depends critically on the reinforcement orientation. The conclusion is
that for routine design it is only slightly conservative, while being radically
simpler (and less prone to error) to design soil nailing ignoring the benefit of
reinforcement bending stiffness altogether. The influence of the reinforcement
bending stiffness is also small compared to the uncertainties, assumptions and
safety margins in design.

6 Further details for the plastic analysis

The plastic analysis presented in this paper has followed that of Jewell and
Pedley (1990), and is a simplified analysis which is slightly conservative. The
conservatism does not influence the conclusions reached about bending
stiffness, however, and the simplified analysis was chosen because of the
benefit of providing results for the plastic case which are directly comparable
with the elastic case, see Figures 6 and 7 for example.

The simplifying assumption is that the axial reinforcement force $(T/T_p)$ does
not affect the ratio $(l/D)^p$, which is calculated for the worst case $T/T_p = 0$,
assuming $M_{\text{max}}^p/M_p = 1$, equation (5).

In practice, axial force, $T/T_p > 0$ reduces the maximum moment which the
reinforcement can support, $M_{\text{max}}^p/M_p < 1$ and hence reduces the ratio $(l/D)^p$.
This can be seen from the rigorous expressions given below for the plastic
analysis of ungrouted and grouted nails respectively,
\[
\left( \frac{l_s}{D} \right)^p = \sqrt{\frac{4\sigma_y}{3\sigma_b^*}} \left( 1 - \frac{T}{T_p} \right) \tag{29}
\]
replaces equation (13) for ungrouted nails, and
\[
\left( \frac{l_s}{D} \right)^p = \sqrt{\frac{4\sigma_y}{3\sigma_b^*}} \left( \frac{D}{D_s} \right) \left( 1 - \frac{T}{T_p} \right) \tag{30}
\]
replaces equation (25) for grouted nails.

The plastic limit to combined axial and shear loading in the grouted nail at 10m depth considered earlier, the central case in Figure 10, is shown in Figure 14, calculated using equation (30). While the rigorous and simplified analyses agree at the two extremes \( T/T_p = 1 \) and 0, the shear force which can be generated in the nail is significantly increased in the intermediate range. The effect of this on the envelope of improved shearing resistance in the soil is rather small, however, as can be seen by comparing Figures 13 and 15. The difference between the results from the simplified and rigorous plastic analyses does not change the conclusions which have been reached.
7 Conclusions

General equations have been presented to describe the influence of reinforcement with bending stiffness on the improvement of shearing resistance in reinforced soil. The agreement found between entirely separate elastic and plastic solutions has provided confidence in the analysis.

Detailed consideration has been given to the benefit which could be safely assumed from grouting the reinforcement bars into the soil. It was argued that without special construction controls it would be prudent to ignore the potential increase in the bending stiffness of the soil nail due to the grout.

The result of the work is summarised in the theoretical curves in Figure 15 for the plastic analysis relevant to grouted nails of typical dimensions at about 10m depth in an excavation or steep slope where $\phi' = 35^\circ$. The improvement in soil shearing resistance is plotted on the $y$ axis in a suitable non-dimensional form.

The improvement in soil shearing resistance stemming only from the axial force in the nail is indicated by the straight solid line in Figure 15. The maximum possible improvement, including the additional benefit from the nail bending stiffness, and for the nail in the most favourable orientation ($\theta = 0^\circ$) with respect to the potential shear surface, is given by the upper line in the figure. This additional benefit is not only rather small but also reduces markedly for more usual orientations between the nail and the potential shear surface in the soil, $\theta = 15^\circ$ to $45^\circ$, as shown by the dotted lines in Figure 15.

The above result leads to the central conclusion of the paper which is that the improvement in soil shearing resistance due to reinforcement bending stiffness is small in comparison to the improvement due to axial force in the reinforcement, and it is also highly variable. Soil nailing design analysis is radically simplified by ignoring the effects of bending stiffness, at the expense of only a marginal conservatism, and this approach is recommended.

Acknowledgements

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References


Notation

\( \gamma \)  
Unit weight of the soil

\( \phi' \)  
Angle of friction of the soil

\( \sigma'_{p} \)  
Limit bearing stress between the soil and the reinforcement

\( (\sigma'_{p})_{\text{design}} \)  
Design value of the limit bearing stress

\( (\sigma_{p})_{\text{max}}^{e} \)  
Maximum bearing stress for the elastic equilibrium

\( (\sigma_{p})_{\text{max}}^{p} \)  
Maximum bearing stress for the plastic equilibrium

\( \sigma'_{v} \)  
Vertical effective stress in the soil

\( \sigma_{y} \)  
Yield stress of the reinforcement

\( \theta \)  
Inclination of the reinforcement with respect to the normal to the shear plane

\( C \)  
Constant in the elastic and plastic analyses

\( D \)  
Diameter of the reinforcement

\( D_{t} \)  
Diameter of the grouted nail

\( E \)  
Young's modulus

\( I \)  
Second moment of area

\( K_{a} \)  
Active earth pressure coefficient of the soil

\( K_{s} \)  
Modulus of subgrade reaction of the soil

\( l_{b} \)  
Minimum required length beyond the point of maximum bending moment

\( l_{e} \)  
Elastic transfer length

\( l_{s} \)  
Distance between points of maximum moment on either side of the shear plane

\( (l_{b}/D)^{e} \)  
Normalised length for elastic equilibrium

\( (l_{b}/D)^{p} \)  
Normalised length for plastic equilibrium

\( L \)  
Minimum length of the reinforcement measured from the shear plane

\( M_{\text{max}} \)  
Maximum available bending moment in the reinforcement

\( M_{\text{max}}^{e} \)  
Elastic bending moment for a given axial force

\( M_{\text{lim}} \)  
Bending moment at the elastic limit of the reinforcement

\( (M_{\text{max}}^{e})_{\text{bearing}} \)  
Maximum elastic bending moment limited by soil bearing stress

\( M_{p} \)  
Fully plastic bending moment of the reinforcement

\( M_{\text{max}}^{p} \)  
Plastic bending moment for a given axial force

\( N_{t} \)  
Bearing capacity factor

\( P_{s} \)  
Normal force acting on a shear surface

\( S \)  
Gross shearing resistance of the unreinforced soil

\( \Delta S \)  
Additional shearing resistance due to the reinforcement

\( T \)  
Axial force in the reinforcement

\( T_{s} \)  
Maximum available shear force in the reinforcement

\( (T_{e}/T_{p})_{\text{lim}} \)  
Limiting normalised shear force from the elastic analysis

\( T_{p} \)  
Fully plastic axial force in the reinforcement

\( (\gamma)' \)  
A parameter corresponding to the elastic analysis

\( \gamma' \)  
A parameter corresponding to the plastic analysis
Appendix A

Derivation of the plastic analysis using the Hansen and Lundgren lateral loading

The limiting lateral loading for a pile across a rupture surface in soil suggested by Hansen and Lundgren (1960) was illustrated in Figure 4. The dimensions $l_1$, the distance between the points on the reinforcement experiencing maximum moment, and $l_2$, the distance from the point of maximum moment to the edge of the lateral loading on the reinforcement, are marked on the figure.

The moment at the intersection between the reinforcement and the shear plane is equal to zero because of symmetry, and the maximum moment occurs at the point of zero shear force, at a distance $l_2/2$ from the end of the reinforcement. The geometry for the loading may be found by taking moments about the centerline for one half of the problem:

$$\frac{\sigma_2 l_1}{2} \left( \frac{l_1}{4} \right) + \frac{\sigma_2 l_2}{2} \left( \frac{l_2}{2} + \frac{l_2}{4} \right) - \left( \frac{l_2}{2} + \frac{3l_2}{4} \right) = 0 \quad \text{...(A1)}$$

which gives:

$$\frac{l_2}{l_1} = \frac{1}{\sqrt{2}} \quad \text{...(A2)}$$

The maximum moment is then found for a nail of width $D$ by taking moments for one of the ends:

$$M_{\text{max}} = \frac{\sigma_2 l_2 D}{2} \left( \frac{3}{4} l_2 - \frac{1}{4} l_2 \right) = \frac{\sigma_2 l_2^2 D}{4} \quad \text{...(A3)}$$

and the maximum shear force by resolving forces:

$$T_c = \frac{\sigma_2 l_2 D}{2} \quad \text{...(A4)}$$

The relation between the maximum shear force and moment is then:

$$T_c = \frac{4M_{\text{max}}}{l_2} \quad \text{...(A5)}$$

The length $l_1$ required to cause plastic bending in the nail in the absence of axial force, $M_{\text{max}} = M_p$, can be determined from equations (A2), (A4) and (A5), and recalling that $M_p = \sigma_s D^3/6$ for circular bar:

$$T_c = \frac{\sigma_2 l_2 D}{2} = \frac{4M_p}{l_2} = \frac{4 \sigma_s D^3}{l_2} \quad \text{...(A6)}$$

giving:

$$\left( \frac{l_2}{D} \right)^2 = \frac{4\sigma_s}{3\sigma_s} \quad \text{...(A7)}$$

The above equation is for the simplified analysis of an ungrouted nail.
If the nail is grouted into a hole of diameter $D_*$, but, for the reasons discussed in Section 4, the limiting plastic moment is the same as for the ungrouted bar, then equation (A7) should be modified as follows:

$$\left( \frac{l}{D} \right)^2 = \frac{4\sigma_y D}{3\sigma_y D_*} \quad \cdots(A8)$$

Finally, in the rigorous plastic analysis it is necessary to take into account the effect of the axial force $T$ in the reinforcement on the maximum moment, $M_{\text{max}} \leq M_p$, equation (5) in the paper, to give:

$$\left( \frac{l}{D} \right)^2 = \frac{4\sigma_y D}{3\sigma_y D_*} \left( 1 - \frac{T}{T_p} \right) \quad \cdots(A9)$$
Figure 1. Limiting combinations of bending moment, M and axial force, T in reinforcement.
Figure 2. Elastic analysis of soil-nail interaction as envisaged by Schlosser (1983). Drawn for the case of $L = 3l_o$. 

\[ l_o = \sqrt[\pi]{\frac{4EI}{K_D}} \]

\[ M_{\max} \]

\[ T_c \]

Lateral stress

Shear force

Bending moment

\[ l_b \]

\[ l_s \]

\[ l_b \]
Figure 3. Plastic analysis of soil-nail interaction proposed by Jewell and Pedley (1990)
\[ \frac{l_b}{l_s} = \frac{1}{\sqrt{2}} \]

Figure 4. Plastic analysis for soil-pile interaction attributed to Hansen and Lundgren (1960)
Figure 5. Comparison of bounding curves for lateral bearing pressure $\sigma'_b/\sigma'_s$ on reinforcement proposed by Jewell et al. (1984) with lateral stresses on rigid piles suggested by Hansen (1961).
Figure 6. Limiting combinations of shear force and axial force of ungrouted reinforcement for the elastic analysis.
Figure 7. Limiting combinations of shear force and axial force of ungrouted reinforcement for the plastic analysis.
Figure 8. Misalignment of a reinforcing bar within the grout will result in different ultimate bending moments of resistance on opposite sides of the shear plane.
Figure 9. Limiting combinations of shear force and axial force of grouted reinforcement for the elastic analysis.
Figure 10. Limiting combinations of shear force and axial force of grouted reinforcement for the plastic analysis.
Figure 11. Force resultants acting on the soil with reinforcement inclined at an angle $\theta$ to a potential shear surface.

\[ S = P_s \tan \phi' \]

\[ \Delta S = T_s (\cos \theta \tan \phi' + \sin \theta) + T_c (\cos \theta - \sin \theta \tan \phi') \]
\[ S = \rho_s \tan \phi' \]

\[ \Delta S = T (\cos \theta \tan \phi' + \sin \theta) + T' (\cos \theta - \sin \theta \tan \phi') \]

Figure 12. Improvement in shearing resistance at a depth of 10m for soil reinforced with grouted nails \((D_g = 0.1m, D = 0.025m)\) inclined at an angle \(\theta\) with respect to the shear surface. Results for the elastic analysis.
Figure 13. Improvement in shearing resistance at a depth of 10m for soil reinforced with grouted nails ($D_g = 0.1m, D = 0.025m$) inclined at an angle $\theta$ with respect to the shear surface. Results for the plastic analysis.
Figure 14 Comparison of limiting plastic forces for a grouted bar for the simplified and rigorous plastic analyses.
Figure 15. Improvement in shearing resistance at a depth of 10m for a soil reinforced with grouted nails ($D_z = 0.1m, D = 0.025m$) inclined at an angle $\theta$ with respect to the shear surface. Results for the rigorous plastic analysis.
Discussion by

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on

Kinematical Limit Analysis for Design of Soil-Nailed Structures

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1 Introduction

In a recent paper, the Writers have drawn attention to what appears to be an error in the current analysis for soil nails under combined axial and lateral loading, Jewell and Pedley (1990a). The apparent consequence is that the magnitude of the maximum shear force calculated in the soil nails can be of the order 10 times greater than that which might actually occur.

This is important as this analysis is being incorporated into design manuals such as Mitchell and Villet (1987) and Elias and Juran (1988). To try and assist a clear and constructive discussion, the Writers have set out revised design equations for both an elastic and a plastic analysis of the forces in soil nails under combined loading. These are presented in a companion paper, Jewell and Pedley (1990b), using the Authors’ symbols and terminology for clarity.

Parametric studies with the revised design equations for typical soil nailing cases have led the Writers to the conclusion that the effects of reinforcement bending stiffness should be ignored for soil nailing design. This is because the improvement in soil stability stemming from bending stiffness appears at best to be very small, and rather variable, compared with the improvement which can be derived from the axial reinforcement force alone, Jewell and Pedley (1990b).

The main difficulties that the Writers have with the existing elastic analysis are highlighted below with reference to the most recent paper, Juran et al (1990). The Writers hope that the discussion of these points, and the Authors’ reply will help to clarify this complex aspect of soil mechanics analysis.
2 Soil nails under combined loading

2.1 Inter-relationship between forces

The Authors acknowledge that the maximum axial force, shear force and bending moment in a soil nail are related, but this appears not to be reflected in their design equations.

The maximum bending moment $M_{\text{max}}$ and maximum shear force $T_c$ in a soil nail are related by the distribution of the lateral loading from the soil. More specifically, they are related through the transfer length $l_o$,

$$l_o = \sqrt[4]{\frac{4EI}{K_sD}} \quad \ldots (1a)$$

or in non-dimensional form, for circular bar,

$$\frac{l_o}{D} = \frac{1}{2} \sqrt[4]{\frac{\pi E}{K_sD}} \quad \ldots (1b)$$

where $D$ is the diameter, $I$ is the second moment of area and $E$ is the Young's modulus of the reinforcement bar, and $K_s$ is the modulus of subgrade reaction for the soil. The transfer length is therefore a function of the relative elastic stiffness of the reinforcement $E$ and of the soil $K_sD$, equation (1b). The Authors' equation (2a) correctly states the relation for the elastic analysis,

$$T_c = \frac{M_{\text{max}}}{0.32l_o} \quad \ldots (2)$$

The capacity of a bar to support bending moment is reduced when the bar also has to carry axial force $T$. In the limit, if the axial force reaches the axial capacity of the bar, $T = A_y\sigma_y = T_y$, then the moment capacity of the bar is reduced to zero, $M_p = 0$. Three envelopes for limiting plastic combinations of axial force $T$ and moment $M$ are shown in Figure 1. It is apparently a relation of this form that has been omitted from the Authors’ analysis which only checks, their equation (7),

$$M_{\text{max}} \leq \frac{M_p}{F_M} \quad \ldots (3)$$

where $F_M$ is a factor of safety, set equal to unity in the Authors’ paper. This check is applied irrespective of the magnitude of the axial force $T$. 
To illustrate analytically the link between the maximum shear and axial forces in a soil nail, it is convenient to use the limiting envelope for a bar of rectangular cross section, because this can be defined as, Calladine (1985),

\[
\frac{M_{\text{max}}}{M_p} + \left(\frac{T}{T_p}\right)^2 = 1 \quad \ldots(4)
\]

which is only slightly conservative for circular bar for which no such simple relation can be derived, Figure 1.

Equation (4) can be combined with equation (2), and recalling that for circular bar \(T_p/M_p = 3\pi/2D\) gives,

\[
\frac{(0.32l_oT_c)(3\pi/2D)}{T_p} + \left(\frac{T}{T_p}\right)^2 = 1 \quad \ldots(5a)
\]

or

\[
\frac{T_c}{T_p} = \frac{0.66}{(l_o/D)} \left(1 - \left(\frac{T}{T_p}\right)^2\right) \quad \ldots(5b)
\]

Equation (5) applies when the properties of the reinforcement bar govern the loading capacity, rather than the lateral soil bearing pressure. The analysis for the latter case is straightforward, but as discussed by Jewell and Pedley (1990b) is found only to govern at very shallow depth.

The typical range for the ratio \(l_o/D\) for soil nails is of the order 5 to 20, (13.6 in the Authors’ worked example) and this allows the likely range for the limiting envelope for combined loading to be drawn, Figure 2. The envelope used by the Authors is shown for comparison, their equation (6), and would appear to be realistic only for ratios \(l_o/D < 1.00\) which are unlikely in soil nailing, equation (1b).

**2.2 Limiting envelope in the earlier work**

In the original work by Schlosser (1983) and Blondeau et al (1984) there was a limiting parabolic curve indicated within the ellipse discussed above, Figure 3. Unfortunately the equation for this curve was not given, although Blondeau et al (1984) gave the expression for the case when \(T = 0\), point X in Figure 3, as,

\[
(T_c)_{T=0} = \frac{1.62M_p}{l_o} + 0.24Dl_oP_l \quad \ldots(6)
\]

where \(P_l\) is the limiting bearing pressure between the soil and the nail. This may be compared with the value from equation (2),
\[(T_e)_{T=0} = \frac{3.12M_p}{I_o} \quad \ldots(7)\]

The Writers suggest that it is an expression such as equation (5b), as illustrated in Figure 2, which describes the limiting load combinations that the parabolic curve in the earlier work sought to describe.

Apparently the parabolic criterion governing combined loading was omitted from the publications in the USA, Mitchell and Villet (1987) and Juran et al (1990).

### 2.3 Benefit of the grout

Schlosser (1983) and Mitchell and Villet (1987) have argued that to allow for the potential increase in bending moment capacity of the soil nail due to the grout surrounding the bar would require construction control on the centralising of each bar, and an integrity for the grouting which would rarely be achieved in practice. They therefore disregard this factor for design purposes, and this approach is supported by Jewell and Pedley (1990b).\(^1\)

By contrast, the Authors have allowed benefit from the grout in calculating the maximum plastic moment \(M_p\) of the nails in their worked example, but have apparently ignored any benefit to the bending stiffness \(EI\) or the transfer length \(l_o\) of the nail. The latter choice is puzzling since for a grouted nail in which the grout is assumed *only to increase the area of contact* between the nail and the soil, Mitchell and Villet (1987) and Jewell and Pedley (1990b) give,

\[
\frac{l_o}{D} = \frac{1}{2} \sqrt{\frac{\pi E}{K_s D_g}} \quad \ldots(7)
\]

where \(D_g\) is the diameter of the grouted hole, from which \(l_o = 0.24m\) for the Authors’ worked example, which is more beneficial than the quoted value \(l_o = 0.34m\) calculated ignoring any effect of the grout.

The Writers have had difficulty matching the increased plastic moment capacity \(M_p = 2.97kNm\) for the grouted bar determined by the Authors, and this might be indicative of the complexity and potential for error introduced by such an allowance. Assuming \(\sigma_s = 168 \times 10^3\ kN/m^2\) for the steel and \(\sigma_g = 21 \times 10^3\ kN/m^2\) for the grout, and for a bar diameter \(D = 0.025m\) and a hole diameter \(D_g = 0.100m\), the Writers found the following maximum ultimate

---

\(^1\) An obvious exception would be for prefabricated nails, grouted into PVC tubing for double protection, in permanent soil nailing applications.
moment capacities (when $T = 0$); for the bar alone $M_c = 0.44 kNm$, allowing for
the grout to act in tension and compression $M_p = 3.88 kNm$, and allowing for
the grout acting only in compression $M_p = 1.85 kNm$.

Because of the above discrepancies in the Authors' worked example, the
points made earlier can be more clearly illustrated by the worked example from
the Appendix on soil nailing in Mitchell and Villet (1987), Section 11.2.3. The
maximum combined loading in the nail in this design example is $T = 20.4 kip$
and $T_c = 8.8 kip$, and the axial capacity of the bar is $T_p = 27 kip$. The transfer
length is $l_o = 7.92 in$ and the bar diameter is $D = 1.54 in$. The limiting envelope
defined by equation (5b) above is shown in Figure 4 where it is compared with
the limiting envelope given in Mitchell and Villet (1987), Figure C-60, and the
force combination allowed for in the design.

2.4 Mohr's circle of stress in the nail

In the Authors' analysis it is suggested that the combination of maximum axial
and shear force in the nail may be determined from a unique Mohr's circle of
stress, Figure 5. The Writers have difficulty with this concept since no such
unique state of stress can exist in the bar; rather the shear stress will vary from a
maximum at the centre of the bar to zero at the outer edge, to meet the
boundary condition of zero complementary shear stress on the free surface of
the bar.

The Authors further suggest that the maximum shear stress in the bar is
mobilised in the direction of the shear surface in the surrounding soil which
intersects the bar.

If this approach is to be recommended it requires a much clearer
justification. In view of the discussion above, however, the Writers cannot see
how this approach through Mohr's circle can be relevant to the determination of
the combined loading in a soil nail.

3 Conclusion

That there can be benefit from reinforcement bending stiffness is not denied by
the Writers, and indeed this is shown in Figures 13 and 15 of Jewell and Pedley
(1990b). However the influence of reinforcement bending stiffness on stability
for the typical applications of soil nailing in excavation and slope stabilisation
works appears to be so small in comparison with that due to the axial
reinforcement forces, and so small in comparison with uncertainties in the soil
and porewater pressure conditions, loadings and factors of safety, that for
design purposes the Writers recommend that the bending stiffness be ignored.
altogether. This brings the additional advantage of a simpler analysis, thereby one less prone to error, and one more amenable to use without recourse to complex computation, and therefore, one which can be easily reviewed and discussed by designers.
References
Figure 1. Limiting combinations of bending moment, $M$, and axial force, $T$, in the reinforcement.
Figure 2. Limiting combinations of shear and axial force. Comparison between Equation (5b) of the discussion and Equation (6) in Juran et al (1990).
Figure 3. Inner parabolic curve, Schlosser (1983), Figure 26, limits the combinations of shear and axial force defined by Equation (6), Jurin et al (1990).
Figure 4. Limiting envelope for shear and axial force combinations defined by Equation (5b), compared with those of design example given in Mitchell and Villet (1987), Figure C-60.
Figure 5. Mohr's circle of stress used to determine the maximum axial and shear forces in a nail, proposed by Schlosser (1983).