STUDIES OF THE CONE PRESSUREMETER TEST IN SAND

by

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In these two papers a summary of the current research work on the cone-pressuremeter at Oxford is presented. The first paper concentrates on the results of a study of the cone-pressuremeter in sand in a large calibration chamber, and includes the details of the tests carried out on a $10cm^2$ pressuremeter. Additional work has been carried out on a $15cm^2$ pressuremeter and also a study of length:diameter ratio effects on a $5cm^2$ pressuremeter.

The second paper concentrates on the results of a study using finite elements, particularly with application to the cone-pressuremeter in sand. Finite element studies are in progress analysing the test as both one-dimensional, (i.e. ignoring length:diameter ratio effects) and as two-dimensional. In the paper in this report only the one-dimensional analyses are presented.
CALIBRATION TESTS OF THE
CONE-PRESSUREMETER IN SAND

F. Schnaid and G.T. Houlsby

SYNOPSIS. A set of tests of the cone-pressuremeter in a large calibration chamber are described. The observed limit pressures, as well as the cone resistance values, are found to be related primarily to the in-situ horizontal stress and the density. The ratio of cone resistance to limit pressure is found to be primarily determined by the sand density. A procedure for deducing the in-situ horizontal stress and density from the results of the cone-pressuremeter tests is described, and is shown to give reasonable values when used to back-analyse the tests.

INTRODUCTION

1. The cone-pressuremeter (CPMT) is a new site investigation device which incorporates a pressuremeter behind a standard cone penetrometer tip. It is a more complex device to interpret than the self-boring pressuremeter because the pressuremeter test is carried out in soil which has been displaced by the penetration of the cone. Nevertheless, the new device has certain advantages, principally in the simplicity and economy of its installation. It is recognized that the cone penetration causes disturbance of the soil, but it is considered that this disturbance should be repeatable, and that the pressuremeter test should therefore be amenable to rational analysis.

2. An analysis of the cone-pressuremeter in clay was presented by Houlsby and Withers (1988), and applied successfully to the results of field tests. An equivalent analysis for the test in sands is not yet developed, although interpretation methods for the CPMT in sands based on large strain finite element analysis of cavity expansion are being developed at Oxford University (Houlsby and Yu, 1990). Since there is also no large body of field data available for cone-pressuremeter tests in sand, interpretation of the test in sand must be based on the results of calibration tests. The purpose of this paper is to examine the results of a set of tests in a large calibration chamber, which were designed to investigate possible methods of interpretation of the cone-pressuremeter. Although information can be obtained from the complete pressure-displacement curve from the pressuremeter test, including unload-reload loops for modulus measurement, this paper will concentrate on the values of the limit pressure obtained from the pressuremeter test and the cone resistance values from the cone test.
TEST PROCEDURE

3. A total of 20 cone-pressuremeter tests in the large calibration chamber have been carried out. In each test the sample was prepared at the required density by raining, the required vertical and horizontal stresses were then applied, and held constant throughout the test. The cone-pressuremeter was pushed into the chamber at a constant speed of 20 mm/s and the cone resistance $q_c$ was measured during penetration. Insertion was stopped when the centre of the pressuremeter was at the midheight of the chamber. A strain controlled pressuremeter test was then performed, in which inflation pressure $\psi$ and radial displacement of the membrane were recorded. The tests were usually taken to 30% hoop strain $\varepsilon = (\Delta R/R_o)$ at which stage a well-defined plateau in the pressure expansion curve had always been reached, corresponding to the limit pressure $\psi_l$.

4. The main results of the tests are given in Table 1. The tests were carried out on Leighton Buzzard 14/25 sand at three relative densities: loose (16% to 27%), medium (60% to 68%) and dense (83% to 89%). The stress states for the tests were selected to give $K = \sigma'_v/\sigma'_c$ values of 0.5, 1.0 and 2.0 at mean effective stresses $p'$ of approximately 50kPa, 100kPa, and 200kPa. There was no attempt to simulate mechanical overconsolidation of the sample in the calibration chamber (c.f. Bellotti et al., 1982). Instead, a range of $K$ values was chosen, enabling the influence of horizontal and vertical stresses on cone resistance and limit pressure to be assessed independently. A similar approach had been previously used at Oxford University to study the cone penetrometer in sands (Houlsby and Hitchman, 1988).

5. All tests were performed on dry sand. Total and effective stresses were therefore identical in the calibration chamber and no distinction is necessary between the measured cone resistance $q_c$ and the total cone resistance $q_t$.

ANALYSIS OF RESULTS

6. The calibration chamber tests on the cone-pressuremeter give measurements of limit pressure $\psi_l$ and cone resistance $q_c$ under conditions of controlled density, vertical stress and horizontal stress. Interpretation of the test depends on understanding of the influence of each of these variables on $q_c$ and $\psi_l$.

7. Houlsby and Hitchman (1988) showed from calibration chamber tests of the cone penetrometer in sand that, for a given density of sand, the tip cone resistance depends on the in-situ horizontal stress, and only to a small extent on the vertical stress. The present work on the CPMT confirms this finding, and a clear relationship between cone resistance and horizontal stress was obtained. The same approach has been applied to the pressuremeter test data. Figure 1 shows that there is an unique relationship between limit pressure and horizontal stress for each of the three densities tested. By comparison, Figure 2 shows the
limit pressure against vertical stress, showing no connection between these quantities. These observations suggest that both the cone resistance and the limit pressure depend primarily on the same variables, i.e. density and horizontal stress.

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<th>$\sigma'_v$</th>
<th>$\sigma'_h$</th>
<th>$\psi_l$</th>
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Table 1, Results of calibration tests of cone pressuremeter

8. It is therefore useful to explore the relationship between $q_e$ and $\psi_l$, which is shown in Figure 3. The horizontal stress has been subtracted from both quantities for consistency with the analysis of tests in clay (Wroth (1988), Houlsby (1988)); this subtraction has only a small effect on the plot. Figure 3 shows that, for a given density, there is a constant ratio between limit pressure and cone resistance and this ratio is independent of stress level and stress ratio. The direct correlation between $q_e$ and $\psi_l$ suggests that the resistance to cone penetration, as well as the limit pressure in the pressuremeter test, is closely controlled by a cavity expansion process. The ratio $(q_e - \sigma'_h)/(\psi_l - \sigma'_h)$ may be a useful variable for deriving soil parameters, in particular the density and the peak friction angle. If a cavity expansion controls both $q_e$ and $\psi_l$, the ratio may not be as sensitive to the effects boundary conditions in the calibration chamber as the independent measurements of cone resistance and limit pressure.
9. The first important application of the CPMT for measuring soil parameters is the estimation of density as a function of $q_c$ and $\psi_i$. The density was obtained by weighing the sand in the chamber at the end of each test and dividing by the volume of the sample. Figure 4 shows the ratio of $(q_c - \sigma_h)/(\psi_i - \sigma_h)$ against relative density, $R_d$. The ratio is sensitive to the relative density and the results can be approximated by a linear equation:
\[ R_d = 9.0 \frac{(q_c - \sigma_h)}{\psi_l - \sigma_h} - 30 \]  

which is applicable for Leighton Buzzard sand with values of \((q_c - \sigma_h)/(\psi_l - \sigma_h)\) in the range of 5 to 14. Calibration chamber data have in the past been extensively used to correlate cone resistance and relative density (Schmertman, 1976 and Lancellotta, 1983). For normally consolidated deposits, \(R_d\) has been expressed as a function of \(q_c\) and \(\sigma'\). As explained above, \(q_c\) is almost completely controlled by the horizontal stress and therefore, for overconsolidated sands, any relationship must account for the value of \(\sigma'\) or \(p'\) since \(K_o\) varies with the overconsolidation ratio. The relationship based on the CPMT results can be considered as a step forward in the prediction of density, as it depends only on the measured values of \(q_c\) and \(\psi\) (the subtraction of \(\sigma_h\) from both quantities represents a small correction for sands, and derived \(R_d\) values are insensitive to a poor estimate of \(\sigma_h\)). Note that the correlation of \((q_c - \sigma_h)/(\psi_l - \sigma_h)\) against \(R_d\) has been tested for a wide range of \(K\) values, but further research should be carried out to validate the correlation in different sands, in order to take into account the influence of other variables such as stiffness.

![Graph](image)

**Figure 3**, Cone resistance against limit pressure

10. By making further use of the results of the cone test, however, it is possible to obtain information about both the in-situ horizontal stress and the density. Recalling that the cone resistance is primarily controlled by the horizontal stress and density, it is found that the results of the calibration chamber tests can be reasonably approximated by the expression:
\( R_d = \frac{1}{3} \left( \frac{q_c - \sigma_h}{\psi_i - \sigma_h} \right) + 10 \)  \[ (2) \]

11. It is important to observe that equation (2), in contrast to equation (1), is expected to be influenced by chamber size effects (Lunne and Parkin, 1982 and Parkin, 1988). Chamber tests in loose and medium sands represent field conditions reasonably well, but tests in dense sand are affected by the proximity of the chamber boundary. It is likely that the cone resistance for the dense tests is underestimated by a factor of approximately two by comparison with field tests. Neglecting the effects of chamber size leads to an overestimation of the density values in the field. An extensive database of approximately 400 tests is currently being analysed, in the context of cylindrical cavity expansion theory, to quantify these effects more precisely. It is emphasised that equation (2) therefore applies only to calibration chamber results, and that the coefficients in the equation would change for field tests.

12. The relationships presented so far can be used to interpret cone-pressuremeter tests. It has been shown that density can be estimated from \((q_c - \sigma_h)/(\psi_i - \sigma_h)\) or directly from \(q_c\), if a reasonable estimate of the horizontal stress can be made. As both quantities \(q_c\) and \(\psi_i\) are controlled by the horizontal stress an alternative approach is to reverse the process to estimate \(\sigma_h'\). The combination of equations (1) and (2) gives \(\sigma_h' = f(q_c, \psi_i)\), expressed as the root of a quadratic equation. Figure 5 shows the comparison between \(\sigma_h'\) estimated from the measured values of \(q_c\) and \(\psi_i\) and \(\sigma_h'\) applied in the calibration chamber. The same data are replotted in Figure 6 in terms of the earth pressure coefficient.
\( K \), (defined as the ratio \( \sigma'_h/\sigma'_v \)). The vertical stress applied to the chamber was used, together with the estimated and applied horizontal stress to compute the \( K \) values. A generally good agreement is observed in the predictions for the range of conditions tested. The horizontal stress was estimated within a factor of 1.3 for every case but one, but with a tendency to underestimate the horizontal stress at high horizontal stress values.

![Graph showing estimated and applied horizontal stress values](image)

**Figure 5, Estimated and applied horizontal stress values**

13. Finally the estimated horizontal stress was used together with equation (1) to calculate the relative density. Figure 7 shows the comparison between \( R_d \) calculated using this method and the values measured for the chamber tests. A generally good agreement between measured and estimated values is observed and relative density can be estimated within an accuracy of about ±4%.

14. Once the relative density and the horizontal stress are known, it is then possible to make an estimate of the peak angle of friction, using for instance the correlations due to Bolton (1986). As an alternative to the above procedure, it is also possible to establish relationships in the form of equations (1) and (2) which give a direct correlation between the results of the cone-pressuremeter tests and the angle of friction. Although the details are not given here, it can be reported that back-analysis of the chamber tests leads to estimates of friction angle to within approximately ±1° when this procedure is used. The discussion in this paper has been made in terms of density rather than friction angle simply because more certainty can be attached to the former quantity in the tests. The two parameters are of course closely related.
15. A comment is appropriate about the possibility of obtaining stiffness measurements representative of in situ conditions from the cone-pressuremeter test. The elastic shear modulus, $G$, is measured by performing unload-reload loops during the pressuremeter test. Preliminary results suggest that the shear modulus obtained from the loops is insensitive to the method of installation and the CPMT provides data which are comparable to those obtained by other pressuremeters (Hughes and Robertson, 1985).
16. The tests described in this paper have all been carried out on a single soil. The more general validity of the correlations described can only be established by further testing on other soils.

CONCLUSIONS

17. Calibration chamber tests have been carried out to establish a basic procedure for the interpretation of the cone-pressuremeter test in cohesionless soils. Assessment of soil properties depends on the relationship between cone resistance $q_c$ and limit pressure $\psi_l$. The ratio $(q_c - \sigma_h)/(\psi_l - \sigma_h)$ is essentially dependent on relative density, $R_d$, and not on stress level or stress ratio. An empirical relationship has been proposed for deriving density and as a function of $q_c$ and $\psi_l$.

18. Test results also show that, for a given density, both the cone resistance and the limit pressure are controlled by the horizontal effective stress. Based on this evidence, a method to estimate $\sigma'_h$ as a function of the measured values of $q_c$ and $\psi_l$ has been proposed as an alternative to the conventional practice of accessing $\sigma'_h$ from the lift-off pressure.

19. CPMT results have been used to measure the elastic shear modulus and to estimate density and horizontal stress. The application of the proposed empirical relationships to field problems must be considered with caution. The values of $q_c$ and $\psi_l$ measured in the calibration chamber are expected to be influenced by the size of the chamber. The influence of sand type and stress history on the test results is also still to be assessed.

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REFERENCES


NOTATION

$e$ Voids ratio
$G$ Shear modulus
$K$ Earth pressure coefficient $\sigma'_v/\sigma'_v$
$p'$ Mean normal effective stress $\frac{1}{3}(\sigma'_v + 2\sigma'_h)$
$q_e$ Cone tip resistance
$R$ Pressuremeter radius
$R_d$ Relative density $(e - e_{\text{min}})/(e_{\text{max}} - e_{\text{min}})$
$\sigma_h$ Horizontal total stress
$\sigma'_h$ Horizontal effective stress
$\sigma'_v$ Vertical effective stress
$\psi$ Pressuremeter inflation pressure
$\psi_l$ Limit pressure
FINITE ELEMENT ANALYSIS
OF THE CONE-PRESSUREMETER TEST

G.T. Houlsby and H.S. Yu

SYNOPSIS  A finite element analysis of the cone pressuremeter test in sand is presented in which the test is idealised as a cylindrical cavity expansion in a Mohr-Coulomb material. A method of estimating strength parameters from the cone pressuremeter test is suggested. Finally, cavity expansion theory is used to explain the effects of chamber diameter on tests of cones and pressuremeters in large calibration chambers.

INTRODUCTION

1. The cone-pressuremeter test is a relatively new in-situ test in which a pressuremeter is combined with a standard cone. The interpretation of the test is more complex than that of the self-boring pressuremeter, because of the influence of the prior cone penetration. An interpretation method for the test in clay has been proposed (Houlsby and Withers, 1988), but interpretation methods for tests in sand have not yet been developed.

2. A research programme aimed at understanding the cone pressuremeter test in sand is currently in progress at Oxford University. The programme includes large scale calibration chamber tests, and also finite element analysis of the test. The latter part of this programme is the subject of this paper. Whilst a comprehensive analysis of the cone pressuremeter test requires a two-dimensional axisymmetric analysis, significant progress can be made by analysing the test as a one-dimensional cavity expansion, and this paper concentrates on this approach.

3. As well as developing methods of interpretation of the test in the field, cavity expansion analysis can be used to aid interpretation of the chamber tests. In particular it is found that the effects of chamber size on cone and pressuremeter tests in chambers can largely be explained by a cavity expansion process.
ANALYSIS OF CAVITY EXPANSION

Finite Element Formulation

4. In the analysis of cylindrical cavity expansion, the expansion is assumed to occur under conditions of plane strain and axial symmetry in a medium of either infinite or finite extent. Because of numerical difficulties in the numerical analysis, the cavity expansion is assumed to start from a small radius instead of zero radius. In order to model this problem of infinite extent using a mesh of finite dimensions, an outer correcting layer is introduced (Burd, 1986).

5. It is well accepted that severe numerical difficulties arise when using the conventional displacement finite element method to analyse incompressible or dilatant solids (Sloan, 1981). These effects are caused by the kinematic constraints imposed on the nodal velocities by the plasticity law, and the consequences of these effects may be divided into two categories. Firstly, in a calculation in which the material stresses are of primary importance, the imposed kinematic constraints produce spurious oscillations in the stresses across an element. Secondly, in a collapse load calculation where the limit load is sought, the excessive kinematic constraints often result in an over-stiff response. Although several approaches have been proposed to deal with this difficulty, none of them has been found to be wholly satisfactory.

6. A detailed theoretical study of the influence of the displacement interpolation function on the excessive kinematic constraints imposed by the incompressibility condition or the constant dilation rate condition has been carried out. A novel displacement interpolation function which is well suited for both one-dimensional and two-dimensional axisymmetric constrained problems has then been used to develop a new finite element method (Yu and Houlsby, 1989; Yu, Houlsby and Burd, 1989).

7. A two-noded element has been used for the case of one-dimensional axisymmetry. In order to evaluate the shape function which links the displacements in the element and those of the nodes, some assumptions about the displacement interpolation are needed. Compared to the conventional approach in which a linear displacement interpolation along an element is assumed, in this study a novel non-linear displacement interpolation is used and is defined by:

\[ u = \frac{C_0}{r} + C_1 r \]  

(1)

where \( C_0 \) and \( C_1 \) are constants for a given element and \( r \) denotes the radius.
8. The rationale behind this approach is that this expansion, unlike the conventional one, is able to capture exactly the displacements for two important cases, i.e. (a) elasticity and (b) incompressibility. The shape function based on the proposed displacement interpolation function, and the details of the associated large strain finite element formulation based on the virtual work principle are not presented here.

**Constitutive Model**

9. A theoretical analysis of the cone pressuremeter test in undrained clay has been proposed by Houlsby and Withers (1988) using the Tresca plasticity model. By contrast, such a simple approach is not yet available for large strain cavity expansion analysis in frictional materials with dilation. Hence, a numerical analysis (e.g. finite element analysis) has to be used in conjunction with any realistic material model used to model sand behaviour. The analyses presented in this paper are for the cone pressuremeter test in sand, because of lack of analytical solutions in this case.

10. Although more sophisticated criteria exist, the Mohr-Coulomb yield criterion is used to model the behaviour of granular materials in this paper. This is mainly because the non-associated Mohr-Coulomb criterion is simple and well established for modelling the dilatancy of sands. Another reason is that some small strain analytical solutions for cavity expansion in a Mohr-Coulomb material exist.

11. The Mohr-Coulomb surface, however, has vertices which give rise to computational difficulties. In finite element computations, a satisfactory method for dealing with these singularities is needed as they are often encountered during the course of an analysis, particularly under conditions of axisymmetry. The approach described by Sloan and Booker (1986) is used to round off corners of the Mohr-Coulomb surface.

12. A backward Euler stress-strain integration rule (de Borst and Vermeer, 1984) is adopted for the stress update calculations at each Gauss point, in order to take advantage of its simplicity and high accuracy for analysis of Mohr-Coulomb plasticity.

**ANALYSIS OF THE CONE PRESSUREMETER TEST**

13. Expansion of a cylindrical cavity in an infinite soil and in a finite soil cylinder has been used to simulate the cone pressuremeter test in the field and in a laboratory calibration chamber respectively. In this study, an expansion ratio of four is applied to the radius of the cavity, which may be assumed to model numerically the expansion of the cone pressuremeter membrane following installation of the instrument. The calculation is typically performed in 400 displacement increments.
Figure 1: Comparison between measured and calculated limit pressures

14. Experimental results for a cone-pressuremeter in a large calibration chamber show a well defined limit pressure after a cavity strain of approximately 20% (Schnaid and Houlsby, 1990). Calculations have been carried out to model as closely as possible the conditions for each test, using measured values of soil shear modulus, and friction and dilation angles estimated using the procedure described by Bolton (1987). The calculated limit pressures are compared with the measured ones in Figure 1. The slight underprediction of limit pressures for dense and medium sands is almost certainly because the end effects on the actual pressuremeter of finite length result in a higher limit pressure than for an ideal, infinitely long, cavity. Investigations of the effects of the finite length of the pressuremeter are in progress at present. The overprediction of limit pressure for loose sands is probably due to use of slightly too high a friction angle in the calculations. Overall the agreement is remarkably good, considering the simplicity of the theoretical model.

15. The recent analysis of the cone pressuremeter test in clay suggests that the difficulties in modelling the disturbance caused by installation of the cone pressuremeter may be overcome by using the plastic unloading curve from the pressuremeter test to derive the soil parameters. The small strain unloading analysis (Houlsby, Clarke and Wroth, 1986) for the Mohr-Coulomb material shows that the plastic unloading slope in a plot of \( \ln \psi \) against \( -\ln(\epsilon_{\text{max}} - \epsilon) \) is primarily controlled by the soil strength parameters and to a small extent by soil stiffness where \( \psi \) is the cavity pressure and \( \epsilon_{\text{max}} \) and \( \epsilon \) denote the maximum and cur-
Figure 2: Example of a numerical cavity expansion-contraction curve

rent cavity strain. The slope $S_d$ of the unloading curve in this plot may be approximately expressed as a function of the angles of friction and dilation as follows:

$$S_d = - \frac{N - \frac{1}{N}}{N + \frac{1}{n}}$$ (2)

where $N = (1 - \sin \phi)/(1 + \sin \phi)$ and $n = (1 - \sin \nu)/(1 + \sin \nu)$, in which $\phi$ and $\nu$ denote the plane strain friction angle and dilation angle respectively. The present large strain finite element analysis using the Mohr-Coulomb yield criterion suggests that the slope of large strain unloading is also controlled by these soil parameters. This dependence may be quantitatively expressed in the same way as the small strain solution. An example of the results of a numerical analysis is plotted as $\ln \psi$ against $-\ln(\epsilon_{\text{max}} - \epsilon)$ in Figure 2, showing that a substantial section of the curve is straight in this plot.

16. Equation (2) is inconvenient in that it involves both the friction and dilation angles. It is convenient to introduce a relationship between these angles, and Rowe's stress dilatancy relationship (Rowe, 1962), which can be written:

$$\sin \nu = \frac{\sin \phi - \sin \phi_{cv}}{1 - \sin \phi sin \phi_{cv}}$$ (3)

is well established empirically, and allows the dilation angle to be eliminated from equation (2), introducing instead the $\phi_{cv}$ value which can be easily determined by independent tests. The result is:
$$\sin \phi = \left( \sin \phi_{cv} + \frac{1 + \sin \phi_{cv}}{S_d} \right) - \sqrt{\left( \sin \phi_{cv} + \frac{1 + \sin \phi_{cv}}{S_d} \right)^2 - 1} \quad (4)$$

17. The above equation may be used to obtain the soil strength parameters from the in-situ cone pressuremeter tests in sand. This relationship for different values of critical state friction angle is plotted in Figure 3.

18. The above relationship has been developed based on the assumption that the cone pressuremeter test can be explained as a cylindrical cavity expansion process. This assumption may be justified either by testing the above proposed correlation using field test data or by comparing the cavity expansion analysis in a finite cylinder with the large calibration chamber test data. The latter approach has been used because of the lack of field data for a cone pressuremeter test. The numerical analysis has been carried out by using some best estimates of the soil parameters for the chamber tests.

19. A typical experimental cone pressuremeter expansion and contraction curve carried out at Oxford by Schnaid is shown in Figure 4 (Schnaid and Houlshby, 1990). A clearly linear unloading curve is defined, which was found generally to be the case for tests on loose sand. For medium and dense sands the slope of the unloading curve on this plot is much less well defined. Although the shape of the theoretical and experimental unloading curves are similar, it is found unfortunately that
the $S_d$ values do not compare well. In general the calculated $S_d$ values are significantly higher than those measured, and in any case (apart from for loose sand) there is a significant scatter in the observed $S_d$ values. The explanation of these topics is at present a subject for further investigation.

**ESTIMATE OF CHAMBER SIZE EFFECTS**

20. One of the aspects of the calibration chamber tests which needs further research is the quantitative estimate of the effects of chamber size on the calibration results. It is well accepted that calibration chamber tests in loose and medium sands represent field conditions reasonably well, but dense tests are significantly effected by the proximity of the chamber boundary (Lunne and Parkin, 1982). Hence, corrections have to be made before applying chamber test results to field conditions.

21. Based on the assumption that the cone pressuremeter test can be explained as a cylindrical cavity expansion process, it may be reasonable to suggest that the chamber size effects can be accounted for by considering the difference between the analysis of cavity expansion in an infinite medium and in a finite cylinder. Whilst extensive cone pressuremeter data in chambers is not available, some insight into this problem can be gained from cone penetrometer tests. By analysing an extensive database of approximately 400 chamber tests on cone penetrometers carried out in Norway, Italy and the U.K., approximate relationships
between the ratio of tip resistance $q_{80}$ measured in a calibration tank of 50 cone diameters to the $q_{60}$ value in smaller tanks of different diameter ratios $C_d$ have been obtained for different ranges of sand density.

22. A comparison is made between the magnitude of the effect predicted by cavity expansion analysis, and the observed chamber diameter effect on the cone test. This comparison is based on the hypothesis that cone penetration resistance is controlled, at least in part, by a cavity expansion process.

23. Figure 5 shows the comparison of these ratios with the ratios calculated for the cylindrical cavity expansion limit pressure, using large strain analysis. The calculations show that a chamber diameter effect would be expected even for loose sands, with an increase in the magnitude of the effect for medium and dense sands. Because of the many different factors which can affect chamber tests (sand density, stiffness, and particle size as well as cone and chamber size), it is difficult to separate out the effect solely of the chamber to cone size. However, the experimental results seem to indicate only a very small effect for chamber to cone diameter ratios greater than 20 for loose and medium sands, but a more significant effect for dense sands. The experimental points for dense sands shown in Figure 5 are approximate only and deduced from a large number of rather scattered test data.

24. The calculations indicate a larger effect than is observed for loose and medium sands. For dense sands the boundary effects of small chambers seems to be even greater than indicated by the calculations. These comparisons do, however, give some confidence that cavity expansion analysis could be used to improve the interpretation of cone and pressuremeter tests in calibration chambers. A similar approach was successfully used by Jewell, Fahey and Wroth (1980).

CONCLUSIONS

25. A relatively simple approach based on cavity expansion theory has been developed to perform an analysis of the cone pressuremeter test both in the field and in a calibration chamber. The proposed method uses large strain theory with a novel finite element formulation which is able to minimize the excessive constraints caused by the incompressibility and constant dilatation rate conditions.

26. Calculations of limit pressures agree well with observed values in a large calibration chamber, except that for dense sands the limit pressure is underestimated, probably due to end effects.

27. In field conditions, a semi-analytical correlation between the soil strength parameters ($\phi, \nu$) and the cone pressuremeter plastic unloading slope $S_d$ has been proposed based on analyses using large strain cavity
expansion model. The application of this correlation to field problems must be considered with caution because verification of the proposed correlation using field test data is still required. Quantitative agreement between analyses and laboratory test results is not good.

28. Together with an extensive database of 400 tests, the proposed numerical method has been used to quantify the chamber size effects. The comparison between the analysis and test data is encouraging, indicating that the assumption that penetration pressures are controlled by a cavity expansion process can at least in part explain observed chamber size effects.

REFERENCES


**NOTATION**

\[ C_0, C_1 \] Constants in displacement interpolation function

\[ C_d \] Chamber to testing device diameter ratio

\[ n, N \] Functions of \( \nu, \phi \)

\[ q_t \] Cone resistance

\[ r \] Radial coordinate

\[ S_d \] Slope of unloading curve in logarithmic plot

\[ u \] Radial displacement

\[ \epsilon \] Tensile hoop strain at pressuremeter surface

\[ \nu \] Angle of dilation

\[ \phi \] Peak angle of friction in plane strain

\[ \phi_{cv} \] Angle of friction at constant volume

\[ \psi \] Pressuremeter pressure

\[ \psi_l \] Pressuremeter limit pressure